Asset Pricing Results in Options Markets: True, Spurious, or Overlooked?*

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Abstract

We show that the rebalancing frequency of delta-hedged option portfolios materially influences biases in measured average returns when underlying prices are subject to microstructure noise. In a controlled simulation framework, we demonstrate that the mean bias of delta-hedged returns increases with hedge frequency, leading standard asset pricing tests to spuriously detect option-return premiums associated with underlying illiquidity. Genuine return premiums related to underlying liquidity and volatility are obscured. Existing bias adjustments fail to correct this distortion. We therefore propose to rebalance the underlying position using lagged deltas, breaking down the mechanical link between hedge ratios and subsequent stock returns.

Keywords: Delta-Hedged Option Returns, Asset Pricing, Microstructure Biases, Volatility Risk Premiums
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1 Introduction

Option returns are widely used in empirical research to analyze option-market behavior and its interdependencies with equity markets.[1] Yet measuring these returns poses significant empirical challenges. Options have highly nonlinear payoffs, exhibit differing return patterns between actively traded and thinly traded contracts, and feature significantly wider bid-ask spreads than stocks. In this context, the importance of a precise and reliable measurement of option returns becomes particularly evident. Wide bid-ask spreads raise concerns that the estimation of risk premiums, as part of expected option returns, may be even more prone to biases resulting from measurement errors in prices than has been previously documented in the literature on stock market asset pricing (Blume and Stambaugh, 1983; Asparouhova et al., 2010).

Adding to the complexity is the fact that the option literature commonly relies on delta-hedged option returns to adjust for the influence of the underlying asset on option price movements. When using these delta-hedged option returns, we not only encounter the well-established *direct mean return (DMR) bias* (originally identified by Blume and Stambaugh (1983) for simple average stock returns) but also a second source of bias, known as the *indirect mean return (IMR) bias*. The IMR bias occurs because hedge ratios are computed from distorted prices (Duarte et al., 2024).[2] While Duarte et al. (2024) find this indirect bias to be relatively small, offering some reassurance, their analysis assumes that no rebalancing of the underlying position occurs between the initiation and closing of the delta-hedged trade. In contrast, many empirical studies employ dynamically hedged option return measures to reduce hedge errors, e.g., Bakshi and Kapadia (2003), Cao and Han (2013), Frazzini and Pedersen (2022), Bali et al. (2023), or Vasquez and Xiao (2024). It remains unclear to what

[1] See, e.g., Bakshi and Kapadia (2003), Goyal and Saretto (2009), Cao and Han (2013), Karakaya (2013), Jones and Shemesh (2018), Choy and Wei (2020), Christoffersen et al. (2018), Frazzini and Pedersen (2022), and Zhan et al. (2022).

[2] Duarte et al. (2024) show that the measurement errors in stock prices, such as those arising from bid-ask spreads transfer to the corresponding option deltas and hedge ratios (defined as the option's delta times the ratio of the underlying price to the option price). These errors induce a spurious covariance between the return of the hedge position and the hedge ratio, which they call *indirect mean return (IMR) bias*.

extent the indirect effect has greater significance in these cases.

To enhance the credibility and robustness of asset pricing results and identified risk premiums in the options market, it is crucial to transparently disclose potential distortions. Are microstructural distortions in options and equity prices under certain circumstances responsible for the perception of risk premiums, even when genuine risk premiums are absent? Do these microstructural distortions impede the accurate identification of extant risk premiums? These fundamental questions have received little attention in empirical research. This gap arises from the inherent opacity of such distortions within empirical option return datasets, making them neither directly observable nor easily separable. Furthermore, we know little about their magnitude and relevance, especially when rebalancing is involved.[3] Using a simulation framework that allows us to disentangle genuine premiums from measurement biases, we contribute to a more accurate assessment of option return determinants.

This paper quantifies the impact of microstructure distortions on asset pricing in options markets. We analyze delta-hedged option returns at varying hedge frequencies to assess the suitability of commonly used return metrics. We formally derive expressions for the direct and indirect mean return biases in delta-hedged option returns, extending Duarte et al. (2024) to general hedge frequencies, and study these biases within a simulation framework. The simulation is designed to replicate the characteristics of widely used empirical option datasets, including OptionMetrics IvyDB, LiveVol Option Trades, LiveVol Option Quotes.[4] To maintain comparability with existing studies, we focus on monthly returns and simulate a liquid options market by examining short-term options.

We begin with a setting where the true option prices originate from Black and Scholes (1973)[5] and decompose delta-hedged option returns into the hedge error (HE) arising from

[3]Signs of distortions are widespread, though. The evidence on volatility risk premiums in stock options is mixed in the empirical literature (Coval and Shumway, 2001; Bakshi and Kapadia, 2003; Carr and Wu, 2009; Driessen et al., 2009). Duarte et al. (2024) highlight that distorted option returns play a significant role in this mixed evidence.

[4]For details on these data sets, we refer to `https://optionmetrics.com/data-products/`, `https://datashop.cboe.com/option-trades/`, and `https://datashop.cboe.com/option-quote-intervals/`.

[5]In this setting, options can be perfectly replicated with an investment in the stock and the money-market account. Hence, the *true* delta-hedged return in continuous time is always zero, making it straightforward

discretization and the components of the mean deviation caused by measurement errors relative to their otherwise identical error-free benchmarks (MR bias). We further divide the total MR bias into three parts: the direct part caused by the raw option return (O-DMR bias), the direct part caused by the underlying return (S-DMR bias), and the indirect part caused by correlations between the underlying return and the hedge ratio (IMR bias). We then examine how portfolio sorting and Fama and MacBeth (1973) regressions with respect to the illiquidity of the underlying stocks may indicate apparent risk premiums, even though the true option prices are unaffected by underlying illiquidity.[6] Finally, we evaluate the effectiveness of Duarte et al.'s 2024 proposed bias correction, developed for their static setting and addressing only the O-DMR bias, and suggest a straightforward correction for the remaining part of the mean return bias.

Next, we shift our focus to settings where genuine risk premiums in option returns are present, aiming to determine whether our methods can accurately detect them. We incorporate hedging costs following Leland (1985) to account for illiquidity premiums, and we consider volatility risk premiums using the stochastic volatility model of Heston (1993). These setups enable us to examine which option return measures are capable of identifying such premiums and whether the proposed adjustments for microstructure biases actually work.

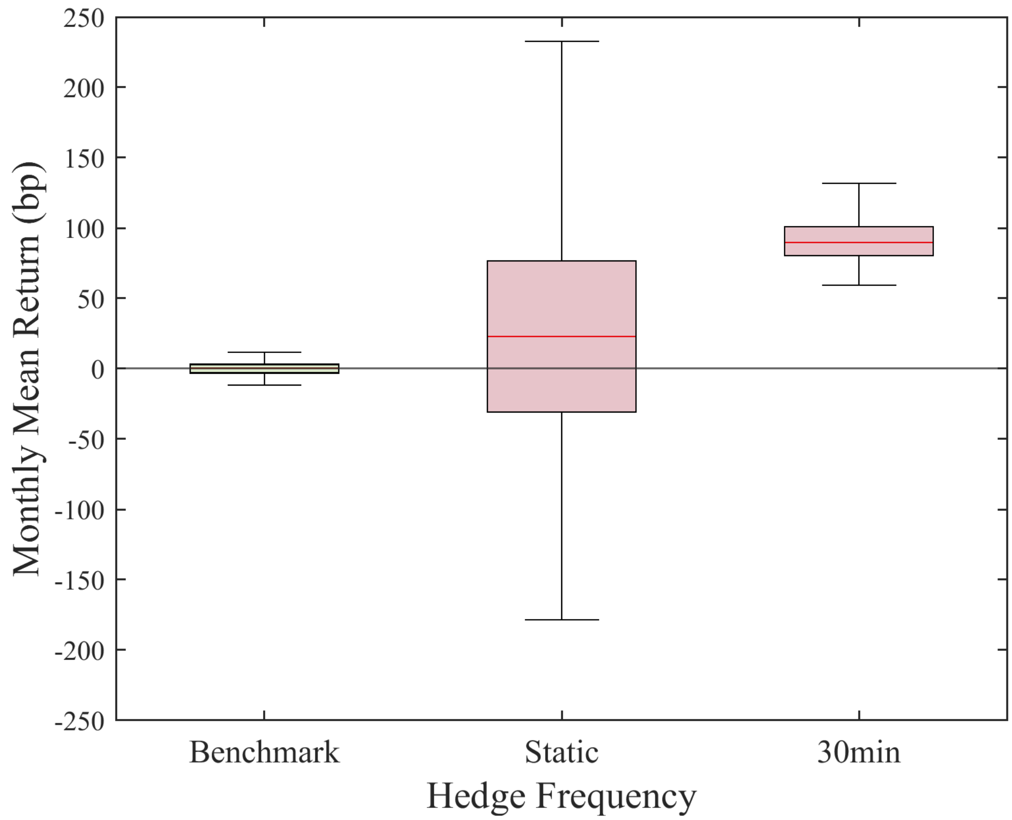
Consistent with Duarte et al. (2024), we observe that measurement errors in option prices result in overall inflated option returns. For low hedge frequencies, which exhibit a much larger standard deviation in monthly returns, the direct part of the bias, especially the one induced by the option (O-DMR bias), dominates. However, as the hedge frequency increases, the standard deviation reduces, but also the size of the S-DMR bias and, more strikingly, the IMR bias, which arises from the error-induced correlation between stock returns and hedge ratios, becomes more pronounced. Figure 1 illustrates this bias-variance trade-off.

to attribute deviations from this benchmark to specific channels. The formal derivation of the mean return bias components, however, is model-free.

[6]In the context of regression analyses, we speak of *regression coefficient (RC)* biases instead of mean return (MR) biases (following Duarte et al., 2024).

Figure 1: **Bias-variance trade-off for different hedge frequencies**

This figure shows the distributions of the monthly mean return for an error-unaffected measure hedged every 30 minutes (*Benchmark*) as well as for two error-affected return measures hedged only at inception (*Static*) or every 30 minutes (*30min*), respectively.



Static delta-hedging, i.e., not rebalancing the position over time, leads to a high standard error in the estimation of the mean, but is roughly centered around the benchmark of 0. In contrast, rebalancing the hedge portfolio every 30 minutes drastically reduces the variance of the estimation but leads to a high upward bias. We find that high-frequency intraday rebalancing can inflate delta-hedged option returns by nearly 100 bp per month, driven primarily by the IMR bias.

We further demonstrate that the impact of the IMR bias on estimating premiums in option returns is substantial and warrants serious attention. It generates a spurious premium of 3% per month for delta-hedged calls on stocks with low liquidity compared to those on stocks with high liquidity. When simulating a sample that incorporates true option return premiums for the liquidity of the underlying, we find that the IMR bias can severely distort regression coefficients – either reversing their sign or dramatically overstating their magnitude – depending on market conditions. These distortions are particularly pronounced for returns calculated with high-frequency rebalancing, where the bias can overshadow any true premium signals. Even in low-frequency measures, a significant portion of the simulated samples fails to detect meaningful return premiums. This pattern holds not only for illiquidity premiums but also for the estimation of volatility risk premiums in the Heston (1993) model. In the case of volatility risk, the IMR bias is most severe for in-the-money options. These findings underscore the need for caution when interpreting empirical evidence, as measurement biases can mask or exaggerate key insights about risk premiums.

To break the mechanical link between microstructure noise in hedge ratios and subsequent stock returns, we propose an adjustment method that employs hedge ratios lagged by one time step.[7] This adjustment is straightforward to implement for both researchers and practitioners. Although it introduces some inaccuracy in the return measure due to the temporal mismatch between hedge ratios and stock returns, we show that this correction resolves the spurious premiums in the Black and Scholes (1973) case and effectively corrects

[7] To align with empirical datasets, we use an intraday lag for intraday-hedged returns and a one-day lag for return measures based on daily end-of-day data.

deviations from the true premiums in the models of Leland (1985) and Heston (1993). In contrast, previously suggested adjustment methods are not able to address these distortions properly.

To better understand the essence of the DMR and IMR biases, consider a simple example. Suppose a stock has an initial and final value of 100, implying a true return of zero. However, the observed initial price has a 50% chance of being either 100.10 or 99.90, reflecting bid-ask spreads. Although this measurement error has a mean of zero, the observed mean return of the stock is biased upward by 0.01 basis points (calculated as $0.5 \cdot \frac{100-99.90}{99.90} + 0.5 \cdot \frac{100-100.10}{100.10}$). These 0.01 basis points exemplify the direct mean return bias (DMR) that dates back to Blume and Stambaugh (1983), who showed that zero-mean noise in prices leads to a strictly positive bias in mean returns. The magnitude of this bias in the stock's mean return is approximately equal to the variance of the relative measurement error in stock prices.

Such a DMR bias arises in raw option returns as well, which is why we distinguish between stock-related S-DMR and option-related O-DMR biases. The bias is more pronounced for options because of their larger bid-ask spreads. For instance, if bid-ask spreads in options trading are ten times larger than in our stock example, the relative error variance is 10^2 times greater. Consequently, the O-DMR bias, approximated by the variance of the relative measurement error, would be about 1 basis point ($0.01 \cdot 10^2$).

Now consider an initially delta-hedged at-the-money call (or put), where the return equals the raw option return minus the hedge ratio times the underlying stock return. Assuming a true hedge ratio of 8 for the call (or -8 for the put), the bias would be approximately 0.92 (1.08) basis points, computed as 1 basis point minus (plus) 8 times 0.01 basis points.

So far, we have focused on the DMR biases in both the option and stock position, while neglecting distortions in the hedge ratios, and thus, the indirect mean return (IMR) bias. This indirect bias arises because measurement errors in the stock affect not only the stock return but also the option's delta, and hence its hedge ratio. If the observed stock price is distorted upward, the hedge ratio of a call also becomes too high, primarily due to its positive gamma. However, the subsequently observed stock return typically moves in the opposite direction. As a result, low stock returns are paired with erroneously high hedge ratios, and vice versa. For a delta-hedged call position, which includes a short position in the stock, this spurious comovement introduces an upward indirect mean return (IMR) bias, although it remains much smaller than the DMR bias (approximately 0.001 basis points in our example).[8]

Therefore, the dominant source of bias in delta-hedged option returns appears to be the O-DMR bias (1 basis point in our example), while the S-DMR bias (0.1 basis points) and the much smaller IMR bias can be safely ignored. However, our simulation study suggests that when the hedge is dynamically adjusted rather than held fixed, the IMR bias quickly becomes the predominant driver. If we hedge daily over the course of a month instead of once at initiation, the total bias in this example can easily exceeds 9 basis points, with the IMR bias accounting for most of this effect.

Although it is intuitive that the biases of the hedge position accumulate with more frequent rebalancing, the magnitude of the IMR bias is far greater than simply multiplying the initial IMR bias by the number of hedge adjustments. The key driver of this phenomenon is the sensitivity of the hedge ratio to stock price changes. In empirical studies, it is common practice to use *practitioner Black-Scholes hedge ratios*, where the observed stock price and implied volatility, derived from observed stock and option prices, are inserted into the Black and Scholes (1973) formula to compute the delta. If the stock price contains noise unrelated to the option price, the implied volatility becomes distorted as well. Consequently, noisy stock-price movements influence the option delta beyond what is captured by the option gamma. The additional channel, which becomes more pronounced for at-the-money options as time to expiration decreases, amplifies the spurious correlation between the hedge ratio and stock returns. Ultimately, this mechanism explains why the IMR bias, though initially

[8]For puts, the hedge ratio is also positively affected, making it less negative in absolute terms. As the hedge portfolio includes a long position in the stock, absolutely smaller hedge ratios are paired with low stock returns and vice versa, which again leads to an upward IMR bias.

negligible, can dominate when rebalancing occurs frequently.

This example therefore challenges the conventional view in the literature that the IMR bias can be ignored. It prompts us to reassess its importance and motivates our paper.

Related Literature

We contribute to the literature on delta-hedged option returns by identifying and quantifying a subtle but economically meaningful bias induced by market microstructure frictions, the indirect mean return (IMR) bias. Using a simulation framework, we analyze this bias in a fully controlled environment that isolates its mechanical origins from other sources of return variation. While Duarte et al. (2024) document that the IMR bias is negligible for static, daily returns, we show that it becomes material once hedge portfolios are frequently rebalanced. Since these returns often inform estimates of expected returns and risk premiums, neglecting the IMR bias can systematically distort inference about options market efficiency and compensation for risk. Our results highlight that what appears as a nonzero or elevated estimated risk premium in high-frequency settings may instead reflect a microstructure-induced measurement artifact. By clarifying this mechanism, we extend the foundational insights of Blume and Stambaugh (1983) on the direct mean return (DMR) bias to the dynamic option trading context, providing guidance for the interpretation of empirical evidence on option returns.

Our study builds on a growing body of research that examines the role of noise and market frictions in the analysis of options markets. Notably, Black (1986) provides a foundational discussion on the pervasive influence of noise across financial markets. More recently, Bliss and Panigirtzoglou (2002) examine the robustness of various estimation methods for option-implied risk neutral probability density functions when prices are distorted by random errors. Hentschel (2003) investigates the impact of noise on the estimation of implied volatility surfaces, while Dennis and Mayhew (2009) analyze how errors in option prices affect the empirical estimation of option pricing models.

Our work also contributes to the literature on the reliability of standard asset pricing tests. For the stock market, Asparouhova et al. (2010) establish that the connection between microstructure noise and stock liquidity implies upward biased estimates of return premiums for illiquidity. Concerning the options market, Branger and Schlag (2008) investigate the reliability of asset pricing tests based on delta-hedged returns with respect to discretization and model misspecifications.

Our study enhances the credibility and robustness of asset pricing outcomes and identified risk premiums in options markets by transparently addressing potential biases in empirical tests. Duarte et al. (2025) show that some stock characteristics, which seem to be priced in the cross-section of individual equity option returns, or the observed illiquidity premium in options, can be distorted by look-ahead bias from an infeasible sample. Goyenko and Zhang (2021) demonstrate that delta-hedged returns computed from end-of-day prices are systematically higher than those based on prices observed at any other time during the trading day and recommend calculating returns from prices observed during the first half of a trading day. We add another layer to these concerns by highlighting the importance of the indirect mean return bias. The broader literature on cross-sectional option returns, including studies by Bakshi and Kapadia (2003), Goyal and Saretto (2009), Cao and Han (2013), Christoffersen et al. (2018), Jones and Shemesh (2018), Frazzini and Pedersen (2022), Zhan et al. (2022), Bali et al. (2023), and Vasquez and Xiao (2024), provides valuable insights into option return dynamics, but often overlooks the potential impact of microstructure biases in dynamic hedging environments. Our research addresses these critical gaps, offers corrections, and helps refine the interpretation of cross-sectional return differences in options. Ultimately, this will lead to a more accurate understanding of the true drivers of option returns.

The remainder of the paper is structured as follows. Section 2 introduces delta-hedged option returns in general and discusses the impact of microstructure biases. Section 3 describes the selected return measures for the analyses in this paper and our simulation approach. Section 4 presents the results of the different simulation analyses and Section 5 concludes.

2 Return Measures and Biases

In this section, we formally introduce delta-hedged option returns and discuss how they depend on microstructure biases extending the work of Duarte et al. (2024) to general hedge frequencies.

General setting and notation: Consider a delta-hedged option return from time t_0 to T, $t_0 < T$. $\widetilde{C}_t$ denotes the price of the option, $\widetilde{S}_t$ the price of the underlying security, and $\widetilde{\Delta}_t$ the delta of the option at time t. r is the constant annual risk-free rate.

The starting point is a delta-hedged option gain in continuous time (e.g., Bakshi and Kapadia, 2003):

$$\widetilde{\Pi}_{\text{cont.}}(t_0, T) = \widetilde{C}_T - \widetilde{C}_{t_0} - \int_{t_0}^{T} \widetilde{\Delta}_u \, d\widetilde{S}_u - \int_{t_0}^{T} r(\widetilde{C}_u - \widetilde{\Delta}_u \widetilde{S}_u) \, du, \tag{1}$$

which is the gain of a portfolio that buys one option, sells delta shares of the underlying stock, and invests the net value at the risk-free rate. The position is rebalanced continuously, leading to a self-financing trading strategy. Because the portfolio is formed at zero cost, (1) can be interpreted as an excess gain over the risk-free rate. In Black and Scholes (1973) type models, in which the dynamics of the underlying asset is governed by a one-dimensional Markov-Itô process, this excess gain is always zero because the option can be perfectly replicated with an investment in the underlying and the risk-free rate.

In practice, continuous trading is not feasible, so researchers resort to discrete rebalancing strategies to assess delta-hedged returns. The options literature usually relies on the following formula which is a straightforward discretization of (1) (e.g., Bakshi and Kapadia, 2003):

$$\widetilde{\Pi}(t_0, T) \quad = \quad \widetilde{C}_T - \widetilde{C}_{t_0} - \sum_{n=0}^{N-1} \widetilde{\Delta}_{t_n} (\widetilde{S}_{t_{n+1}} - \widetilde{S}_{t_n}) - \sum_{n=0}^{N-1} (t_{n+1} - t_n) r(\widetilde{C}_{t_n} - \widetilde{\Delta}_{t_n} \widetilde{S}_{t_n}). \tag{2}$$

The corresponding portfolio is set up at t_0, rebalanced in discrete time at $t_1 < \cdots < t_{N-1}$, with $t_0 < t_1$, and the gain is realized at $t_N = T$, with $t_{N-1} < t_N$. To simplify the notation, we follow Duarte et al. (2024) and assume an interest rate of zero. The final return is obtained by dividing $\widetilde{\Pi}(t_0, T)$ by the initial option price $\widetilde{C}_{t_0}$:

$$\widetilde{\pi}(t_0, T) \quad = \quad \frac{\widetilde{C}_T - \widetilde{C}_{t_0}}{\widetilde{C}_{t_0}} \quad - \quad \sum_{n=0}^{N-1} \quad \underbrace{\left(\widetilde{\Delta}_{t_n} \frac{\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}_{\text{hedge ratio}} \quad \frac{\widetilde{S}_{t_{n+1}} - \widetilde{S}_{t_n}}{\widetilde{S}_{t_n}}. \tag{3}$$

In (3), a delta-hedged option return consists of two components: the simple buy-and-hold return of the option and the compounded return of the dynamically rebalanced hedge portfolio. The latter is the aggregate of N individual returns, each determined by the current *hedge ratio* and the stock return over the respective period.

Next, we introduce measurement errors in prices. Following Duarte et al. (2024), we assume that prices for options and their underlyings at time t are observed with a multiplicative measurement error:

$$C_t = \widetilde{C}_t \cdot (1 + \varepsilon_{C,t}), \quad S_t = \widetilde{S}_t \cdot (1 + \varepsilon_{S,t}),$$

where $\widetilde{C}_t$ and $\widetilde{S}_t$ describe the *true* prices of an option (call or put) and its underlying, C_t and S_t describe the observed (noisy) prices. The error terms $\varepsilon_{C,t}$ and $\varepsilon_{S,t}$ are assumed to have a zero mean. In addition, they are assumed to be stochastically independent over time, across assets, and independent of true prices.

Based on this simple model, Duarte et al. (2024) show that the expected value of the observed delta-hedged return is a biased estimate of the true return (*mean return (MR) bias*). They divide this bias into two components:

The first component is the *direct mean return (DMR) bias* and was documented for the first time by Blume and Stambaugh (1983) for the stock market. This bias describes the effect of measurement errors on the raw returns of assets. It leads to an upward distortion of these quantities that is approximately the size of the variance of the relative error. Since this type of bias exists for both option and stock returns, with distinct properties, we further distinguish between *Option-DMR (O-DMR)* bias and *Stock-DMR (S-DMR)* bias.

The second main component of the bias is the *indirect mean return (IMR) bias*, which was identified by Duarte et al. (2024). The IMR bias only applies to hedged option portfolios and is caused by the microstructure noise dependencies that affect the return of the underlying and the respective hedge ratio. Note that not only is the return of the stock position measured with error, but also the number of shares (i.e., the delta) computed using an erroneous price is incorrect due to the same measurement error. Together, both errors induce a spurious covariance between the otherwise independent return of the stock from t_n to t_{n+1} and the hedge ratio at t_n, leading to an additional upward bias in the delta-hedged option return.

The right-hand side of Expression (4) formalizes both the DMR and the IMR bias. The expression extends the approximation formula for delta-hedged option portfolios' mean return bias of Duarte et al. (2024) to general hedge frequencies:[9]

$$
\begin{aligned}
\mathbb{E}\left[\pi(t_0,T)-\widetilde{\pi}(t_0,T)\right] \quad \approx \quad & \underbrace{\underbrace{\mathbb{E}\left[\varepsilon^2_{C,t_0}\right]}_{\text{O-DMR bias}} \underbrace{-\sum_{n=0}^{N-1}\mathbb{E}\left[\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right]}_{\text{S-DMR bias}}}_{\text{DMR bias}} \\
& \underbrace{-\sum_{n=0}^{N-1}\operatorname{Cov}\left(\frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\varepsilon_{S,t_n}\ ,\ -\varepsilon_{S,t_n}\right)}_{\text{IMR bias}}
\end{aligned}
\tag{4}
$$

with

$$
-\sum_{n=0}^{N-1}\operatorname{Cov}\left(\frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\varepsilon_{S,t_n}\ ,\ -\varepsilon_{S,t_n}\right) = \sum_{n=0}^{N-1}\mathbb{E}\left[\frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right]. \tag{5}
$$

Note that (4) and (5) hold for both call and put options.

The first part of (4) corresponds to the direct component of the bias and consists of two terms. We observe that errors affecting the raw option return result in an upward bias approximately equal to the variance of the respective option price error at time t_0 (O-DMR

[9]The derivation can be found in Appendix A.

bias). Similarly, the expected simple stock returns from t_n to t_{n+1} are biased upwards by the variance of the stock price error at t_n. In the case of a put option, which has a negative delta, this S-DMR bias introduces an additional upward bias. Conversely, for a delta-hedged call option, the short position in the underlying asset reduces the upward bias that originates solely from the option position.

The second part of (4) provides an approximation of the indirect component of the bias. In simple terms, when the true stock price $\widetilde{S}_{t_n}$ is distorted by a measurement error ε_{S,t_n}, the observed simple stock return deviates by approximately $-\varepsilon_{S,t_n}$ from the true stock return. This measurement error ε_{S,t_n} in the stock price also affects the calculation of the delta at time t_n, which in turn influences the hedge ratio. The expression $\frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\varepsilon_{S,t_n}$ approximates the change in the hedge ratio due to the difference between the erroneous stock price and the true price.[10] The IMR bias is then determined by adding all the corresponding covariances. Looking at (5), we get a clearer understanding of this indirect bias. This bias, like the DMR bias, explicitly depends on the positive variance of the stock price errors. In addition, it depends on the sensitivity of the hedge ratio to changes in the underlying price while keeping the option price constant. The sensitivity of at-the-money options rises as maturity approaches, as small movements in the underlying generate larger adjustments in delta, through increasing gamma, and trigger stronger responses in implied volatility.

Note that both the S-DMR bias and the IMR bias depend on how frequently the position in the underlying asset is rebalanced. This implies that microstructure noise can impact different measures of option returns differently, suggesting that the choice of measure is crucial. Unlike the DMR bias of a simple asset return, which can be calculated with minimal assumptions, the entire right-hand side of (4) cannot be evaluated analytically. Therefore, it is essential to quantitatively assess the relative magnitudes of the MR bias components and their sensitivity to hedge frequency in a controlled simulation framework.

Duarte et al. (2024) provide a comprehensive discussion on two additional biases: the

[10] In Appendix A, we provide details on these approximations.

sample selection (SS) bias and the correlated errors in variables (CEIV) bias.[11] The first proves to be negligible for at-the-money options, so we will not investigate it further here. To address the second, we employ the approach proposed by Duarte et al. (2024) and introduce a one-day lag to the independent variables when conducting portfolio sorts or regressions. This methodology has also previously been used in the empirical literature, for instance, by Goyal and Saretto (2009) or Christoffersen et al. (2018).

3 Methodology

This section reviews measures of monthly delta-hedged option returns documented in the literature and outlines the simulation approach used in our analysis. Additional details are provided in Appendix C.

3.1 Variety of Delta-Hedged Returns

The options literature lacks a universally agreed definition for computing delta-hedged returns. One distinguishing factor is the chosen hedge frequency, which refers to the frequency with which the underlying position is rebalanced. The measures that are usually implemented based on daily end-of-day quote data for options and underlyings comprise *static* and *daily* delta hedging.[12] Historically, researchers have widely utilized end-of-day data due to its ease of handling, particularly considering limitations in computing power, which nowadays is not as big an issue. Goyenko and Zhang (2021) find that returns computed with end-of-day quotes are systematically biased compared to returns based on quotes captured

[11]The SS bias arises when certain observations are excluded from the final sample due to measurement errors (such as spurious no-arbitrage violations of mid-prices) encountered during data filtering. Duarte et al. (2024) demonstrate that this bias is negligible when examining liquid at-the-money (ATM) options. The CEIV bias occurs when independent variables in a portfolio sort or regression analysis are affected by the same measurement errors as the dependent variable, i.e., the option return.

[12]Goyal and Saretto (2009), Broadie et al. (2009), Byun and Kim (2016), and Zhan et al. (2022) are examples for the use of static hedging, while Cao and Han (2013), Constantinides et al. (2013), Karakaya (2013), Choy and Wei (2020), Choy and Wei (2022), Frazzini and Pedersen (2022), Bali et al. (2023), Tian and Wu (2023), or Vasquez and Xiao (2024) rely on daily rebalancing.

at any other time during the trading day. The reason is demand pressure that affects end-of-day prices, which motivates the use of high-frequency intraday data. Other examples of recent studies that rely on data at a higher frequency than daily are Christoffersen et al. (2018), who use aggregated intraday prices of option trades, Muravyev and Ni (2020), who compute open-to-close and overnight returns, and Goyenko and Zhang (2021), who compute daily returns starting at different hours during the trading day.

Using end-of-day data, we implement the widely used *static* and *daily* hedging strategies, along with *weekly* and *bi-weekly* rebalancing. This setup allows us to examine how hedge frequency shapes microstructure biases, with the weekly and bi-weekly cases naturally positioned between the two extremes of hedging once per month and rebalancing each day. In addition to these four measures based on daily data, we examine three different intraday measures in our simulation environment. For the first measure, the hedge position is rebalanced every 30 minutes (*30min*), which is the highest frequency in our setting. The second measure is calculated by rebalancing every time a trade of the option occurs (*Tradetimes*). Researchers could implement this measure based on the LiveVol Option Trades dataset that provides trade prices and option and underlying quotes at the time of a trade. The third measure relying on intraday data is inspired by Christoffersen et al. (2018). In their main analysis, they match daily averages of option trade prices with end-of-day data for deltas and stocks. We adapt this measure to our monthly setting with daily rebalancing of the hedge position using end-of-day data and averages of intraday option prices on the first and last day of the month (*DailyAVG*).

All of these measures are compared to a benchmark return that comes closest to the continuous unbiased version assumed in standard option pricing models. We compute our benchmark return using prices without measurement errors (*true* data) at the highest frequency of rebalancing, i.e., every 30 minutes. For the Black and Scholes (1973) model, the delta-hedged benchmark return is known to be 0 with continuous rebalancing. Therefore, the small difference between our reported benchmark return and 0 is solely due to hedge errors caused by discretization. A summary of all the returns analyzed can be found in Table 1.

Table 1: **Overview of delta-hedged return measures**

This table summarizes the different monthly return measures that are considered throughout our paper. *EOD* stands for 'end-of-day' and *quote* refers to the quoted mid-price. The *Benchmark*-measure is computed without measurement errors. *DailyAVG* describes the return inspired by Christoffersen et al. (2018).

Measure	**Option Prices**	**Deltas**	**Rebalancing**
Benchmark	True	True	Every 30 minutes
30min	Quote	Quote	Every 30 minutes
Tradetimes	Quote	Quote	Every time a trade occurs
DailyAVG	Daily average quote at tradetimes	EOD quote	Daily
Daily	EOD quote	EOD quote	Daily
Weekly	EOD quote	EOD quote	Weekly
Bi-Weekly	EOD quote	EOD quote	Every two weeks
Static	EOD quote	EOD quote	None after setup

3.2 Simulation Model

To create a database for the analysis of delta-hedged returns, we use a simulation framework. This approach is particularly suitable because empirical samples often contain inherent biases that cannot be observed or controlled. In a simulation environment, we can generate *true* data and add errors separately. This allows us to compare different return measures that rely on noisy data with a common unbiased benchmark. The main structure of the simulation procedure is outlined below. For details, please refer to Appendix C.

The simulation approach can be summarized as follows: 1000 (independent) samples of data are created, each sample representing a single simulation trial. Each sample consists of a cross section of 500 stocks with corresponding options. The assets are observed over a 120-month period. Return measures are then computed for each sample, and results are averaged across all simulation trials. This methodology allows for robust results.

The dataset for each simulation trial comprises *true* and noisy data for options and their corresponding underlying stocks. The true prices and deltas are computed using the model proposed by Black and Scholes (1973) for the initial part of our analyses, and the models introduced by Leland (1985) and Heston (1993) for the later part. The noisy market data mimics the observed option prices provided by OptionMetrics and LiveVol by adding error terms to the true quantities. Resembling a sample of options on all S&P 500 stocks over 120 months, we get a total of 60,000 stock months in each simulated sample. Within each month, there are 22 trading days. Each day consists of 14 intraday time points. This corresponds to one observation taken every 30 minutes over a day with six and a half hours of trading per day, which is the usual length of the trading period for the US equity options market. At the beginning of the first month in each sample, all stocks have an initial value of 100. The stocks' values are reset to 100 at the beginning of every subsequent month.[13] The strikes of these options are also set to 100, ensuring that they are initially at-the-money (ATM). For every month, a corresponding call and put option are simulated that expire after 35 trading days. This is meant to approximate a time-to-maturity of one full month plus the time until the third Friday of the following month, which is the standard monthly expiration date for most option classes. This approach allows us to simulate options that are relatively liquid, similar to the empirical samples often used in the literature.

We now outline the procedure within a single simulation trial. First, we generate stock returns to obtain the monthly price paths for each stock. True stock returns are assumed to follow a standard market model for our baseline setting in the Black and Scholes (1973) model, as well as for the Leland (1985) model:

$$r_t^i = \beta_i \cdot r_t^m + \xi_t^i \tag{6}$$

r_t^i describes the return of stock i at time t. r_t^m is the normally distributed return of the market. β_i denotes the sensitivity of stock i towards the market return and is drawn from

[13] The starting value of 100 for stock prices is chosen for normalization purposes. The level of the initial stock price does not influence delta-hedged returns because the option price is homogeneous of degree one in the stock price.

a uniform distribution between 0.5 and 1.5, i.e., the average beta is equal to 1. The stock-specific error term ξ_t^i is normally distributed with a zero mean.

The stock data for the stochastic volatility model of Heston (1993), which are used for the analyses in Section 4.4.2, are generated using the following processes:

$$r_t^i = \lambda_1 \cdot V_{t-1}^i \cdot \Delta t + \sqrt{V_{t-1}^i} \cdot \xi_t^{\prime,i} \cdot \sqrt{\Delta t} \tag{7}$$

$$V_t^i = V_{t-1}^i + \kappa(\theta - V_{t-1}^i) \cdot \Delta t + \sigma \sqrt{V_{t-1}^i} \cdot \left(\rho \cdot \xi_t^{\prime,i} + \sqrt{1-\rho^2} \cdot \xi_t^{\prime\prime,i}\right) \cdot \sqrt{\Delta t} \tag{8}$$

Here, r_t^i represents the return, and V_t^i denotes the instantaneous variance of stock i at time t. The stochastic processes $\xi_t^{\prime,i}$ and $\xi_t^{\prime\prime,i}$ follow a standard normal distribution, and Δt denotes the simulated step size of our data, which is set to 30 minutes. The parameterization is based on the specifications provided in Duarte et al. (2024).

Next, we simulate noisy stock prices to resemble the prices for the underlying stocks found in standard option datasets. Note that liquidity suppliers often do not set quotes symmetrically around the fundamental value of the stocks, leading to noisy mid prices (for a discussion, see Hagströmer, 2021). We generate observed stock prices by adding a random error to the true prices. The error is assumed to follow a symmetric triangular distribution with lower and upper limits that correspond to -0.5 and +0.5 times the relative bid-ask spread of the stock (Duarte et al., 2024):

$$S_t^i = \widetilde{S}_t^i \cdot \left(1 + \varepsilon_{S,t}^i\right) \tag{9}$$

S_t^i is the noisy stock price and $\widetilde{S}_t^i$ the true price of stock i at time t. $\varepsilon_{S,t}^i$ stands for the triangular-distributed random error.

After simulating the stock data, we generate data for options using a similar approach as with stocks. To introduce noise, we add an error term to each option's true value:

$$C_t^i = \widetilde{C}_t^i \cdot \left(1 + \varepsilon_{C,t}^i\right) \tag{10}$$

C_t^i describes the option's observed price and $\widetilde{C}_t^i$ is the true price of option i at time t based on Black and Scholes (1973), Leland (1985), or Heston (1993). For the calculation of $\widetilde{C}_t^i$, $\widetilde{S}_t^i$ is used. Like for stocks, the error term $\varepsilon_{C,t}^i$ is drawn from a symmetric triangular distribution between -0.5 and +0.5 times the option's relative bid-ask spread.

To determine the relative bid-ask spreads of options, we use the model and the parameters of Duarte et al. (2024). They rely on a linear regression motivated by the findings of Fontnouvelle et al. (2003). Details can be found in Appendix D.

Finally, we calibrate trading frequencies depending on an option's time to maturity and bid-ask spread according to a linear regression model.[14] To this end, we use data on short-term at-the-money options on S&P 500 stocks from OptionMetrics and LiveVol. We estimate the expected number of trades per day and apply an exponential distribution to simulate the occurrences of option trades.

To provide an overview of the simulated dataset, Table 2 presents summary statistics for the base case, in which option prices are computed using the model of Black and Scholes (1973). The statistics are computed separately within each sample and then averaged across all simulation trials. Stock returns exhibit an average volatility of 44% which closely matches the implied volatility of the noisy prices for both calls and puts. The distribution of option deltas, with averages of 0.52 for calls and -0.48 for puts, indicates that the sample is concentrated around at-the-money options. A clear difference in magnitude arises when comparing bid-ask spreads across asset classes. The average bid-ask spread for stocks is 0.20% with only 1% of observations exceeding 1.03%. In contrast, simulated options display substantially wider bid-ask spreads, with an average of approximately 14%. These figures align with prior empirical evidence documenting the relatively high transaction cost in options markets. Finally, trade frequencies differ markedly between call and put options. On average, call options are traded 6.14 times per day, whereas put options trade only 3.85 times per day.

[14]Details are presented in Appendix E.

Table 2: **Summary of the simulated dataset for the Black and Scholes (1973) model**

This table shows summary statistics of the dataset according to the model of Black and Scholes (1973). Implied volatilities and deltas correspond to noisy prices. The statistics are calculated for each sample separately and then averaged over all simulation trials. The characteristics for stocks are constant over time within each trial, i.e., the statistics are computed directly over all stocks. For options, the statistics are taken cross-sectionally over the mean values per month.

		Mean	Std	$q_{0.01}$	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$q_{0.99}$
Stocks	Beta	1.00	0.29	0.51	0.75	1.00	1.25	1.49
	Volatility (%)	44.34	12.99	20.90	33.24	44.03	55.46	67.20
	Bid-Ask Spread (%)	0.20	0.21	0.02	0.08	0.14	0.25	1.03
Calls	IV (%)	44.10	12.97	20.70	33.02	43.78	55.19	67.09
	Delta	0.52	0.18	0.15	0.38	0.52	0.67	0.87
	Bid-Ask Spread (%)	13.91	9.15	2.89	7.60	11.49	17.56	46.60
	Trades per Day	6.14	0.59	4.49	5.79	6.19	6.54	7.34
Puts	IV (%)	44.07	12.93	20.73	33.03	43.75	55.11	67.05
	Delta	-0.48	0.18	-0.85	-0.62	-0.48	-0.33	-0.13
	Bid-Ask Spread (%)	14.05	8.90	3.12	7.91	11.76	17.65	45.73
	Trades per Day	3.85	0.46	2.66	3.56	3.86	4.16	4.86

4 Results

In this section, we present the main results of our simulation analyses. We start by discussing the effect of microstructure noise on all implemented measures in general. Next, we evaluate asset pricing tests with respect to the liquidity of the underlying and introduce bias adjustments. The last part extends the Black and Scholes (1973) simulation framework to include risk premiums in delta-hedged option returns.

4.1 Statistical Properties of Different Delta-Hedged Return Measures

Table 3 reports summary statistics for our seven monthly return measures, with results for call options in Panel (a) and for put options in Panel (b). For all but one measure, mean returns are positive, indicating deviations from the theoretical benchmark of zero that would prevail under continuous trading. The largest average deviations occur for measures based on high-frequency intraday data. Focusing on call options, we find an upward bias of 92 bp when the hedge portfolio is rebalanced every 30 minutes and 46 bp when adjustments are made at trade times. Measures relying solely on end-of-day data exhibit smaller positive biases, ranging from 24.6 bp for the *Static* measure to 17.1 bp for weekly rebalancing. Put options display a very similar pattern. Overall, the mean bias declines as the hedge frequency decreases, although lower rebalancing intensity comes at the cost of reduced hedge accuracy, reflected in higher average standard deviations.

The columns *HE (hedge error)*, *O-DMR Bias*, *S-DMR Bias*, and *IMR Bias* in Table 3 present a decomposition of the mean error. We assess the magnitude of each component by sequentially introducing deviations from the benchmark return. To isolate the bias due to discretization (the hedge error), we compute each measure using its respective rebalancing scheme on noise-free data and evaluate the deviation from zero. The O-DMR bias component is obtained by recalculating each measure using simple returns of options based on noisy data, while keeping hedge ratios and stock returns error-free. To quantify the S-DMR bias, we add noise to the stock returns but still use noise-free hedge ratios. Finally, the incremental effect of the IMR bias is captured by applying measurement errors to both hedge ratios and raw asset returns.

Using the *true* data, free of measurement errors, we observe positive hedge errors except for the *DailyAVG* measure. This result aligns with the findings of Branger and Schlag (2008), who theoretically show that in the model of Black and Scholes (1973) expected hedge errors are positive. As the equity premium is positive, deltas generally increase over the course of a month. If deltas are not promptly adjusted after price changes, they remain too low on average, resulting in a too small short position (for calls) or too large long position (for puts) in the underlying. This, in turn, leads to an upward-biased delta-hedged return. Increasing the hedge frequency adjusts the delta upward more quickly and mitigates this bias.

All remaining components reflect biases resulting from microstructure noise in our data. Consistent with the discussion in Section 2, the O-DMR bias remains constant across measures for a given option type, as this bias depends only on the error in the option price at inception of the portfolio, not on hedge frequency. Puts exhibit a slightly higher O-DMR bias of 13.52 bp compared to calls with 12.87 which can be explained by the on average higher bid-ask spread for put options in our simulations. The exceptions to this pattern are *Tradetimes* and *DailyAVG*. The former has a different starting point of return calculation for each option due to the stochastic simulation of trade times, which can result in a different option error variance at inception. The latter is discussed in more detail below.

For call options in Panel (a), the S-DMR bias is negative across all measures. Because call deltas are positive, the short position in the underlying reverses the upward bias inherent in the simple stock return. In contrast, the put options in Panel (b) exhibit a positive bias reflecting the long position in the stock. The absolute magnitude of this bias increases with hedge frequency, rising from 0.04 bp for *Static* to about 8-9 bp for *30min*. For the IMR bias, we observe pronounced differences across measures, with a substantial increase as hedge frequency rises for both call and put options. While negligible relative to other sources of bias at weekly or lower hedge frequency, the IMR reaches almost 6 bp for daily rebalancing, roughly half the magnitude of the DMR bias and more than one quarter of the total bias. Its importance intensifies sharply for intraday measures: when rebalancing every 30 minutes, the indirect component surpasses 72 bp for puts and 88 bp for calls, corresponding to 76% and 96% of the total bias, respectively.

Unlike all other measures, the *DailyAVG* measure exhibits a downward-biased mean relative to the zero benchmark. This negative mean bias is caused by a large discretization error. According to Jensen's inequality, the raw return computed from daily average prices is lower than the average raw return computed from individual price observations:

$$\frac{\frac{1}{n}\sum_{j=1}^{n}\widetilde{C}_{T,j}}{\frac{1}{m}\sum_{i=1}^{m}\widetilde{C}_{0,i}} \leq \frac{1}{m \cdot n}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\widetilde{C}_{T,j}}{\widetilde{C}_{0,i}}. \tag{11}$$

In expectation, monthly option returns are largely invariant to the specific time points chosen on the first and last trading days of the month. Applying the expectation operator to both sides therefore implies that the *DailyAVG* measure exhibits a downward bias compared to using end-of-day prices when computing expected option returns. The DMR and IMR biases partially offset this discretization bias of the *DailyAVG* measure, resulting in a total deviation of -73 bp for calls and -44 bp for puts. The O-DMR bias is the smallest among all measures because averaging option prices reduces the variance of the measurement error. In contrast, the standard deviation of the *DailyAVG* measure exceeds 2000 bps, compared with slightly below 1200 bps for our other daily-rebalanced measure for both calls and puts. The high standard deviation is due to the mismatch between option prices and hedge ratios on the first and last day of the month. Option prices C_{t_0} and C_T are computed from daily averages of intraday prices, whereas hedge ratios and stock returns are based on end-of-day prices. The mismatch causes over- or under-hedging and thus generates highly volatile payoffs when the portfolio is unwound.

In contrast to the setting in Duarte et al. (2024), the IMR bias is far from negligible. It becomes particularly pronounced under high-frequency rebalancing and emerges as the dominant source of deviation from the error-free benchmark. The magnitude of this bias is similar for call and put options. Therefore, we focus on call options in the remainder of this study and report corresponding results for puts in Appendix G.

4.2 Spurious Return Premiums

Biases in returns complicate the economic interpretation of asset pricing results. They can lead to spurious inferences if price effects are mistakenly attributed to economic factors when the true source is measurement bias. Conversely, microstructure noise can mask genuine price effects, hindering the identification of existing risk premiums. In this section, we address the first issue. Section 4.3 presents adjustment methods for the observed microstructure biases, and Section 4.4 investigates the identification of existing premiums in option returns.

Our simulated data based on the Black and Scholes (1973) model is well suited to examine

Table 3: **Biases of different return measures**

This table shows statistical properties of all implemented option return measures for call and put options. Mean, standard deviation, and biases are given in basis points. All values are computed separately for each sample and then averaged across samples. The hedge error (HE) measures the deviation of the respective noise-free return from zero. The O-DMR bias is the difference between the respective noise-free return and the corresponding measure in which the raw option returns are affected by errors. The S-DMR bias is the incremental deviation when raw underlying returns are also noisy. The IMR bias is the additional change due to error-affected hedge ratios.

(a) Call options

	Mean	Std	HE	O-DMR Bias	S-DMR Bias	IMR Bias
Benchmark	0.20	278.00	0.20	0	0	0
30min	92.12	709.40	0.20	12.87	-9.48	88.53
Tradetimes	45.89	743.56	0.07	11.19	-4.20	38.84
DailyAVG	-73.03	2460.49	-81.26	3.29	-0.60	5.54
Daily	18.71	1160.36	1.12	12.87	-0.68	5.40
Weekly	17.13	2384.18	4.06	12.87	-0.14	0.34
Bi-Weekly	18.41	3326.54	5.92	12.87	-0.08	-0.30
Static	24.62	4669.47	12.20	12.87	-0.04	-0.41

(b) Put options

	Mean	Std	HE	O-DMR Bias	S-DMR Bias	IMR Bias
Benchmark	0.30	276.52	0.30	0	0	0
30min	94.70	736.45	0.30	13.52	8.18	72.70
Tradetimes	35.32	864.25	1.00	11.24	2.27	20.81
DailyAVG	-44.55	2193.87	-55.30	5.16	0.53	5.07
Daily	21.31	1179.77	1.37	13.52	0.60	5.83
Weekly	19.95	2383.21	4.53	13.52	0.13	1.77
Bi-Weekly	20.84	3312.21	5.96	13.52	0.07	1.29
Static	26.89	4624.68	12.45	13.52	0.04	0.88

the extent to which commonly used asset pricing methods, such as portfolio sorts or Fama and MacBeth (1973) regressions, may falsely identify price effects when the true prices contain no risk premium. Specifically, we investigate which option-return measures are particularly prone to indicating a spurious premium driven by underlying illiquidity, even though true option prices are unaffected by the underlying's illiquidity.

Panel (a) of Table 4 reports average returns of delta-hedged calls sorted by the illiquidity of the underlying, measured by the bid-ask spread. All return measures exhibit a positive return spread between the portfolios with the highest and lowest average bid-ask spread. The magnitude of this apparent premium increases sharply with hedge frequency, ranging from 2 bp for static hedging to 22 bp for daily rebalancing, and even reaching 300 bp per month for 30-minute rebalancing. The *high-minus-low (H-L) return* is statistically significant at the 5% level in virtually all samples for the two intraday rebalancing measures, *30min* and *Tradetimes*. For daily rebalancing, significant H-L returns are found in 32.8% of the samples, while for lower hedge frequencies the proportion falls below 6%.[15]

The observed spurious premiums can be fully attributed to the biases documented in Table 3. The discretization error does not differ between stocks with high and low bid-ask spreads. However, in our simulation model, the illiquidity of the underlying asset affects the magnitude of microstructure errors: their variances increase with the bid-ask spreads. As a result, the associated biases also rise with the stock's bid-ask spread, generating positive high-minus-low returns. This finding suggests that asset pricing tests associating option returns with underlying illiquidity are prone to false positives in the presence of microstructure noise.

We identify the indirect component of the mean return bias as the primary driver of the spuriously positive premiums. Both the DMR and the IMR bias depend on the underlying's liquidity over the variance of errors in prices. For call options, the direct component of the bias is negatively related to the stock's bid-ask spread, whereas the indirect bias is positively

[15] We test whether high-minus-low returns differ from zero using a t-test with Newey-West standard errors. Although our simulated data do not require this correction, we apply it for consistency with the empirical literature, where it is standard practice. Because the Newey-West test is robust to autocorrelation and heteroscedasticity, it imposes fewer assumptions on the data but may have lower power in our setting.

related (see Equations (4) and (5)). As a result, the positive high-minus-low returns are driven by the IMR bias, which dominates the direct component across all measures.[16]

Next, we estimate return premiums using Fama and MacBeth (1973) regressions. Duarte et al. (2024) refer to the biases induced by microstructure noise in such regressions as *regression coefficient (RC) biases*, which can be viewed as a generalization of the MR bias. As with mean portfolio returns, RC biases can be decomposed into direct and indirect components, which affect regression coefficients in ways analogous to their impact on mean returns.

Panel (a) of Table 5 reports the findings of our regression analysis, focusing on the slope coefficient. Consistent with the portfolio-sort results, the coefficients on the bid-ask spread are positive (except for static hedging) and increase substantially with hedge frequency: 0.03 for bi-weekly hedging and 10.90 for 30-minute rebalancing. The latter implies that a 1 bp increase in the bid-ask spread of the stock raises the monthly delta-hedged call return by 10.90 bps. As argued earlier, these effects can be attributed to the indirect bias component.[17] Regarding the statistical significance, measures with intraday rebalancing (*Tradetimes* and *30min*) exhibit positively significant coefficients at the 5% level in all simulated samples. Significance declines for lower frequencies: daily rebalancing yields positively significant coefficients in 73.8% of samples, while for lower frequencies significance occurs in only in about 5% of trials. This pattern reflects two effects. First, the bias is smaller for less frequent rebalancing, producing smaller regression coefficients. Second, variance of the delta-hedged returns is higher at lower frequencies, leading to larger standard errors and lower t-statistics.

4.3 Bias Adjustments

Given the substantial impact of the IMR bias on statistical inference, it is natural to ask whether this bias can be effectively accounted for in option returns. While well-established

[16] We also conduct this portfolio sort with measures for calls that isolate the DMR bias, i.e., employing unbiased hedge ratios but noise-affected raw returns for both options and the underlying. In this case, we observe close to zero or negative high-minus-low-returns for all measures, which is in line with the negative influence of the stock bid-ask spread on the DMR bias.

[17] Another regression analysis using return measures that isolate the direct bias component yields slightly negative coefficients for all measures, consistent with the portfolio-sort discussion.

Table 4: **Returns of option portfolios sorted by the bid-ask spread of the underlying in the model of Black and Scholes (1973)**

This table shows average monthly returns of delta-hedged at-the-money call portfolios sorted by the bid-ask spread of the underlying. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	0.28	0.09	0.22	0.17	0.26	-0.02	-0.01	2.7	2.9
30min	14.48	19.92	31.68	61.04	333.47	318.99	27.88	100	0
Tradetimes	11.83	14.30	18.91	32.04	152.36	140.53	14.16	99.9	0
DailyAVG	-77.17	-78.75	-76.32	-75.43	-57.48	19.69	0.68	10.3	0.6
Daily	13.86	13.23	14.50	16.34	35.61	21.75	1.52	32.8	0
Weekly	16.91	16.20	16.04	16.64	19.88	2.97	0.10	3.8	2.3
Bi-Weekly	18.76	17.96	17.19	18.34	19.81	1.05	0.03	3	2.3
Static	23.69	26.06	23.40	24.24	25.70	2.02	0.03	2.7	2.7

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	0.28	0.09	0.22	0.17	0.26	-0.02	-0.01	2.7	2.9
30min	2.52	7.81	19.71	48.91	320.39	317.87	29.01	100	0
Tradetimes	1.23	3.54	8.53	21.40	141.61	140.38	14.97	100	0
DailyAVG	-9.47	-11.55	-8.81	-8.41	9.59	19.07	0.70	10.3	0.6
Daily	1.89	1.15	2.53	4.27	23.49	21.60	1.62	36.4	0
Weekly	4.82	4.21	4.13	4.39	7.68	2.86	0.11	3.6	2.1
Bi-Weekly	6.46	6.04	4.98	6.02	7.28	0.81	0.02	3.4	2.3
Static	11.15	13.91	10.92	11.44	12.95	1.81	0.03	2.7	2.8

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	0.28	0.09	0.22	0.17	0.26	-0.02	-0.01	2.7	2.9
30min	0.79	0.30	0.41	0.50	0.38	-0.41	-0.05	2.4	2.9
Tradetimes	0.33	-0.12	-0.07	-0.01	-0.39	-0.73	-0.07	2.7	2.9
DailyAVG	-8.98	-10.94	-8.82	-10.45	-10.49	-1.52	-0.05	2.6	3.4
Daily	2.18	1.77	2.30	2.14	1.63	-0.56	-0.02	3	2.6
Weekly	5.21	5.12	4.89	3.90	3.64	-1.57	-0.05	2.6	3.4
Bi-Weekly	6.95	6.83	5.03	6.43	5.62	-1.33	-0.03	2.2	2.6
Static	10.24	12.95	9.96	11.05	11.39	1.16	0.02	3.3	3.2

Table 5: **Cross-sectional regressions of option returns on underlying bid-ask spreads in the model of Black and Scholes (1973)**

This table shows the results of univariate cross-sectional Fama and MacBeth (1973) regressions of delta-hedged at-the-money call returns on the bid-ask spreads of the underlying stocks. We only report statistics for the estimated slope coefficient. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) contains the results obtained through an ordinary least squares (OLS) procedure. In Panel (b), a weighted least squares (WLS) regression is performed with weights proportional to the one-day-lagged gross return of the option. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the slope coefficient is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	Mean	Std	Avg. t-stat	*(+)	*(-)
Benchmark	0.00	0.05	0.01	3.4	3.2
30min	10.90	0.38	29.63	100	0
Tradetimes	4.81	0.24	20.18	100	0
DailyAVG	0.69	0.47	1.54	33	0.2
Daily	0.75	0.24	3.12	73.8	0
Weekly	0.11	0.47	0.25	4.5	1.1
Bi-Weekly	0.03	0.64	0.04	3	2.7
Static	-0.01	0.89	-0.02	2.6	2.4

(b) O-DMR-bias adjusted

	Mean	Std	Avg. t-stat	*(+)	*(-)
Benchmark	0.00	0.05	0.01	3.4	3.2
30min	10.85	0.37	30.57	100	0
Tradetimes	4.80	0.23	21.04	100	0
DailyAVG	0.66	0.44	1.55	33.6	0.2
Daily	0.75	0.23	3.30	76.7	0
Weekly	0.12	0.44	0.27	4.9	0.9
Bi-Weekly	0.04	0.60	0.06	3.3	2.5
Static	0.00	0.83	-0.01	2.9	2.4

(c) DMR- and IMR-bias adjusted

	Mean	Std	Avg. t-stat	*(+)	*(-)
Benchmark	0.00	0.05	0.01	3.4	3.2
30min	-0.01	0.17	-0.06	2.3	3.2
Tradetimes	-0.01	0.20	-0.07	3	3.4
DailyAVG	-0.05	0.50	-0.11	1.4	3.7
Daily	-0.02	0.35	-0.06	2.5	3
Weekly	-0.04	0.51	-0.08	2.3	2.5
Bi-Weekly	-0.03	0.65	-0.06	2.2	3.6
Static	-0.03	0.87	-0.04	3.1	2.9

corrections exist for the DMR bias, no methods specifically targeting the IMR bias are currently available. This gap likely reflects the limited recognition and importance of the IMR bias in prior studies. Our approach primarily targets the IMR bias but also addresses the hedge-frequency-dependent component of the DMR bias (S-DMR bias). Together, these two biases constitute a single hedge-frequency-dependent bias that arises when calculating expected hedge portfolio returns with noisy data. In practice, the S-DMR and IMR bias appear together because, given asset prices, the hedge ratio multiplied by the underlying return can be computed as $\frac{\Delta_{t_n}}{C_{t_0}}(S_{t_{n+1}} - S_{t_n})$. Nonetheless, we treat the S-DMR bias separately, following Duarte et al. (2024), to maintain consistency in comparison.

In their study, Duarte et al. (2024) propose weighted portfolio sorts and regressions to address microstructure biases. Specifically, they recommend the weighting scheme introduced by Asparouhova et al. (2010) for the stock market, which weights returns from time t_0 to t_1 by the one-day lagged gross return of the asset $\frac{C_{t_0}}{C_{t_{-1}}}$. This weighting mitigates upward bias by inducing a negative correlation between returns and weights; when the option price at t_0 is high due to positive noise, the subsequent return is low but receives a high weight; conversely, when the price is low, the subsequent return is high, but the weight is low. As a consequence, high returns are down-weighted and low returns are up-weighted, ensuring that the average weighted return is adjusted downward. This mechanism effectively mitigates the direct mean return bias arising from errors in the option price (O-DMR bias).

It is important to note that this procedure does not influence the S-DMR bias or the indirect component of the mean return bias. For daily static returns, this correction is sufficient because these components, especially the IMR bias, are negligible (Duarte et al., 2024). However, as we have demonstrated, the impact of the IMR bias becomes significantly more pronounced when considering higher hedge frequencies for monthly returns.

To assess the magnitude and relevance of the remaining bias, we apply the lagged gross-return weighting to our asset pricing tests on stock liquidity. Panel (b) of Table 4 presents the results for the portfolio sort and Panel (b) of Table 5 presents the regression results.

After applying the weighting, average portfolio returns decrease, while the return spread and statistical significance remain largely unchanged. Regression coefficients for intraday returns also remain highly significant. This weighting corrects only the first component of the DMR bias (essentially the bias in raw option returns) and has limited effect. In particular, the bias adjustment is independent of hedge frequency and stock illiquidity, which explains why the asset pricing tests continue to produce unreliable results.

To address the spurious premiums caused by the indirect bias, we propose a simple correction that can be easily implemented by researchers and practitioners. Instead of using the contemporaneous delta at time t_n multiplied by the stock return from t_n to t_{n+1}, we use the lagged delta from the previous time step. This approach breaks the correlation between the hedge ratio and the subsequent stock return that is induced by microstructure noise. The lag length depends on the available data: for intraday data, we use a lag of 30 minutes or one trade; for end-of-day data used in the daily, weekly, bi-weekly, and static rebalancing measures, we use a lag of one day. This procedure ensures independence between microstructure noise in the stock return and in the hedge ratio, thereby mitigating the hedge-frequency-dependent component of the MR bias, primarily the IMR bias. An analytical justification of this approach is provided in Appendix B.

Panels (c) of Tables 4 and 5 report the results of this correction. After applying the adjustment, the high-minus-low returns are close to zero and significant in only about 5% of samples across all measures. Similarly, in the regression analysis, the average coefficients are close to zero and statistically significantly in only about 5-6% of simulation trials, corresponding to the size of the test.

4.4 Identifying Actual Return Premiums

Up to this point, all observed return premiums were spurious, arising solely from measurement errors in the data that induced return biases. In the following, we introduce genuine premiums into the true option prices.

4.4.1 Hedging Costs

Instead of generating true option prices that follow the Black and Scholes (1973) model and, therefore, contain no premium related to illiquidity of the underlying, we now incorporate such a premium directly into the true option prices. Leland (1985) provides a classic model of option hedging with transaction costs, offering a straightforward method to generate these option prices while consistently reflecting cross-sectional differences in the illiquidity of the underlying stocks. He derives an expression for an option's implied variance in the presence of costs associated with trading the underlying (see Leland, 1985, Equation (13)):

$$\widehat{\sigma}_i^2 = \sigma_i^2 \cdot \left(1 - \frac{BA_S^i}{\sigma_i}\sqrt{\frac{2}{\pi \cdot 1/264}}\,\mathrm{sign}(p_i)\right) \tag{12}$$

Here, σ_i and BA_S^i denote the Black and Scholes (1973) implied volatility and relative bid-ask spread of stock i, respectively. A positive value of p_i represents a long position in the option, while a negative value represents a short position.[18] Interpreting p_i as the net position of the representative market maker in the option on stock i, the implied volatility of the option in equation (12) reflects the cost the market maker incurs while hedging her option position on a daily basis.

In line with demand-based option pricing theory and empirical evidence (Gârleanu et al., 2009; Fournier and Jacobs, 2020; Kanne et al., 2023), when a market maker holds a net short position due to positive demand, option prices, as expressed in equation (12), are positively related to the bid-ask spread of the underlying security. This relation implies a negative premium in subsequent option returns associated with the underlying's liquidity. Conversely, when the market maker holds a net long position, higher bid-ask spreads lead to higher option returns.

We use (12) to compute true option prices and simulate two sets of option prices based on random market maker positions. In the first set, the market maker is long with a probability

[18] Leland (1985) analyzes only the case of a long position in a single option. For a generalization, see Hoggard et al. (1994).

of 25% for each option, whereas in the second set this probability increases to 75%. These assumptions imply that, on average, the true option prices in the first setting embed a negative premium for illiquidity, while in the second setting they contain a positive one.

Following standard asset pricing methodology, we apply portfolio sorts to assess how well our delta-hedged option return measures capture the true illiquidity premium. Table 6 shows results for the first scenario, where the market maker is long in 25% of the simulated options. Panel (a) reports the unadjusted results starting with the error-free benchmark. In contrast to the previous analyses based on Black and Scholes (1973) prices, the *true* return in continuous time is no longer zero, as the option cannot be perfectly replicated using only the stock and a money market account. In this scenario, the mean high-minus-low portfolio return equals -83 bp per month and is significantly different from zero in almost all samples. Ideally, all other measures should align with this benchmark.

For the error-affected measures, the average high-minus-low returns for portfolios sorted by stock bid-ask spread approximately match the benchmark when the hedging frequency is weekly or lower, although they are often not significantly different from zero. With daily or intraday rebalancing, however, the mean return increases with hedge frequency and even turns positive for the intraday measures. This pattern arises from the indirect bias discussed in Section 4.2 and Table 4. As hedge frequency increases, this bias grows, leading to an upward distortion that is positively correlated with the stock bid-ask spread, thereby masking the true negative relationship between returns and hedging costs. For a rebalancing frequency of every 30 minutes, the average return premium for stock illiquidity is 2.3% per month and statistically significant across all simulated samples.

As in our previous analyses, we apply corrections for microstructure noise in two steps. Panel (b) of Table 6 presents portfolio sorts with return weights proportional to the one-day lagged gross returns of the options. The results exhibit similar patterns to the unadjusted case. Notably, the high-frequency measures still exhibit the incorrect signs for the high-minus-low returns.

In Panel (c), we apply our delta-lagging method to mitigate the hedge-frequency-dependent bias components, particularly the IMR bias. As a result, the sizes of the return premiums now closely align with the benchmark across all measures. However, this improvement comes at the cost of increased variability in measured returns. Specifically, the standard errors for all measures with at most daily rebalancing increase slightly relative to Panel (b), owing to the timing mismatch between the lagged deltas used in the hedging strategy and the corresponding stock returns.

In the second scenario, we consider a different benchmark return. Here, market makers are net long in 75% of the simulated options, resulting in a positive true premium for the illiquidity of the underlying stock. Consequently, the true return premium and the IMR bias move in the same direction. Panel (a) of Table 7 shows that the high-minus-low returns of all measures display the correct sign relative to the benchmark. However, the magnitude of the premium differs substantially across rebalancing frequencies. While the returns based on end-of-day data are closest to the benchmark, the intraday measures strongly overestimate the effect. The same pattern as in the previous analysis holds for statistical significance: low-frequency measures exhibit greater volatility and hence lower significance, whereas the *30min* measure yields significant results in all samples, and the *Static* measure detects the correct premium in only 55.8% of simulation trials.

When applying the bias adjustments, Panels (b) and (c) of Table 7 again show that the lagged gross-return weighting alone does not effectively correct for the biased mean returns in the high-frequency measures. By contrast, our delta-lagging approach eliminates the upward bias, bringing all measures much closer to the benchmark on average.

4.4.2 Volatility Risk

The question of whether volatility risk is priced in stock options remains a central topic in the empirical options literature. Duarte et al. (2024) show that microstructure biases can substantially distort statistical inference on volatility risk premiums and demonstrate

Table 6: **Returns of option portfolios sorted by the bid-ask spread of the underlying in the model of Leland (1985) (market maker long in 25% of all options)**

This table shows average monthly returns of delta-hedged at-the-money call portfolios sorted by the bid-ask spread of the underlying. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-9.72	-19.39	-29.99	-46.50	-92.67	-82.95	-19.97	0	99.9
30min	4.30	0.40	1.11	13.35	229.84	225.53	17.94	100	0
Tradetimes	1.73	-5.60	-11.41	-14.80	55.58	53.85	4.70	88.3	0
DailyAVG	-86.95	-96.99	-104.69	-119.18	-71.60	15.35	-1.99	0.1	49.5
Daily	3.51	-6.31	-15.77	-30.23	-58.40	-61.91	-4.33	0.1	96.6
Weekly	6.66	-3.33	-14.24	-30.08	-73.58	-80.24	-2.76	0	75.4
Bi-Weekly	8.64	-1.59	-13.13	-28.43	-73.61	-82.25	-2.05	0	51.9
Static	13.45	6.50	-7.10	-22.66	-68.14	-81.59	-1.45	0	30

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-9.72	-19.39	-29.99	-46.50	-92.67	-82.95	-19.97	0	99.9
30min	-7.32	-10.89	-9.66	3.10	218.13	225.45	18.96	100	0
Tradetimes	-8.32	-15.40	-20.47	-23.43	47.30	55.62	5.26	92.4	0
DailyAVG	-18.58	-28.42	-35.40	-48.86	-72.32	-53.74	-1.98	0	49
Daily	-8.15	-17.57	-26.55	-40.37	-67.58	-59.44	-4.43	0	97.4
Weekly	-5.11	-14.51	-24.91	-40.39	-82.78	-77.68	-2.86	0	77.8
Bi-Weekly	-3.32	-12.71	-24.13	-38.82	-83.14	-79.82	-2.12	0	55
Static	1.34	-4.91	-18.29	-33.58	-77.81	-79.15	-1.50	0	31.7

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-9.72	-19.39	-29.99	-46.50	-92.67	-82.95	-19.97	0	99.9
30min	-9.19	-18.40	-28.64	-44.29	-89.75	-80.56	-9.47	0	100
Tradetimes	-9.45	-18.67	-29.10	-44.41	-89.21	-79.77	-7.23	0	100
DailyAVG	-17.96	-27.80	-35.52	-51.07	-91.59	-73.63	-2.36	0	65
Daily	-7.71	-16.93	-26.86	-42.66	-88.57	-80.86	-3.77	0	95.2
Weekly	-4.68	-13.63	-24.29	-40.83	-86.63	-81.95	-2.58	0	69.4
Bi-Weekly	-2.91	-11.85	-24.18	-38.46	-84.75	-81.84	-2.00	0	50.5
Static	0.44	-5.85	-19.26	-33.92	-79.28	-79.72	-1.46	0	29.7

Table 7: **Returns of option portfolios sorted by the bid-ask spread of the underlying in the model of Leland (1985) (market maker long in 75% of all options)**

This table shows average monthly returns of delta-hedged at-the-money call portfolios sorted by the bid-ask spread of the underlying. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	10.55	20.61	33.02	53.23	138.65	128.10	28.32	100	0
30min	24.61	40.59	64.76	115.46	538.17	513.56	35.62	100	0
Tradetimes	21.91	34.27	51.55	85.44	320.06	298.16	25.49	100	0
DailyAVG	-67.98	-59.52	-45.62	-25.66	151.05	219.04	4.82	99	0
Daily	23.82	33.75	47.37	69.79	178.63	154.81	10.21	100	0
Weekly	26.99	36.77	48.93	69.94	159.22	132.23	4.38	98.1	0
Bi-Weekly	28.99	38.55	50.11	71.65	158.86	129.87	3.12	82.5	0
Static	33.84	46.80	56.31	77.73	164.78	130.94	2.23	55.8	0

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	10.55	20.61	33.02	53.23	138.65	128.10	28.32	100	0
30min	12.13	27.61	51.31	100.96	512.32	500.18	37.49	100	0
Tradetimes	10.94	22.65	39.62	72.27	298.95	288.00	26.77	100	0
DailyAVG	-1.06	6.16	19.11	37.45	131.70	132.76	4.75	98.8	0
Daily	11.30	20.81	33.94	55.43	158.85	147.56	10.55	100	0
Weekly	14.36	23.90	35.58	55.37	139.63	125.27	4.47	98.3	0
Bi-Weekly	16.16	25.73	36.41	57.00	138.86	122.70	3.18	84	0
Static	20.85	33.67	42.40	62.49	144.55	123.70	2.27	57.8	0

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	10.55	20.61	33.02	53.23	138.65	128.10	28.32	100	0
30min	10.24	19.96	31.78	51.32	131.50	121.26	13.57	100	0
Tradetimes	9.80	19.33	30.75	50.26	129.74	119.94	10.41	100	0
DailyAVG	-0.43	6.79	18.96	35.12	107.77	108.20	3.37	89.3	0
Daily	11.74	21.44	33.58	53.00	132.68	120.94	5.47	100	0
Weekly	14.79	24.78	36.20	54.91	134.75	119.96	3.67	92.7	0
Bi-Weekly	16.56	26.59	36.33	57.33	136.76	120.19	2.86	77.4	0
Static	19.94	32.70	41.39	62.08	142.68	122.73	2.17	56.4	0

that, once these biases are accounted for, volatility risk is negatively priced in equity options. Extending this discussion, we examine the estimation of volatility risk premiums using monthly delta-hedged returns under different hedge frequencies, emphasizing once again the importance of the indirect component of the mean-return and regression-coefficient biases.

We consider a setting in which true option prices are generated from the stochastic volatility model of Heston (1993). This model features a stochastic mean-reversion process for the instantaneous variance of stock returns in addition to the geometric Brownian motion for the stock price. We adopt the parameterization of Duarte et al. (2024) for the dynamics of the stock price and instantaneous variance under the physical probability measure

$$\begin{aligned} dS_t &= (r + \lambda_1 V_t) S_t \, dt + \sqrt{V_t} S_t \, dB^{\mathbb{P}}_{1,t}, \\ dV_t &= \kappa(\theta - V_t)\, dt + \sigma \sqrt{V_t} \left(\rho \, dB^{\mathbb{P}}_{1,t} + \sqrt{1-\rho^2}\, dB^{\mathbb{P}}_{2,t} \right). \end{aligned} \tag{13}$$

In this framework, the instantaneous delta-hedged option returns can be written as

$$\mathbb{E}_t\left[\frac{dC_t}{C_t}\right] - \left(\Delta_t \frac{S_t}{C_t}\right) \cdot \mathbb{E}_t\left[\frac{dS_t}{S_t}\right] = \mathbb{E}_t\left[\frac{\nu_t}{C_t}\lambda^{\sigma}_t \, dt\right], \tag{14}$$

where ν_t denotes the option vega (the sensitivity of the option price to volatility) and λ^{σ}_t is the time-dependent volatility risk premium, which is linear in $\sqrt{V_t}$. Motivated by equation (14), we estimate the sign of the market price of volatility risk by sorting delta-hedged option portfolios on the ratio $\frac{\nu_t \sqrt{V_t}}{C_t}$, which we refer to as *vola-elasticity*.[19] We simulate option prices using the parameter values of Duarte et al. (2024). Details of the simulation are provided in Appendix C, and the theoretical model is described in Appendix F.

Following our approach from the previous section, we use a sample of liquid at-the-money options to estimate the sign of the volatility risk premium via portfolio sorts. Two adjustments are required relative to our earlier simulations. First, to account for the *correlated-errors-in-variables (CEIV) bias* that arises when measurement errors in prices affect both the

[19] In our analysis, this variable is computed from noisy values. We use the Black and Scholes (1973)-implied volatility to approximate $\sqrt{V_t}$ and compute the vega with the Black-Scholes formula.

dependent and independent variables, we lag the independent variable by one day relative to the computation of the delta-hedged return, as suggested by Duarte et al. (2024). Second, to ensure sufficient dispersion of the vola-elasticity, we simulate a broader strike range by drawing the initial option moneyness, $\frac{K}{S}$, from the interval $(0.85, 1.15)$.

Table 8 presents the results for call options. Panel (a) reports the unadjusted results, beginning with the error-free benchmark. As vola-elasticity increases, mean returns decline. Specifically, the L-portfolios yield an average return of roughly -80 basis points per month, while the H-portfolios average -861 basis points per month. This monotonic pattern is consistent with the chosen parametrization that implies a negative volatility risk premium.

For the error-affected measures, all individual portfolios, except those based on the *DailyAVG* measure[20], exhibit an upward bias in mean returns compared to the benchmark. The bias varies across portfolios, causing distortions in the H-L portfolios, and the magnitude of the bias increases with hedge frequency. The pattern suggests that the frequency-dependent IMR bias is a primary driver of this effect.

We apply corrections for microstructure biases in Panel (b) of Table 8, where we use lagged gross-return weighting to address the O-DMR bias, following Duarte et al. (2024). This adjustment reduces mean returns across all measures and portfolio ranks. The downward correction is frequency independent, averaging about 2 basis points for the low-rank portfolios (L-portfolios) and increases gradually to around 34 basis points for the high-rank portfolios (H-portfolios). The portfolio-rank dependence arises because, in our simulation model, option bid-ask spreads are positively related to the option's moneyness (see Appendix D). Consequently, the vega-elasticity correlates with the option's bid-ask spread, and the positive O-DMR bias naturally increases with portfolio rank. As a result, this correction mitigates part of the bias in H-L returns.

Applying the delta-lagging correction in Panel (c) to address the IMR bias produces an additional downward adjustment that brings all portfolio returns including the high-

[20]We discussed the source of the downward bias in *DailyAVG* earlier, hence, we omit it here and focus on the remaining measures.

Table 8: **Option returns sorted by vola-elasticity in the model of Heston (1993)**

This table shows average monthly returns of delta-hedged at-the-money call portfolios sorted by their vola-elasticity. Option moneyness, $\frac{K}{S}$, lies between 0.85 and 1.15. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-79.45	-168.62	-304.80	-533.30	-860.85	-781.40	-4.33	0	98.3
30min	-5.50	-88.97	-212.39	-429.03	-710.69	-705.19	-3.91	0.1	95.5
Tradetimes	-43.55	-129.35	-258.09	-477.36	-778.37	-734.81	-4.02	0	97
DailyAVG	-101.37	-202.82	-352.52	-599.54	-930.91	-829.54	-3.53	0	90.3
Daily	-71.42	-158.32	-289.38	-509.40	-803.99	-732.57	-3.55	0	92.3
Weekly	-75.33	-162.86	-293.27	-514.71	-809.42	-734.10	-2.59	0	66.5
Bi-Weekly	-77.33	-165.67	-296.37	-520.76	-818.28	-740.95	-2.13	0	52.3
Static	-76.48	-163.60	-295.02	-519.72	-820.57	-744.09	-1.71	0	37.8

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-79.45	-168.62	-304.80	-533.30	-860.85	-781.40	-4.33	0	98.3
30min	-7.43	-91.16	-215.64	-435.50	-744.05	-736.61	-4.22	0.1	97.5
Tradetimes	-45.00	-130.59	-259.10	-478.76	-792.18	-747.18	-4.24	0	97.8
DailyAVG	-83.72	-172.53	-305.26	-527.20	-840.41	-756.69	-3.43	0	88.6
Daily	-73.13	-160.37	-292.57	-515.81	-836.74	-763.60	-3.82	0	96
Weekly	-77.00	-164.88	-296.52	-521.16	-843.10	-766.09	-2.79	0	73.8
Bi-Weekly	-78.95	-167.55	-299.54	-527.14	-851.80	-772.85	-2.29	0	57.9
Static	-78.09	-165.57	-298.25	-526.60	-855.24	-777.15	-1.83	0	42.6

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-79.45	-168.62	-304.80	-533.30	-860.85	-781.40	-4.33	0	98.3
30min	-77.85	-164.93	-296.98	-518.52	-837.69	-759.84	-4.22	0	97.8
Tradetimes	-77.51	-164.13	-295.34	-515.44	-831.78	-754.26	-4.09	0	97.1
DailyAVG	-88.00	-177.02	-309.35	-531.72	-845.03	-757.03	-2.96	0	79.8
Daily	-77.84	-165.14	-296.81	-520.59	-841.39	-763.54	-3.14	0	84.3
Weekly	-78.71	-166.74	-298.96	-526.90	-850.73	-772.03	-2.49	0	64.4
Bi-Weekly	-80.21	-169.37	-303.39	-535.43	-864.46	-784.26	-2.13	0	52.1
Static	-79.97	-168.27	-304.52	-541.53	-876.46	-796.50	-1.75	0	38.6

minus-low returns close to the benchmark. This refinement is particularly important for the high-frequency measures, where the IMR bias is especially pronounced.

An important observation from this analysis is that each measure, whether corrected or not, accurately identifies the sign of the modeled volatility risk premium. Low-frequency measures display weaker statistical significance, with only 37-74% of the samples showing significant results. High-frequency measures, by contrast, identify a significant negative volatility risk premium in more than 95% of simulation trials. This evidence indicates that although microstructure biases noticeably affect mean portfolio returns, they do not materially distort the inference about the sign of the volatility risk premium. A key reason for this robustness is the weak correlation between vola-elasticity and the total MR bias, which allows for a reliable estimation of the volatility risk premium across different measures.

However, this robustness result should not lead to false confidence. The limited impact of the IMR bias in this setting arises because the factors driving the bias are largely independent of the sorting variable used in the previous analysis. This highlights a general principle: the IMR bias remains negligible when it is weakly correlated with the sorting variable but can substantially distort results when such dependence exists.

To illustrate this point, we extend our simulation framework to introduce a correlation between stock volatility and bid-ask spreads, which are modeled as independent in Duarte et al. (2024). Empirical evidence supports such a relationship (e.g., Hou and Loh, 2016). Specifically, we model the stock bid-ask spread, which governs microstructure noise, as a linear function of stock volatility:

$$BA^i_{S,t} = a + b \cdot \sqrt{V^i_t}. \tag{15}$$

The parameters a and b are calibrated such that the mean and variance of the stock bid-ask spread match those in the independent baseline scenario described in Appendix C.

Imposing (15) generates different relationships between the stock bid-ask spread and vola-elasticity depending on option moneyness. Figure 2 shows how volatility affects vola-elasticity across moneyness. For in-the-money (ITM) calls ($\frac{K}{S} < 1$), higher volatility increases vola-elasticity, while for out-of-the-money (OTM) calls ($\frac{K}{S} > 1$), it decreases it. Because of the positive relation of the bid-ask spread and volatility in (15), vola-elasticity rises with stock illiquidity for ITM options but falls for OTM options. This implies that both the magnitude and direction of the MR bias, particularly its indirect component, vary across option categories. With a negative volatility risk premium, ITM options are most affected; the negative relationship between vola-elasticity and expected option return is partially offset by the positive link to the indirect mean return bias. To quantify these effects, we extend our analysis beyond ATM options to include ITM options as well. We use a strike range where $\frac{K}{S}$ varies from 0.7-0.85 for ITM calls (1.15-1.3 for ITM puts, see Appendix G).[21]

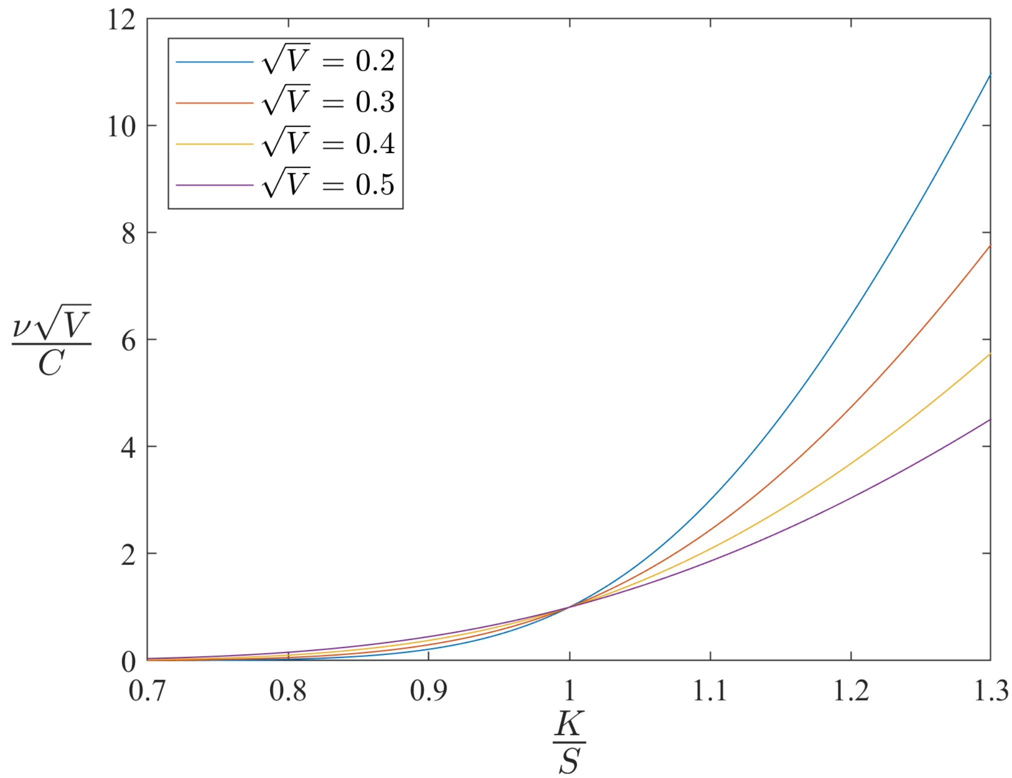
In contrast to the base scenario in Table 8, where stock illiquidity was assumed to be independent of volatility, Table 9 reports results under a positive illiquidity-volatility relationship as specified in (15). For ATM options, all measures correctly identify the sign of the volatility risk premium, consistent with our base scenario, and the bias in the high-minus-low return remains small relative to the effect size. As illustrated in Figure 2, the relationship between the IMR bias and the sorting variable is ambiguous within our ATM strike range from 0.85 to 1.15. We find an upward bias that roughly increases with hedge frequency but is weaker for high-frequency measures than in Table 8, indicating a slightly stronger contribution from options with $\frac{K}{S} < 1$. However, the microstructure bias adjustments, lagged gross-return weighting in Panel (b) and additional delta-lagging in Panel (c), again effectively mitigate this distortion.

For ITM delta-hedged call options, the results in Table 9 tell a different story. Here, the IMR bias is positively correlated with the sorting variable. In this case, high-frequency measures either misidentify the sign of the volatility risk premium (*30min*) or yield weak

[21]In unreported results, we also examine OTM options. They show that OTM calls alone provide a noisy estimate of the volatility risk premium. Even the benchmark exhibits weak statistical significance highlighting the inherently high return variance of OTM options. As expected, the impact of the IMR bias in this case does not lead to a wrongly estimated sign of the VRP.

Figure 2: **Vola-elasticity for varying option moneyness**

This figure shows the relationship of *vola-elasticity*, $\frac{\nu\sqrt{V}}{C}$, to option moneyness for call options in the model of Heston (1993). We use the parametrization described Appendix C and different levels for stock volatility and do not include microstructure noise. To approximate the true volatility and the option vega, we use the formulas from the Black and Scholes (1973) model.



significance for a negative premium (*Tradetimes*). In line with our earlier results for stock illiquidity in the model of Leland (1985), lagged gross-return weighting does not sufficiently adjust for this bias. By contrast, the delta-lagging correction successfully removes it, allowing all measures to recover the correct sign and magnitude comparable to the benchmark.

5 Conclusion

When computing delta-hedged option returns, the choice of rebalancing frequency is critical when data is observed with errors. Although high-frequency data allows for more accurate hedge updates, microstructure noise induces a substantial mean-return bias. Using a controlled simulation environment, we show that the *indirect* component of this bias, which was introduced by Duarte et al. (2024) and is due to spurious correlations between hedge ratios and stock returns, increases with hedge frequency and dominates the direct component. Because this IMR bias is primarily driven by the liquidity of the underlying asset, we use portfolio sorts and cross-sectional regressions to test for return premiums associated with this variable. Our findings indicate that high-frequency measures are particularly biased toward identifying such premiums.

To address the specific challenges of delta-hedged option returns affected by microstructure noise, we propose a simple yet effective correction: lagging the delta by one time step relative to the underlying returns. This adjustment effectively reduces the spurious statistical significance previously observed in analyses, providing a clearer understanding of the true relationships at play.

We leverage the model of Leland (1985), which embeds an intrinsic option-return premium associated with the illiquidity of the underlying asset. Our findings reveal that the IMR bias leads to significant deviations in illiquidity premiums compared to a benchmark free of errors. These deviations worsen at higher hedge frequencies. While low-frequency measures provide the most accurate premiums, they often lack statistical significance, par-

Table 9: **VRP estimation for different moneyness categories**

This table shows the results of univariate portfolio sorts of delta-hedged calls on the vola-elasticity for different moneyness categories. Ranges for option moneyness, $\frac{K}{S}$, are (0.85,1.15) for at-the-money (ATM) and (0.7,0.85) for in-the-money (ITM) options. Stock bid-ask spreads are simulated according to (15). Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	ATM				ITM			
	H-L	Avg. t	*(+)	*(-)	H-L	Avg. t	*(+)	*(-)
Benchmark	-781.38	-4.33	0	98.3	-38.83	-5.84	0	100
30min	-730.53	-4.02	0	97.1	12.10	1.52	33.8	0.1
Tradetimes	-743.11	-4.06	0	97.3	-14.91	-1.98	0	48.9
DailyAVG	-831.22	-3.54	0	90.3	-48.71	-1.87	0	44.2
Daily	-734.23	-3.55	0	92.8	-35.10	-3.94	0	95.9
Weekly	-734.30	-2.59	0	66.7	-37.69	-2.74	0	72
Bi-Weekly	-740.90	-2.13	0	52.1	-38.53	-2.23	0	55.1
Static	-743.97	-1.71	0	38	-37.70	-1.72	0	38.5

(b) O-DMR-bias adjusted

	H-L	Avg. t	*(+)	*(-)	H-L	Avg. t	*(+)	*(-)
Benchmark	-781.38	-4.33	0	98.3	-38.83	-5.84	0	100
30min	-761.93	-4.34	0	98.4	11.68	1.49	33.5	0.1
Tradetimes	-740.82	-4.06	0	97.3	-14.83	-1.97	0	48.7
DailyAVG	-758.37	-3.43	0	88.8	-39.63	-1.56	0	33.8
Daily	-765.30	-3.83	0	96	-35.29	-4.04	0	96.6
Weekly	-766.34	-2.79	0	73.8	-37.87	-2.81	0	73.4
Bi-Weekly	-772.88	-2.29	0	57.8	-38.67	-2.29	0	56.8
Static	-777.18	-1.83	0	42.4	-37.83	-1.76	0	40.2

(c) DMR- and IMR-bias adjusted

	H-L	Avg. t	*(+)	*(-)	H-L	Avg. t	*(+)	*(-)
Benchmark	-781.38	-4.33	0	98.3	-38.83	-5.84	0	100
30min	-759.86	-4.22	0	97.8	-38.73	-5.26	0	99.7
Tradetimes	-740.62	-3.88	0	95.8	-38.51	-4.93	0	99.7
DailyAVG	-757.18	-2.96	0	80.2	-42.53	-1.61	0	34.6
Daily	-763.63	-3.14	0	84.4	-38.14	-3.27	0	86.6
Weekly	-772.07	-2.49	0	64.3	-36.76	-2.33	0	59.6
Bi-Weekly	-784.18	-2.13	0	52.3	-35.66	-1.85	0	43.1
Static	-796.50	-1.75	0	38.6	-30.74	-1.27	0	26.1

ticularly in bi-weekly and static rebalancing scenarios. After applying our adjustments for microstructure noise, estimation errors are significantly reduced across all measures. This makes high-frequency measures particularly compelling, as they become more reliable and capable of yielding robust, statistically significant results.

Extending the analysis to the Heston (1993) model, we estimate the volatility risk premium via portfolio sorts. For at-the-money options, the bias remains minor due to a weak correlation between the sorting variable and microstructure errors. However, when volatility and bid-ask spreads are endogenously linked, the bias becomes pronounced, particularly for in-the-money options, where high-frequency hedging can even invert the estimated sign. The delta-lagging adjustment effectively corrects the bias, restoring both the sign and magnitude of the volatility risk premium.

Overall, our findings underscore the importance of accounting for microstructure-induced correlations in delta-hedged return estimation. We recommend implementing delta-lagging whenever high-frequency rebalancing is used, particularly in studies sorting or regressing on illiquidity-related characteristics. This adjustment ensures more accurate identification of return premia and enhances the reliability of empirical asset pricing tests based on option returns.

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A MR Bias Approximation Formula

As in Section 2, we consider delta-hedged option returns calculated from time t_0 to T with $t_0 < T$. Rebalancing of the position in the underlying takes place discretely at $t_1 < \cdots < t_{N-1}$ with $t_0 < t_1$, and the return is realized at $t_N = T$ with $t_{N-1} < t_N$. $\widetilde{C}_t$, $\widetilde{S}_t$ are the *true* (unobserved) values of the option and stock prices at time t. The observed counterparts are given by $C_t = \widetilde{C}_t \cdot (1 + \varepsilon_{C,t})$ and $S_t = \widetilde{S}_t \cdot (1 + \varepsilon_{S,t})$. The delta of the option implied by the given option and stock prices is denoted by $\Delta(S, C)$. For brevity, we write $\widetilde{\Delta}_t \coloneqq \Delta(\widetilde{S}_t, \widetilde{C}_t)$ and $\Delta_t \coloneqq \Delta(S_t, C_t)$.

Following Duarte et al. (2024), we assume that the risk-free rate is zero. Therefore, the observed delta-hedged return can be expressed as

$$\pi(t_0, T) \quad = \quad \frac{C_T - C_{t_0}}{C_{t_0}} \quad - \quad \sum_{n=0}^{N-1} \quad \left(\Delta_{t_n} \frac{S_{t_n}}{C_{t_0}} \right) \quad \frac{S_{t_{n+1}} - S_{t_n}}{S_{t_n}}.$$

Accordingly, we denote the return based on error-free values as $\widetilde{\pi}(t_0, T)$.

For the derivation of the approximation formula, we rely on the same assumptions as Duarte et al. (2024):

1. Price errors have a mean of zero.

2. Price errors are independent over time and between different securities.

3. Price errors are independent of true prices.

4. $\widetilde{C}_t - \widetilde{C}_{t+\tau} \to 0$ and $\widetilde{S}_t - \widetilde{S}_{t+\tau} \to 0$ as $\tau \to 0$.

Step 1 - Return of the option position (see Blume and Stambaugh, 1983):

$$\begin{aligned}
\mathbb{E}\left[\frac{C_T - C_{t_0}}{C_{t_0}}\right] &= \mathbb{E}\left[\frac{\widetilde{C}_T(1+\varepsilon_{C,T}) - \widetilde{C}_{t_0}(1+\varepsilon_{C,t_0})}{\widetilde{C}_{t_0}(1+\varepsilon_{C,t_0})}\right] \\
&= \mathbb{E}\left[\frac{\widetilde{C}_T}{\widetilde{C}_{t_0}}\frac{(1+\varepsilon_{C,T})}{(1+\varepsilon_{C,t_0})} - 1\right] \\
&= \mathbb{E}\left[\left(\frac{\widetilde{C}_T - \widetilde{C}_{t_0}}{\widetilde{C}_{t_0}} + 1\right)\frac{1+\varepsilon_{C,T}}{1+\varepsilon_{C,t_0}} - 1\right] \\
&= \left(\mathbb{E}\left[\frac{\widetilde{C}_T - \widetilde{C}_{t_0}}{\widetilde{C}_{t_0}}\right] + 1\right)\mathbb{E}\left[\frac{1}{1+\varepsilon_{C,t_0}}\right] - 1 \\
&\approx \left(\mathbb{E}\left[\frac{\widetilde{C}_T - \widetilde{C}_{t_0}}{\widetilde{C}_{t_0}}\right] + 1\right)\left(1 + \mathbb{E}\left[\varepsilon^2_{C,t_0}\right]\right) - 1 \\
&= \mathbb{E}\left[\frac{\widetilde{C}_T - \widetilde{C}_{t_0}}{\widetilde{C}_{t_0}}\right]\left(1 + \mathbb{E}\left[\varepsilon^2_{C,t_0}\right]\right) + \mathbb{E}\left[\varepsilon^2_{C,t_0}\right] \\
&\approx \mathbb{E}\left[\frac{\widetilde{C}_T - \widetilde{C}_{t_0}}{\widetilde{C}_{t_0}}\right] + \mathbb{E}\left[\varepsilon^2_{C,t_0}\right]
\end{aligned}$$

The fourth equality holds because error terms are independent of true prices and across time and all errors have an expectation of zero. The approximation following this is obtained from a second-order Taylor approximation of $\frac{1}{1+x}$ around zero:

$$\frac{1}{1+x} \approx 1 - x + x^2$$

By omitting the product of the expected return and the variance of the measurement error (following Blume and Stambaugh (1983)), we obtain the final approximation.

Step 2 - Return of the position in the underlying (based on Duarte et al., 2024): We proceed in the same way as Duarte et al. (2024) to derive the approximation for the error-affected return of the position in the underlying but need to distinguish between two different cases because we are considering general hedge frequencies. This distinction arises from the fact that the *hedge ratio* at time t_0, denoted as $\beta^C_{S,t_0} \coloneqq \Delta(S_{t_0}, C_{t_0})\frac{S_{t_0}}{C_{t_0}}$, is calculated based on the stock price and option price at t_0 alone. In contrast, the hedge ratio at a later time $t_n > t_0$, denoted as $\beta^C_{S,t_n} \coloneqq \Delta(S_{t_n}, C_{t_n})\frac{S_{t_n}}{C_{t_0}}$, depends not only on the stock and option prices at t_n, but also on the option price at time t_0.

Let $n = 0$ and define

$$H_0(S_{t_0}, C_{t_0}, S_{t_1}) \coloneqq \beta^C_{S,t_0} \cdot \frac{S_{t_1} - S_{t_0}}{S_{t_0}}$$

A second-order Taylor approximation yields:

$$\begin{aligned}
H_0(S_{t_0}, C_{t_0}, S_{t_1}) \quad \approx \quad & H_0(\widetilde{S}_{t_0}, \widetilde{C}_{t_0}, \widetilde{S}_{t_1}) \\
+ \quad & \frac{\partial H_0}{\partial S_{t_0}} \widetilde{S}_{t_0} \varepsilon_{S,t_0} + \frac{\partial H_0}{\partial C_{t_0}} \widetilde{C}_{t_0} \varepsilon_{C,t_0} + \frac{\partial H_0}{\partial S_{t_1}} \widetilde{S}_{t_1} \varepsilon_{S,t_1} \\
+ \quad & \frac{\partial^2 H_0}{\partial S_{t_0} \partial C_{t_0}} \widetilde{S}_{t_0} \varepsilon_{S,t_0} \widetilde{C}_{t_0} \varepsilon_{C,t_0} \\
+ \quad & \frac{\partial^2 H_0}{\partial S_{t_0} \partial S_{t_1}} \widetilde{S}_{t_0} \varepsilon_{S,t_0} \widetilde{S}_{t_1} \varepsilon_{S,t_1} \\
+ \quad & \frac{\partial^2 H_0}{\partial C_{t_0} \partial S_{t_1}} \widetilde{C}_{t_0} \varepsilon_{C,t_0} \widetilde{S}_{t_1} \varepsilon_{S,t_1} \\
+ \quad & \frac{1}{2} \frac{\partial^2 H_0}{(\partial S_{t_0})^2} \widetilde{S}^2_{t_0} \varepsilon^2_{S,t_0} + \frac{1}{2} \frac{\partial^2 H_0}{(\partial C_{t_0})^2} \widetilde{C}^2_{t_0} \varepsilon^2_{C,t_0} + \frac{1}{2} \frac{\partial^2 H_0}{(\partial S_{t_1})^2} \widetilde{S}^2_{t_1} \varepsilon^2_{S,t_1}
\end{aligned}$$

By taking expectations, most of the terms vanish because all errors have a mean of zero by assumption:

$$\begin{aligned}
\mathbb{E}\left[H_0(S_{t_0}, C_{t_0}, S_{t_1})\right] \quad \approx \quad & \mathbb{E}\left[H_0(\widetilde{S}_{t_0}, \widetilde{C}_{t_0}, \widetilde{S}_{t_1})\right] \\
+ \quad & \mathbb{E}\left[\frac{1}{2} \frac{\partial^2 H_0}{(\partial S_{t_0})^2} \widetilde{S}^2_{t_0}\right] \mathbb{E}\left[\varepsilon^2_{S,t_0}\right] \\
+ \quad & \mathbb{E}\left[\frac{1}{2} \frac{\partial^2 H_0}{(\partial C_{t_0})^2} \widetilde{C}^2_{t_0}\right] \mathbb{E}\left[\varepsilon^2_{C,t_0}\right] \\
+ \quad & \mathbb{E}\left[\frac{1}{2} \frac{\partial^2 H_0}{(\partial S_{t_1})^2} \widetilde{S}^2_{t_1}\right] \mathbb{E}\left[\varepsilon^2_{S,t_1}\right]
\end{aligned}$$

The partial derivatives evaluate to

$$\begin{aligned}
\frac{\partial^2 H_0}{(\partial S_{t_0})^2} &= \frac{\partial^2 \beta^C_{S,0}}{(\partial S_{t_0})^2}\frac{S_{t_1}-S_{t_0}}{S_{t_0}} - 2\frac{\partial \beta^C_{S,0}}{\partial S_{t_0}}\frac{S_{t_1}}{S^2_{t_0}} + 2\beta^C_{S,0}\frac{S_{t_1}}{S^3_{t_0}}, \\
\frac{\partial^2 H_0}{(\partial C_{t_0})^2} &= \frac{\partial^2 \beta^C_{S,0}}{(\partial C_{t_0})^2}\frac{S_{t_1}-S_{t_0}}{S_{t_0}}, \\
\frac{\partial^2 H_0}{(\partial S_{t_1})^2} &= 0.
\end{aligned}$$

Using the fourth assumption that differences in prices converge to zero as the difference in time becomes smaller, we obtain

$$\begin{aligned}
\mathbb{E}\left[H_0(S_{t_0}, C_{t_0}, S_{t_1})\right] \quad \approx \quad & \mathbb{E}\left[H_0(\widetilde{S}_{t_0}, \widetilde{C}_{t_0}, \widetilde{S}_{t_1})\right] \\
- \; & \mathbb{E}\left[\widetilde{\beta}^C_{S,0}\right]\mathbb{E}\left[\varepsilon^2_{S,t_0}\right] + \mathbb{E}\left[\frac{\partial \widetilde{\beta}^C_{S,0}}{\partial S_{t_0}}\widetilde{S}_{t_0}\right]\mathbb{E}\left[\varepsilon^2_{S,t_0}\right].
\end{aligned}$$

Now let $n > 0$ and define

$$H(S_{t_n}, S_{t_{n+1}}, C_{t_0}, C_{t_n}) = \beta^C_{S,t_n} \cdot \frac{S_{t_{n+1}} - S_{t_n}}{S_{t_n}}$$

Applying a second-order Taylor approximation yields:

$$\begin{aligned}
\mathbb{E}\left[H(S_{t_n}, S_{t_{n+1}}, C_{t_0}, C_{t_n})\right] \quad \approx \quad & \mathbb{E}\left[H(\widetilde{S}_{t_n}, \widetilde{S}_{t_{n+1}}, \widetilde{C}_{t_0}, \widetilde{C}_{t_n})\right] \\
+ \quad & \mathbb{E}\left[\frac{1}{2}\frac{\partial^2 H}{(\partial S_{t_n})^2}\widetilde{S}^2_{t_n}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right] \\
+ \quad & \mathbb{E}\left[\frac{1}{2}\frac{\partial^2 H}{(\partial S_{t_{n+1}})^2}\widetilde{S}^2_{t_{n+1}}\right]\mathbb{E}\left[\varepsilon^2_{S,t_{n+1}}\right] \\
+ \quad & \mathbb{E}\left[\frac{1}{2}\frac{\partial^2 H}{(\partial C_{t_0})^2}\widetilde{C}^2_{t_0}\right]\mathbb{E}\left[\varepsilon^2_{C,t_0}\right] \\
+ \quad & \mathbb{E}\left[\frac{1}{2}\frac{\partial^2 H}{(\partial C_{t_n})^2}\widetilde{C}^2_{t_n}\right]\mathbb{E}\left[\varepsilon^2_{C,t_n}\right]
\end{aligned}$$

The partial derivatives evaluate to

$$
\begin{aligned}
\frac{\partial^2 H}{(\partial S_{t_n})^2} &= \frac{\partial^2 \beta^C_{S,t_n}}{(\partial S_{t_n})^2}\frac{S_{t_{n+1}}-S_{t_n}}{S_{t_n}} - 2\frac{\partial \beta^C_{S,t_n}}{\partial S_{t_n}}\frac{S_{t_{n+1}}}{S^2_{t_n}} + 2\beta^C_{S,t_n}\frac{S_{t_{n+1}}}{S^3_{t_n}},\\
\frac{\partial^2 H}{(\partial S_{t_{n+1}})^2} &= 0,\\
\frac{\partial^2 H}{(\partial C_{t_0})^2} &= \frac{\partial^2 \beta^C_{S,t_n}}{(\partial C_{t_0})^2}\frac{S_{t_{n+1}}-S_{t_n}}{S_{t_n}},\\
\frac{\partial^2 H}{(\partial C_{t_n})^2} &= \frac{\partial^2 \beta^C_{S,t_n}}{(\partial C_{t_n})^2}\frac{S_{t_{n+1}}-S_{t_n}}{S_{t_n}}.
\end{aligned}
$$

Assuming again $\widetilde{S}_{t_{n+1}} - \widetilde{S}_{t_n} \to 0$, we obtain:

$$
\begin{aligned}
\mathbb{E}\left[H(S_{t_n}, S_{t_{n+1}}, C_{t_0}, C_{t_n})\right] \quad &\approx \quad \mathbb{E}\left[H(\widetilde{S}_{t_n}, \widetilde{S}_{t_{n+1}}, \widetilde{C}_{t_0}, \widetilde{C}_{t_n})\right]\\
&- \ \mathbb{E}\left[\widetilde{\beta}^C_{S,t_n}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right] + \mathbb{E}\left[\frac{\partial \widetilde{\beta}^C_{S,t_n}}{\partial S_{t_n}}\widetilde{S}_{t_n}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right]
\end{aligned}
$$

Plugging the results of both steps into the definition of our delta-hedged return (2), we arrive at

$$
\begin{aligned}
\mathbb{E}\left[\pi(t_0,T) - \widetilde{\pi}(t_0,T)\right] \quad &\approx \quad \mathbb{E}\left[\varepsilon^2_{C,t_0}\right] - \sum_{n=0}^{N-1}\mathbb{E}\left[\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right]\\
&+ \quad \sum_{n=0}^{N-1}\mathbb{E}\left[\frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right]\\
&= \quad \mathbb{E}\left[\varepsilon^2_{C,t_0}\right] - \sum_{n=0}^{N-1}\mathbb{E}\left[\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right]\\
&+ \quad \sum_{n=0}^{N-1} -\operatorname{Cov}\left(\frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\varepsilon_{S,t_n}\ ,\ -\varepsilon_{S,t_n}\right).
\end{aligned}
$$

The equality holds because all error terms have zero mean and are independent of true quantities.

We show in the following that the terms inside the covariance can be interpreted as deviations of the hedge ratio and the stock return affected by stock measurement errors from their error-free counterparts.

For the hedge ratio, we consider a first-order Taylor approximation of the function f with $f(S_{t_n}) := \frac{\Delta(\widetilde{C}_{t_n}, S_{t_n}) S_{t_n}}{\widetilde{C}_{t_0}}$ around the true value of the stock price $\widetilde{S}_{t_n}$ evaluated at $\widetilde{S}_{t_n}(1+\varepsilon_{S,t_n})$:

$$
\begin{aligned}
\frac{\Delta(\widetilde{C}_{t_n}, S_{t_n}) S_{t_n}}{\widetilde{C}_{t_0}} &\approx \frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}} + \frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}(\widetilde{S}_{t_n}(1+\varepsilon_{S,t_n}) - S_{t_n}) \\
&= \frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}} + \frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\varepsilon_{S,t_n} \\
\Leftrightarrow \frac{\Delta(\widetilde{C}_{t_n}, S_{t_n}) S_{t_n}}{\widetilde{C}_{t_0}} - \frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}} &\approx \frac{\partial\left(\frac{\widetilde{\Delta}_{t_n}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}}\widetilde{S}_{t_n}\varepsilon_{S,t_n}
\end{aligned}
$$

For the stock return, we consider again a first-order Taylor approximation. We expand the function $g(S_{t_n}) := \frac{\widetilde{S}_{t_{n+1}} - S_{t_n}}{S_{t_n}}$ around the true value of the stock price $\widetilde{S}_{t_n}$ and evaluate it at $\widetilde{S}_{t_n}(1+\varepsilon_{S,t_n})$:

$$
\begin{aligned}
\frac{\widetilde{S}_{t_{n+1}} - S_{t_n}}{S_{t_n}} &\approx \frac{\widetilde{S}_{t_{n+1}} - \widetilde{S}_{t_n}}{\widetilde{S}_{t_n}} - \frac{\widetilde{S}_{t_{n+1}}}{\widetilde{S}^2_{t_n}}(\widetilde{S}_{t_n}(1+\varepsilon_{S,t_n}) - \widetilde{S}_{t_n}) \\
&= \frac{\widetilde{S}_{t_{n+1}} - \widetilde{S}_{t_n}}{\widetilde{S}_{t_n}} - \underbrace{\frac{\widetilde{S}_{t_{n+1}}}{\widetilde{S}_{t_n}}}_{\approx 1}\varepsilon_{S,t_n} \\
\Leftrightarrow \frac{\widetilde{S}_{t_{n+1}} - S_{t_n}}{S_{t_n}} - \frac{\widetilde{S}_{t_{n+1}} - \widetilde{S}_{t_n}}{\widetilde{S}_{t_n}} &\approx -\varepsilon_{S,t_n}
\end{aligned}
$$

B Analytical Justification of IMR-Bias Adjustment

In light of formula (4), we can see the general effect of our proposed delta-lagging technique in the following way. We replace Δ_{t_n}, which is used to rebalance the hedge position at time t_n, by $\Delta_{t_{m(n)}}$ with $t_{m(n)} < t_n$. The partial derivative appearing in the IMR bias term then simplifies because the used option delta does no longer depend on the stock price at t_n:

$$\frac{\partial\left(\frac{\widetilde{\Delta}_{t_{m(n)}}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right)}{\partial S_{t_n}} = \frac{\widetilde{\Delta}_{t_{m(n)}}}{\widetilde{C}_{t_0}}$$

Using the right-hand side of (5) as a representation of the IMR bias, we can approximate the total mean return bias of the measure with the adjusted delta as

$$\begin{aligned}\mathbb{E}\left[\pi^{\text{adj.}}(t_0,T) - \widetilde{\pi}^{\text{adj.}}(t_0,T)\right] \quad &\approx \quad \mathbb{E}\left[\varepsilon^2_{C,t_0}\right] - \sum_{n=0}^{N-1}\mathbb{E}\left[\frac{\widetilde{\Delta}_{t_{m(n)}}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right] \\ &\qquad + \sum_{n=0}^{N-1}\mathbb{E}\left[\frac{\widetilde{\Delta}_{t_{m(n)}}\widetilde{S}_{t_n}}{\widetilde{C}_{t_0}}\right]\mathbb{E}\left[\varepsilon^2_{S,t_n}\right]\end{aligned}$$

Clearly, the two sums cancel and we are left with

$$\mathbb{E}\left[\pi^{\text{adj.}}(t_0,T) - \widetilde{\pi}^{\text{adj.}}(t_0,T)\right] \quad \approx \quad \mathbb{E}\left[\varepsilon^2_{C,t_0}\right].$$

It should be mentioned here that $\widetilde{\pi}^{\text{adj.}}(t_0,T)$ differs from our usual error-free benchmark in the way that the delta used to compute the hedge ratio does not match the stock return. This inevitably induces an inaccuracy of the delta hedge. However, this effect has been proven to be negligible in our simulation environment.

C Summary of the Simulation Procedure

General setting:

- 1000 simulation trials
- Cross section: 500 stocks
- Time dimension: 120 months, 22 trading days per month, 14 time points per day, i.e., 6.5h of trading per day and one observation every 30 minutes
- Options: Each month, one call and one put option with a maturity of 35 days are generated for every stock.
- Risk-free rate: 0

In total, 18,480,000,000 observations are simulated for each option type (1000 simulation trials x 500 stocks x 120 months x 22 trading days per month x 14 time points per day).

Simulation of stock data:

1. Procedure to obtain true stock data for the models of Black and Scholes (1973) and Leland (1985):[22]
   - β_i: Draw a market beta from a uniform distribution between 0.5 and 1.5.
   - σ_{ξ^i}: Draw the standard deviation of the stock-specific error (see the market model in (6)) from a uniform distribution with lower bound 0.27% and upper bound 1.07%. The bounds are obtained by scaling the daily bounds of 1% and 4% to intraday values, i.e., dividing by $\sqrt{14}$.

[22] Our procedure for generating stock data in our setting for the models of Black and Scholes (1973) and Leland (1985) follows an earlier version of Duarte et al. (2024) titled "Very noisy option prices and inference regarding option returns" from 2020.

- σ_i: The annual stock volatility can be calculated as

$$\sigma_i = \sqrt{12 \cdot 22 \cdot 14} \cdot \sqrt{(\beta_i \cdot \sigma^m)^2 + \sigma^2_{\xi^i}}.$$

  The market standard deviation σ^m equals 0.27%, which corresponds to 1% per day.

- r^m_t: Draw the return of the market index from a normal distribution with mean 0.0021% (0.03% per day) and standard deviation $\sigma^m = 0.27\%$ (1% per day).

- ξ^i_t: Draw the idiosyncratic stock error from a normal distribution with mean zero and variance $\sigma^2_{\xi^i}$.

- $\widetilde{S}^i_t$: Calculate the fair price for stock i by using the initial price of 100 for every month and the market model for the return r^i_t from equation (6):

$$r^i_t = \beta_i \cdot r^m_t + \xi^i_t$$

2. Procedure to obtain true stock data for the model of Heston (1993):[23]

- V^i_t: Draw the instantaneous variance of the stock return according to the following mean reversion process:

$$V^i_t = V^i_{t-1} + \kappa(\theta - V^i_{t-1}) \cdot \Delta t + \sigma\sqrt{V^i_{t-1}} \cdot \left(\rho \cdot \xi^i_{1,t} + \sqrt{1-\rho^2} \cdot \xi^i_{2,t}\right) \cdot \sqrt{\Delta t}$$

  The initial variance every month is simulated using the steady-state distribution which is a gamma distribution with shape parameter $\frac{2\kappa\theta}{\sigma^2}$ and scale parameter $\frac{\sigma^2}{2\kappa}$. The parameters are $\kappa = 4.5891$, $\theta = 0.1444$, $\sigma = 0.5$ and $\rho = -0.4$. The stochastic movements $\xi^i_{1,t}$ and $\xi^i_{2,t}$ are drawn from standard normal distributions that are correlated across stocks, namely $\mathrm{Corr}(\xi^i_{1,t}, \xi^j_{1,t}) = \mathrm{Corr}(\xi^i_{2,t}, \xi^j_{2,t}) = 0.5,\ i \neq j$. Δt

[23] Our procedure for generating stock data in our setting for the model of Heston (1993) follows Duarte et al. (2024).

denotes the simulated step size which is 30 minutes, i.e., $1/(12 \cdot 22 \cdot 14)$.

- $\widetilde{S}_t^i$: Calculate the fair price of stock i by using an initial price of 100 for every month and the following evolution of the stock return:

$$r_t^i = \lambda_1 \cdot V_{t-1}^i \cdot \Delta t + \sqrt{V_{t-1}^i} \cdot \xi_{1,t}^i \cdot \sqrt{\Delta t}$$

V_t^i, $\xi_{1,t}^i$ and Δt are as described above and we choose $\lambda_1 = 0.554$.

3. Procedure to obtain noisy stock data for all models:

- BA_S^i: Draw the natural logarithm of the relative bid-ask spread from a normal distribution with a mean of -6.5674 and a standard deviation of 0.7308.

- $\varepsilon_{S,t}^i$: Draw the measurement error of stock i from a symmetric triangular distribution with lower and upper limits that correspond to -0.5 and +0.5 times the stock's relative bid-ask spread BA_S^i.

- S_t^i: Calculate the observed price for stock i according to

$$S_t^i = \widetilde{S}_t^i \cdot \left(1 + \varepsilon_{S,t}^i\right).$$

Simulation of option data:

The following quantities are simulated for each option and time point (500 stocks x 60 months x 22 days x 14 intraday time points) in a single simulation trial:

- K_i: Draw a strike price for each option. The strikes are set to 100 for each option for the models of Black and Scholes (1973) and Leland (1985). For the model of Heston (1993), the strikes are randomly drawn uniformly from a range of $[70, 85]$ for ITM calls/OTM puts, $[85, 115]$ for ATM options, or $[115, 130]$ for OTM calls/ITM puts.

- $\widetilde{C}_t^i$: Calculate the fair option price using the model of Black and Scholes (1973), Leland (1985), or Heston (1993). The true stock price $\widetilde{S}_t^i$ is used for this calculation.

- $BA^i_{C,t}$: Draw the option's relative bid-ask spread with the linear regression model presented in Appendix D (see (16) and (17)).

- $\varepsilon^i_{C,t}$: Draw a measurement error from a symmetric triangular distribution with lower and upper limits that correspond to -0.5 and +0.5 times the option's relative bid-ask spread.

- C^i_t: Calculate the observed option prices as

$$C^i_t = \widetilde{C}^i_t \cdot \left(1 + \varepsilon^i_{C,t}\right).$$

- Implied volatilities: Calculate option-implied volatilities with fair prices for options and stocks, as well as based on and observed option and stock prices, by inverting the Black and Scholes (1973) formula. For true data simulated according to the model of Black and Scholes (1973), this calculation can be omitted since the implied volatility is simply the already calculated volatility of the underlying stock σ_i, which is constant over time. When simulating data according to the models of Leland (1985) and Heston (1993), the option-implied volatilities change by assumption over time. For the Leland (1985) model, we calculate *true* implied volatilities according to equation (12). To obtain the *true* implied volatilities in the Heston (1993) model as well as the implied volatilities for noisy data across all three models, we rely on a simple bisection method with boundary values of 0.001 and 9 (see Engle et al., 2010).

- $\widetilde{\Delta}^i_t$, Δ^i_t: Calculate option deltas corresponding to the true price of the option and the underlying stock ($\widetilde{\Delta}^i_t$) and related to the observed prices (Δ^i_t) by plugging the respective implied volatilities together with all other required parameters into the Black and Scholes (1973) formulas.

- Trading times: Decide for every option at every time point if a trade occurs. The respective probability is calculated according to the method presented in Appendix E (see (18) and the explanations below).

D Simulation of Option Bid-Ask Spreads

We rely on Duarte et al. (2024) for our procedure of simulating market data for options and also apply their model for the generation of bid-ask spreads. The model imposes a relation between the option's relative bid-ask spread and various characteristics of the option itself and its underlying stock:

$$\begin{aligned}
\log(BA^i_{C,t}) &= \beta_0 \\
&\quad +\beta_1 \cdot \widetilde{C}^i_t \\
&\quad +\beta_2 \cdot \mathbb{1}(\widetilde{C}^i_t < 2) \\
&\quad +\beta_3 \cdot \mathbb{1}(5 \leq \widetilde{C}^i_t < 10) \\
&\quad +\beta_4 \cdot \mathbb{1}(10 \leq \widetilde{C}^i_t < 20) \\
&\quad +\beta_5 \cdot \mathbb{1}(20 \leq \widetilde{C}^i_t) \\
&\quad +\beta_6 \cdot BA^i_S \\
&\quad +\beta_7 \cdot \widetilde{\Delta}^i_t \\
&\quad +\beta_8 \cdot \widetilde{\Gamma}^i_t \\
&\quad +\beta_9 \cdot IV_i \\
&\quad +\eta_i + \eta_t + \eta_{i,t}
\end{aligned} \tag{16}$$

$\widetilde{C}^i_t$ is the true value of option i at time t, BA^i_S and IV_i are the bid-ask spread and the Black and Scholes (1973) implied volatility of the corresponding stock. $\widetilde{\Delta}^i_t$ and $\widetilde{\Gamma}^i_t$ describe the option delta and gamma that are calculated using the true quantities. The coefficient values are taken from Duarte et al. (2024) and can be found in Table A1.[24]

The three error terms are drawn from independent centered normal distributions. The standard deviation of the firm error η_i and the time error η_t are given in Table A1. The standard deviation of the idiosyncratic error $\eta_{i,t}$ is calculated based on the following model

[24] In contrast to Duarte et al. (2024), we only have two indices, i and t, since we do not consider multiple options per underlying.

for the variance that uses the same variables as in (16):

$$\begin{aligned}
\mathrm{Var}(\eta_{i,t}) \quad = \quad & \beta_0' \\
& +\beta_1' \cdot \widetilde{C}_t^i \\
& +\beta_2' \cdot \mathbb{1}(\widetilde{C}_t^i < 2) \\
& +\beta_3' \cdot \mathbb{1}(5 \leq \widetilde{C}_t^i < 10) \\
& +\beta_4' \cdot \mathbb{1}(10 \leq \widetilde{C}_t^i < 20) \\
& +\beta_5' \cdot \mathbb{1}(20 \leq \widetilde{C}_t^i) \\
& +\beta_6' \cdot BA_S^i \\
& +\beta_7' \cdot \widetilde{\Delta}_t^i \\
& +\beta_8' \cdot \widetilde{\Gamma}_t^i \\
& +\beta_9' \cdot IV_i
\end{aligned} \tag{17}$$

The respective coefficient values can be found in Table A1.

As this model is fitted on daily data, we keep all time-dependent errors constant per day to avoid implausibly strong fluctuation of option bid-ask spreads within a single day.

Table A1: **Relative bid-ask spread of options: Model coefficients**

This table shows the values for the coefficients in (16) and (17). All values are taken from Duarte et al. (2024), Table A6. $\widetilde{C}_t^i$ describes the value of option i at time t and IV_i the Black and Scholes (1973) implied volatility of the corresponding underlying stock. BA_S^i stands for the relative bid-ask spread of the underlying stock, while $\widetilde{\Delta}_t^i$ and $\widetilde{\Gamma}_t^i$ denote the delta and gamma of the option.

	Coefficients of (16)		**Coefficients of** (17)	
	Calls	Puts	Calls	Puts
Intercept	-1.5918	-1.63460	0.68862	0.57739
$\widetilde{C}_t^i$	-0.03424	-0.04411	0.01370	0.01834
$\mathbb{1}(\widetilde{C}_t^i < 2)$	0.21301	0.26549	-0.00000	0.02556
$\mathbb{1}(5 \leq \widetilde{C}_t^i < 10)$	0.08138	0.11991	0.06302	0.07486
$\mathbb{1}(10 \leq \widetilde{C}_t^i < 20)$	0.14566	0.24220	0.10108	0.09999
$\mathbb{1}(20 \leq \widetilde{C}_t^i)$	0.25699	0.39351	-0.09957	-0.10067
BA_S^i	-0.44066	-0.45837	0.44052	0.39363
$\widetilde{\Delta}_t^i$	-1.36109	1.23748	-0.56042	0.45008
$\widetilde{\Gamma}_t^i$	0.50579	0.01704	0.06013	0.22383
IV_i	-0.03319	-0.08338	0.22336	0.24150
Standard deviation of η_t	0.3060			
Standard deviation of η_i	0.4203			

E Simulation of Trade-Time Points

To simulate option trades, we fit a linear regression model for the log number of option trades per day depending on the option's time to maturity and relative bid-ask spread. We calibrate the model for short-term at-the-money options on S&P 500 stocks between January 2004 and October 2017 from OptionMetrics and the LiveVol option trades dataset. We select calls and puts with standard expiration and the shortest maturity (up to one month) for each stock and trading day to estimate the following regression for calls and puts separately:

$$\log(\text{num-trades}_{i,t} + 1) = \widetilde{\beta}_0 + \widetilde{\beta}_1 \cdot \text{ttm}_{i,t} + \widetilde{\beta}_2 \cdot BA^i_{C,t} + \widetilde{\eta}_{i,t} \tag{18}$$

$\text{num-trades}_{i,t}$ stands for the number of trades per day, $\text{ttm}_{i,t}$ for time to maturity and $BA^i_{C,t}$ denotes the relative bid-ask spread of option i at time t. The coefficients are $\widetilde{\beta}_{0,\text{Calls}} = 1.9776$, $\widetilde{\beta}_{1,\text{Calls}} = 0.0110$ and $\widetilde{\beta}_{2,\text{Calls}} = -1.0546$ in the case of call options. For puts, the respective values are $\widetilde{\beta}_{0,\text{Puts}} = 1.6978$, $\widetilde{\beta}_{1,\text{Puts}} = 0.0025$ and $\widetilde{\beta}_{2,\text{Puts}} = -0.9131$. $\widetilde{\eta}_{i,t}$ is an error term.

We apply an exponential distribution to simulate the occurrences of option trades. Therefore, the probability that an option trade at time t for stock i will occur in the next 30 minutes is given by $1 - \exp(-\frac{\text{num-trades}_{i,t}}{14})$.[25]

[25] The expected number of trades was fitted on a daily level and has to be divided by 14 because our simulation model comprises 14 intraday time points.

F Details on the Heston (1993) Model

The dynamics of the stock price and the instantaneous variance of the stock return under the risk neutral measure are given by

$$\begin{aligned} dS_t &= rS_t\,dt + \sqrt{V_t}S_t\,dB^{\mathbb{Q}}_{1,t}, \\ dV_t &= \kappa^*(\theta^* - V_t)\,dt + \sigma\sqrt{V_t}\,\left(\rho\,dB^{\mathbb{Q}}_{1,t} + \sqrt{1-\rho^2}\,dB^{\mathbb{Q}}_{2,t}\right) \end{aligned} \tag{19}$$

where $B^{\mathbb{Q}}_{1,t}$ and $B^{\mathbb{Q}}_{2,t}$ describe standard Brownian motions.

The change of measure according to

$$\begin{aligned} dB^{\mathbb{Q}}_{1,t} &= dB^{\mathbb{P}}_{1,t} + \lambda_1\sqrt{V_t}, \\ dB^{\mathbb{Q}}_{2,t} &= dB^{\mathbb{P}}_{2,t} + \lambda_2\sqrt{V_t} \end{aligned} \tag{20}$$

leads to the dynamics under the physical measure

$$\begin{aligned} dS_t &= (r + \lambda_1 V_t)S_t\,dt + \sqrt{V_t}S_t\,dB^{\mathbb{P}}_{1,t}, \\ dV_t &= \kappa(\theta - V_t)\,dt + \sigma\sqrt{V_t}\,\left(\rho\,dB^{\mathbb{P}}_{1,t} + \sqrt{1-\rho^2}\,dB^{\mathbb{P}}_{2,t}\right) \end{aligned} \tag{21}$$

where $\kappa = \kappa^* - \sigma\left(\rho\lambda_1 + \sqrt{1-\rho^2}\lambda_2\right)$ and $\theta = \dfrac{\theta^*\kappa^*}{\kappa}$.

Following Duarte et al. (2024), we choose $\kappa^* = 3$, $\theta^* = 0.47^2$, $\sigma = 0.5$, $\rho = -0.4$, $\lambda_1 = 0.554$ and $\lambda_2 = -3.226$ which leads to $\kappa = 4.5891$ and $\theta = 0.38^2$.

Duarte et al. (2024) further show that the instantaneous expected delta-hedged return of an option C can be expressed as

$$\mathbb{E}_t\left[\frac{dC_t}{C_t}\right] - \left(\Delta_t\frac{S_t}{C_t}\right)\cdot\mathbb{E}_t\left[\frac{dS_t}{S_t}\right] = \mathbb{E}_t\left[\frac{\nu_t}{C_t}\lambda^{\sigma}_t\,dt\right], \tag{22}$$

where ν_t denotes the vega of the option and $\lambda^{\sigma}_t = \dfrac{\sigma}{2}\left(\rho\lambda_1 + \sqrt{1-\rho^2}\lambda_2\right)\cdot\sqrt{V_t}$ is the time-dependent volatility risk premium.

G Results for Put Options

Table A2: **Returns of option portfolios sorted by the bid-ask spread of the underlying in the model of Black and Scholes (1973)**

This table shows average monthly returns of delta-hedged at-the-money put portfolios sorted by the bid-ask spread of the underlying. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	0.41	0.17	0.38	0.26	0.30	-0.11	-0.03	2.8	3.7
30min	16.71	22.51	33.90	63.30	337.10	320.40	29.96	100	0
Tradetimes	13.76	14.77	18.31	26.66	103.09	89.33	8.14	100	0
DailyAVG	-49.26	-49.32	-48.64	-46.61	-28.90	20.37	0.78	13.8	0.1
Daily	16.38	15.88	16.99	18.87	38.44	22.06	1.52	33.5	0
Weekly	19.62	19.55	19.13	18.98	22.48	2.86	0.10	3.2	2.2
Bi-Weekly	21.06	21.20	19.59	20.53	21.80	0.75	0.02	3.6	2.2
Static	26.31	29.13	25.38	25.90	27.74	1.43	0.02	2.7	2.6

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	0.41	0.17	0.38	0.26	0.30	-0.11	-0.03	2.8	3.7
30min	2.65	8.31	19.82	48.87	320.80	318.14	31.59	100	0
Tradetimes	2.04	3.06	6.54	14.81	90.79	88.74	8.56	100	0
DailyAVG	-11.19	-11.47	-10.56	-8.88	8.94	20.13	0.82	15.1	0.1
Daily	2.22	1.67	3.02	4.70	24.31	22.09	1.63	36.7	0
Weekly	5.22	4.91	4.91	4.79	8.41	3.20	0.12	3.6	2
Bi-Weekly	6.63	6.38	5.48	6.15	7.88	1.25	0.03	3.7	2
Static	11.51	14.03	11.11	11.43	13.39	1.88	0.03	2.8	2.7

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	0.41	0.17	0.38	0.26	0.30	-0.11	-0.03	2.8	3.7
30min	0.96	0.76	0.77	0.76	0.81	-0.15	-0.02	2.4	2.5
Tradetimes	2.26	1.46	1.85	1.85	1.95	-0.30	-0.02	2.9	3
DailyAVG	-10.64	-10.66	-10.89	-11.00	-11.25	-0.61	-0.02	3.6	2.1
Daily	2.78	2.60	2.78	2.59	2.57	-0.21	-0.01	3	2
Weekly	6.62	7.09	6.75	5.83	5.88	-0.73	-0.03	2.2	3.4
Bi-Weekly	9.75	9.95	8.00	9.44	8.96	-0.79	-0.02	2.1	2.5
Static	16.45	19.03	16.01	16.96	17.56	1.12	0.02	3.1	2.9

Table A3: **Cross-sectional regressions of option returns on underlying bid-ask spreads in the model of Black and Scholes (1973)**

This table shows the results of univariate cross-sectional Fama and MacBeth (1973) regressions of delta-hedged at-the-money put returns on the bid-ask spreads of the underlying stocks. We only report statistics for the estimated slope coefficient. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) contains results obtained through an ordinary least squares (OLS) procedure. In Panel (b), a weighted least squares (WLS) regression is performed with weights proportional to the one-day-lagged gross return of the option. (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	Mean	Std	Avg. t-stat	*(+)	*(-)
Benchmark	-0.00	0.05	-0.01	3.3	3
30min	11.00	0.34	33.98	100	0
Tradetimes	3.07	0.21	14.35	100	0
DailyAVG	0.71	0.42	1.77	39.7	0.2
Daily	0.76	0.24	3.21	74.8	0
Weekly	0.12	0.46	0.27	6	1.6
Bi-Weekly	0.05	0.64	0.06	3.6	2.4
Static	0.01	0.88	-0.00	3.5	2.6

(b) O-DMR-bias adjusted

	Mean	Std	Avg. t-stat	*(+)	*(-)
Benchmark	-0.00	0.05	-0.01	3.3	3
30min	10.92	0.33	35.31	100	0
Tradetimes	3.05	0.20	14.94	100	0
DailyAVG	0.69	0.39	1.84	41.5	0
Daily	0.77	0.23	3.41	78.8	0
Weekly	0.13	0.44	0.32	6.3	1.7
Bi-Weekly	0.06	0.60	0.09	3.7	2.4
Static	0.02	0.84	0.01	3.2	2.7

(c) DMR- and IMR-bias adjusted

	Mean	Std	Avg. t-stat	*(+)	*(-)
Benchmark	-0.00	0.05	-0.01	3.3	3
30min	0.00	0.16	0.02	3.8	2.7
Tradetimes	0.00	0.23	0.00	3.2	2.3
DailyAVG	-0.02	0.47	-0.04	3.7	4.1
Daily	-0.00	0.35	-0.01	2.6	2.9
Weekly	-0.02	0.51	-0.04	2.5	2.9
Bi-Weekly	-0.01	0.66	-0.03	2.3	2.7
Static	-0.01	0.88	-0.02	3.2	3.3

Table A4: **Returns of option portfolios sorted by the bid-ask spread of the underlying in the model of Leland (1985) (market maker long in 25% of all options)**

This table shows average monthly returns of delta-hedged at-the-money put portfolios sorted by the bid-ask spread of the underlying. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-9.55	-19.28	-29.81	-46.32	-92.49	-82.94	-20.16	0	99.9
30min	6.73	2.88	3.55	15.66	234.66	227.93	19.25	100	0
Tradetimes	3.82	-4.27	-11.64	-19.87	8.18	4.36	0.12	13.3	10.5
DailyAVG	-58.73	-67.47	-77.16	-90.25	-115.97	-57.24	-2.23	0.1	59.3
Daily	6.13	-3.42	-13.20	-27.80	-55.08	-61.22	-4.22	0.1	96.4
Weekly	9.49	-0.05	-11.12	-27.62	-70.51	-80.00	-2.77	0	76.2
Bi-Weekly	11.07	1.46	-10.67	-25.88	-71.20	-82.27	-2.07	0	52.5
Static	16.20	9.60	-4.83	-20.49	-65.63	-81.83	-1.47	0.2	29.9

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-9.55	-19.28	-29.81	-46.32	-92.49	-82.94	-20.16	0	99.9
30min	-7.00	-10.57	-9.60	2.83	219.47	226.47	20.54	100	0
Tradetimes	-7.52	-15.20	-22.18	-29.89	-1.61	5.91	0.36	15.8	8.3
DailyAVG	-20.04	-28.40	-37.15	-49.61	-74.22	-54.18	-2.23	0.1	59.1
Daily	-7.70	-16.88	-26.24	-40.32	-66.70	-58.99	-4.34	0	97
Weekly	-4.61	-13.93	-24.34	-40.30	-81.99	-77.38	-2.84	0	77.5
Bi-Weekly	-3.04	-12.53	-23.84	-38.89	-82.50	-79.47	-2.11	0	54.2
Static	1.91	-4.80	-18.28	-33.80	-77.17	-79.08	-1.50	0.1	30.3

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-9.55	-19.28	-29.81	-46.32	-92.49	-82.94	-20.16	0	99.9
30min	-8.83	-17.98	-28.41	-44.27	-89.33	-80.51	-9.34	0	99.9
Tradetimes	-7.45	-16.81	-26.45	-42.62	-86.98	-79.52	-5.91	0	100
DailyAVG	-19.44	-27.84	-37.38	-51.77	-93.71	-74.27	-2.56	0	70.2
Daily	-7.09	-16.20	-26.35	-42.46	-87.72	-80.63	-3.73	0	94.6
Weekly	-3.07	-11.71	-22.49	-39.36	-84.53	-81.45	-2.56	0	69.7
Bi-Weekly	0.05	-8.77	-21.30	-35.75	-81.61	-81.66	-1.99	0	49.9
Static	6.82	0.15	-13.46	-28.44	-73.27	-80.09	-1.45	0.1	29.7

Table A5: **Returns of option portfolios sorted by the bid-ask spread of the underlying in the model of Leland (1985) (market maker long in 75% of all options)**

This table shows average monthly returns of delta-hedged at-the-money put portfolios sorted by the bid-ask spread of the underlying. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	10.68	20.66	33.14	53.21	138.70	128.02	28.50	100	0
30min	26.99	42.96	67.04	117.32	539.11	512.12	38.33	100	0
Tradetimes	23.89	35.34	50.93	79.38	258.17	234.27	19.20	100	0
DailyAVG	-39.63	-29.77	-17.72	3.79	108.02	147.65	5.36	99.9	0
Daily	26.38	36.55	49.83	71.94	181.43	155.04	10.15	100	0
Weekly	29.76	39.97	51.95	72.08	162.00	132.24	4.43	97.8	0
Bi-Weekly	31.36	41.51	52.42	73.90	160.82	129.47	3.15	84.1	0
Static	36.52	49.76	58.40	79.46	166.81	130.29	2.26	58.8	0

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	10.68	20.66	33.14	53.21	138.70	128.02	28.50	100	0
30min	12.52	28.05	51.58	100.81	510.59	498.07	40.81	100	0
Tradetimes	11.67	22.66	37.62	65.00	236.13	224.46	20.00	100	0
DailyAVG	-2.36	6.51	17.89	37.46	131.54	133.89	5.33	100	0
Daily	11.81	21.63	34.48	55.77	160.48	148.67	10.50	100	0
Weekly	14.93	24.62	36.42	55.76	141.49	126.56	4.51	98.6	0
Bi-Weekly	16.51	26.06	36.95	57.26	140.52	124.01	3.20	84.4	0
Static	21.49	33.90	42.67	62.54	146.25	124.76	2.29	60.3	0

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	10.68	20.66	33.14	53.21	138.70	128.02	28.50	100	0
30min	10.68	20.52	32.26	51.65	132.98	122.30	13.52	100	0
Tradetimes	11.73	21.03	33.23	51.71	131.60	119.87	8.59	100	0
DailyAVG	-1.75	7.08	17.65	35.20	107.69	109.43	3.66	94.1	0
Daily	12.43	22.33	34.36	53.55	134.65	122.22	5.49	100	0
Weekly	16.48	26.88	38.32	56.76	138.18	121.71	3.70	93.2	0
Bi-Weekly	19.62	29.88	39.57	60.54	141.25	121.63	2.87	77.8	0
Static	26.45	38.95	47.63	68.18	150.57	124.12	2.17	56.9	0

Table A6: **Option returns sorted by vola-elasticity in the model of Heston (1993)**

This table shows average monthly returns of delta-hedged at-the-money put portfolios sorted by their vola-elasticity. Option moneyness, $\frac{K}{S}$, is between 0.85 and 1.15. Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-101.88	-206.45	-357.99	-548.58	-805.51	-703.62	-4.69	0	99.2
30min	-17.75	-120.60	-264.78	-447.06	-661.36	-643.61	-4.23	0	96.9
Tradetimes	-73.16	-174.76	-320.26	-501.78	-736.25	-663.08	-4.21	0	97.2
DailyAVG	-110.33	-222.00	-383.77	-587.61	-844.71	-734.38	-3.55	0	91.1
Daily	-92.09	-193.48	-340.27	-521.57	-744.90	-652.81	-3.60	0	91.3
Weekly	-97.22	-198.97	-345.12	-528.22	-753.94	-656.71	-2.47	0	62.7
Bi-Weekly	-98.58	-201.30	-348.35	-534.58	-769.66	-671.08	-2.04	0	48.9
Static	-98.95	-201.04	-346.96	-527.14	-762.60	-663.66	-1.63	0	35.9

(b) O-DMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-101.88	-206.45	-357.99	-548.58	-805.51	-703.62	-4.69	0	99.2
30min	-21.06	-125.35	-271.35	-460.51	-707.07	-686.01	-4.66	0	99.1
Tradetimes	-75.61	-177.96	-323.98	-509.30	-758.87	-683.26	-4.52	0	98.3
DailyAVG	-99.13	-202.19	-352.11	-545.80	-801.06	-701.94	-3.60	0	91.5
Daily	-95.00	-197.91	-346.39	-534.62	-789.63	-694.63	-3.96	0	95.6
Weekly	-100.13	-203.39	-351.16	-541.14	-797.77	-697.64	-2.70	0	69.4
Bi-Weekly	-101.51	-205.76	-354.25	-547.43	-812.46	-710.95	-2.22	0	54.4
Static	-101.96	-205.62	-352.90	-539.63	-804.33	-702.36	-1.76	0	39.7

(c) DMR- and IMR-bias adjusted

	1 (L)	2	3	4	5 (H)	H-L	Avg. t-stat	*(+)	*(-)
Benchmark	-101.88	-206.45	-357.99	-548.58	-805.51	-703.62	-4.69	0	99.2
30min	-100.03	-202.05	-350.24	-536.90	-790.68	-690.65	-4.55	0	98.9
Tradetimes	-98.71	-199.43	-345.57	-528.16	-778.47	-679.75	-4.11	0	96.6
DailyAVG	-104.19	-206.86	-355.93	-548.87	-804.03	-699.84	-2.97	0	79.3
Daily	-100.38	-202.69	-350.01	-537.88	-792.80	-692.42	-3.09	0	80.7
Weekly	-100.15	-202.69	-348.15	-535.92	-792.58	-692.42	-2.39	0	59.8
Bi-Weekly	-100.46	-203.79	-349.07	-537.06	-798.09	-697.63	-2.00	0	46.7
Static	-99.31	-201.39	-342.10	-519.67	-777.54	-678.23	-1.59	0	34.7

Table A7: **VRP estimation for different moneyness categories**

This table shows the results of univariate portfolio sorts of delta-hedged puts on the vola-elasticity for different moneyness categories. Ranges for option moneyness, $\frac{K}{S}$, are (0.85,1.15) for at-the-money (ATM) and (1.15,1.3) for in-the-money (ITM) options. Stock bid-ask spreads are simulated according to (15). Average returns are in basis points. The benchmark corresponds to a hedge frequency of every 30 minutes. Panel (a) presents equally weighted portfolios. In Panel (b), the portfolio weights are the one-day lagged gross returns of the options. Panel (c) additionally lags the deltas by one time step. The last two columns report the percentage of simulated samples in which the H-L portfolio return is positive (*(+))/negative (*(-)) and significantly different from 0 at the 5% level (Newey-West test with five lags).

(a) Unadjusted

	ATM				**ITM**			
	H-L	Avg. t	*(+)	*(-)	H-L	Avg. t	*(+)	*(-)
Benchmark	-703.64	-4.69	0	99.2	-58.54	-5.68	0	100
30min	-626.97	-4.11	0	96.1	15.88	1.37	28.2	0
Tradetimes	-657.01	-4.17	0	96.5	-34.27	-3.08	0	83.9
DailyAVG	-733.44	-3.55	0	91.1	-61.63	-2.29	0	59.8
Daily	-651.76	-3.60	0	91.3	-51.13	-3.99	0	97.1
Weekly	-656.55	-2.47	0	63.2	-55.29	-2.84	0	74.6
Bi-Weekly	-670.85	-2.04	0	49.3	-56.18	-2.26	0	56.2
Static	-663.49	-1.63	0	35.8	-56.57	-1.77	0	39

(b) O-DMR-bias adjusted

	H-L	Avg. t	*(+)	*(-)	H-L	Avg. t	*(+)	*(-)
Benchmark	-703.64	-4.69	0	99.2	-58.54	-5.68	0	100
30min	-668.99	-4.52	0	98.5	15.17	1.34	27.6	0
Tradetimes	-655.30	-4.19	0	96.7	-34.06	-3.07	0	83.9
DailyAVG	-701.03	-3.59	0	91.5	-55.23	-2.12	0	53.4
Daily	-693.57	-3.95	0	95.5	-51.76	-4.12	0	97.6
Weekly	-697.44	-2.70	0	69.3	-55.95	-2.93	0	77.5
Bi-Weekly	-710.74	-2.22	0	54.4	-56.87	-2.33	0	58.3
Static	-702.20	-1.75	0	39.7	-57.31	-1.83	0	41.2

(c) DMR- and IMR-bias adjusted

	H-L	Avg. t	*(+)	*(-)	H-L	Avg. t	*(+)	*(-)
Benchmark	-703.64	-4.69	0	99.2	-58.54	-5.68	0	100
30min	-690.75	-4.55	0	98.8	-56.51	-5.32	0	100
Tradetimes	-658.41	-3.85	0	95.1	-55.27	-4.67	0	99.4
DailyAVG	-699.83	-2.97	0	79.6	-59.67	-2.14	0	55
Daily	-692.37	-3.09	0	80.6	-57.20	-3.46	0	89.1
Weekly	-692.40	-2.39	0	59.7	-58.69	-2.61	0	69.1
Bi-Weekly	-697.47	-2.00	0	46.8	-61.11	-2.22	0	55.6
Static	-678.00	-1.59	0	34.8	-65.77	-1.89	0	44.9