Separate Risk from Optionality

Liuren Wu,* Yang Xu,†

Abstract

This paper models the joint stock and bond return behavior within a structural model framework. The framework disentangles the stock and bond's exposures to firm value variation from their exposures to the embedded call optionality. The factor portfolio targeting a unit exposure to the asset return risk generates a significantly positive average excess return. The factor portfolio targeting a unit optionality exposure to the volatility variation generates a significantly negative average excess return. The separation of asset return risk from optionality sheds light on the distress puzzle in the stock and bond markets, the bet-against-beta anomaly, and the volatility premium.

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*Zicklin School of Business, Baruch College, The City University of New York, One Bernard Baruch Way, New York, NY 10010, USA. E-mail: liuren.wu@baruch.cuny.edu.

†Memorial University of Newfoundland, Faculty of Business Administration, Morrissey Rd, St. John's, NL A1B 3X9, Canada. E-mail: y.xu@mun.ca.

Investors are averse to risk, but they love optionality. Risk is often measured by the degree of variation in future outcomes; whereas optionality manifests itself in a convex payoff structure. Because high optionality often comes hand in hand with high risk, the two effects can work against each other and generate seemingly abnormal market pricing behaviors. One such example is the puzzling observation that stocks on companies with high default risk have delivered anomalously low average excess returns (Campbell, Hilscher, and Szilagyi 2008), whereas bonds with high default risk have abnormally wide credit spreads (Huang and Huang 2012).

This paper proposes to disentangle the optionality effect from the risk effect in a company's stock and bond. We start with the classic structural model of Merton (1974), which treats a company's stock as a call option on the company's firm value. Given its stylized nature, we do not use the model to generate bottom-up pricing implications on the bond credit spread or the equity risk premium; but rather, we use the model as a top-down value representation function to highlight the major risk exposures of a company's stock and bond. Under this model representation, the stock has a positive delta exposure to the underlying firm value variation. A high level of the firm's asset return risk translates into a high level of stock return risk through the delta exposure. Meanwhile, the stock also embeds positive optionality due to the convex call option payoff structure on the firm value. This optionality affects the stock return variation through the call option's positive vega exposure and the variation of the asset return volatility.

The bond of the firm can be regarded as portfolio that is long the firm asset and short the call option. As such, the bond also has positive delta exposure that complements the positive delta exposure of the stock; nevertheless, as the bond is short the call option, its vega optionality exposure is negative, exactly the opposite of the stock's positive vega optionality exposure.

The optionality in the stock and the bond of a company is chiefly induced by financial leverage and the nature of limited liability on the company's equity owners. The effect is small when a company has little debt and is far from default. As a company's debt increases, its distance to default shortens, its default probability increases, and the embedded call optionality becomes stronger. In this case, positive investor preference for optionality can drive up the value of a stock with high embedded optionality, and drive down the stock return. Meanwhile, since the bond embeds a short option exposure, the same investor preference for optionality can drive down the bond value and widen the credit spread.

Through the Merton (1974) structural model representation, we attribute the return on the stock and the bond of a company to their respectively delta and vega exposures, multiplied by the corresponding asset return and the asset return volatility change, respectively. Taking expectation on the return attribution, we attribute the stock and bond return risk premiums to the asset return risk premium and the optionality risk premium, the contribution of each dictated by the respective risk exposures. When risk aversion leads to a positive asset return risk premium but preference for optionality leads to a negative optionality risk premium, stock return risk premium is to increase with its positive delta exposure but decline with its positive optionality exposure; whereas bond return risk premium is to increase with both its positive delta exposure and its negative optionality exposure.

Based on the structural return attribution analysis, we propose a joint stock and bond return factor model that links the cross-sectional variation of the stock and bond returns to the variation in their estimated delta and vega exposures. A cross-sectional regression on the return factor model generates excess returns on two factor portfolios. The first portfolio targets a unit delta exposure to the firm asset return but zero exposure to the optionality. The second portfolio targets a unit vega optionality exposure to asset return volatility variation but zero delta exposure to the firm value variation.

We perform empirical analysis on the stocks and bonds on US publicly traded companies. At the end of each month during our sample period, we implement the Merton (1974) model on each company in our selected universe and compute the delta and vega exposures of the stock and the bond of the company. We construct the joint stock-bond return factor model with the structurally estimated delta and vega exposures. Model estimation shows that the asset return risk portfolio with zero optionality exposure generates a significantly positive average excess return, consistent with investor risk aversion. By contrast, the delta-neutral vega optionality portfolio generates a significantly negative average excess return, reflecting strong investor preference for optionality.

Once the delta and vega exposures are neutralized, the average intercept estimate of the joint return factor model becomes statistically insignificant. Allowing separate intercepts for the bond and stock excess returns leads to insignificant average intercept estimates for both markets.

The structural separation of the linear asset return risk from the optionality risk sheds light on the seemingly abnormal behaviors of stock and bond returns in relation to their default risk exposures. On the one hand, default risk models can hardly generate credit spreads wide enough to match market observations on corporate bonds (e.g., Eom, Helwege, and Huang 2004; Huang and Huang 2012); on the other hand, stocks with high default risk have delivered anomalously low average excess returns (e.g., Dichev 1998; Griffin and Lemmon 2002; Campbell, Hilscher, and Szilagyi 2008). We show that the opposite optionality exposures in stocks and bonds, coupled with investor preference for optionality, creates a compounding effect on the expected bond return risk premium but an attenuating effect on stock return risk premium. The optionality exposure lowers the expected stock excess return for companies with high default risk but raises the expected bond excess return of the same company.

Investor preference for optionality shows up in many different forms. The most direct manifestation is in the options market, where investors are willing to pay more than the breakeven price to buy options. As a result, writing options has been found to generate positive delta-hedged average excess returns (e.g., Bakshi and Kapadia 2003; Cao and Han 2013). Selling variance swaps, which can be replicated by a portfolio of options across the whole spectrum of strikes, has also been found to generate positive risk premiums (Carr and Wu 2009). Since the chief exposure of a delta-hedged option position is the return volatility variation through the vega exposure, the well-documented negative volatility risk premium in the options market is mainly a reflection of the positive investor preference for optionality and fundamentally an optionality risk premium. Compared to the options market, the optionality embedded in stocks and bonds is much less direct. Through a structural model setting and a joint stock-bond return factor model, we are able to disentangle the optionality effect from the asset return risk exposure and separately identify the asset return risk premium and the option risk premium.

The literature has also attempted to identify the optionality or volatility exposure in the stock market by statistically estimating the stock return sensitivity on some volatility index, such as the VIX index (e.g., Ang, Hodrick, Xing, and Zhang 2006; Harvey, Liu, and Zhu 2016; Barinov and Chabakauri 2023). Given the excess return series on the two factor portfolios extracted from our joint return factor model, we also estimate the statistical return beta of each stock and bond on the two factor portfolios. We show that the statistical beta estimates are highly correlated with the structural beta constructs. Cross-sectional Fama and MacBeth (1973)-type return regressions on the beta estimates also generate significantly positive risk premiums on the asset return beta and significantly negative risk premiums on the optionality beta.

The embedded optionality has also contributed to other documented bond and stock return behaviors. Based on structural model implementations, investors have designed capital structure arbitrage strategies that are long bond and short the stock of the same company to delta hedge when the bond's credit spread is found to be wider than model valuation (e.g., Yu 2006; Duarte, Longstaff, and Yu 2007). Such a delta-neutral strategy is effectively short the option and loads negatively on the vega exposure. Therefore, although the strategy is designed to capture bond mispricing, its investment performance is at least partially driven by the positive risk premium on the strategy's negative optionality exposure.

In the stock market, Frazzini and Pedersen (2014) find that high-beta stocks generate lower risk-adjusted average excess returns. They propose a betting-against-beta (BAB) strategy to benefit from the pricing anomaly. The strategy is long low beta stocks and short high beta stocks while maintaining beta neutrality. Since high beta names tend to have high optionality, we conjecture that the strategy embeds a short optionality exposure. We regress the BAB portfolio returns against the returns on our optionality factor portfolio and show that the BAB portfolio has a significantly negative loading on the optionality factor. Once the optionality exposure is hedged, the average excess return on the BAB portfolio is no longer statistically significant.

This paper starts with the premise that investors are averse to risk but love optionality. Investor risk aversion is a common assumption in classic asset pricing theories. The literature has also proposed several theoretical frameworks that can generate the optionality-loving behavior. In particular, the cumulative prospect theory of Tversky and Kahneman (1992) captures this optionality-loving behavior by generating risk aversion for gains of high probability and losses of low probability but risk seeking for losses of high probability and gains of low probability. Barberis and Huang (2008) apply the theory to explain why stocks with lottery-like payoffs tend to be overpriced and generate lower returns. Kumar (2009) finds that individual investors prefer stocks with lottery features and that the demand for lottery-like stocks increases during economic downturns. Eraker and Ready (2015) show that investors also tend to overpay for lottery-like stocks in the OTC market. Conrad, Kapadia, and Xing (2014) highlight the role of lottery-like payoffs in explaining the distress puzzle in the stock market. These findings provide support to our starting premise.

This paper uses the classic Merton (1974) structural model as a top-down value representation function to highlight the two major risk sources underlying the stock and bond of a company, i.e., the underlying firm value variation and the asset return volatility movement. Different from this top-down analysis, historical efforts focus on improving the classic structural model from bottom up by proposing different firm value dynamics and capital structure mechanisms to better match the observed credit spread behaviors (e.g., Geske 1977; Longstaff and Schwartz 1995; Leland and Toft 1996; Collin-Dufresne and Goldstein 2001). More recently, Bhamra, Kuehn, and Strebulaev (2010) embed a structural model inside a dynamic consumption-based asset pricing model to jointly explain the equity and credit risk premiums. The two approaches complement each other. The bottom-up modeling efforts offer structural mechanisms to generate the observed risk premiums, whereas our top-down analysis focuses on risk magnitude and risk exposure construction and directly estimates the risk premium of each risk source from cross-sectional return factor model constructions. Our approach builds a bridge between classic structural modeling and empirical return factor models.

1. Structural Decomposition of Stock and Bond Risk Exposures

This section starts with the Merton (1974) model, where the firm has a single zero-coupon bond as its debt with a principal amount of D and expiry at time T. The equity of the firm can be regarded as a call option on the firm value with the option's strike price equal to the debt principal and the option's maturity matching the bond expiry. The debt of the firm can be regarded as a portfolio that is long the firm asset and short the call option.

1.1 Value representation and risk exposures

The Merton (1974) model is highly stylized with well-known biases in its model-implied credit spreads (Eom, Helwege, and Huang 2004; Huang and Huang 2012). Nevertheless, the model implications can still be very effective in differentiating the cross-sectional credit spread variations of different companies (Bai and Wu 2016). The model can also generate reasonably accurate risk sensitivities and hedging ratios (Schaefer and Strebulaev 2008). In this paper, we do not take the Merton model at its literal meaning, but use it as a top-down value representation function to summarize the major risk sources of the underlying securities of the company and to compute their risk exposures. This approach is similar to how the industry and the academia in the options market have been using the Black and Scholes (1973) and Merton (1973) (BMS) model, not as a bottom-up option pricing model, but as a top-down value representation function to summarize the major risk sources of an option contract (e.g., Carr and Wu 2020; Wu and Zhang 2025).

Based on the Merton model representation, we can represent the value of the stock in a company via the BMS pricing equation,

$$S(t, F_t, \sigma_t) = F_t N(d_t) + De^{-r_t\tau} N(d_t - \sigma_t\sqrt{\tau}), \tag{1}$$

with r_t denoting the instantaneous riskfree rate, τ the option time to maturity, σ_t the asset return volatility, $N(\cdot)$ the cumulative normal function, and d_t the standardized moneyness measure often referred to as the distance to default,

$$d_t = \frac{\ln(F_t/D) + r_t\tau + \frac{1}{2}\sigma_t^2\tau}{\sigma_t\sqrt{\tau}}. \tag{2}$$

Equation (1) represents the stock value variation in terms of the calendar time t and its two major risk sources, i.e., variations in the underlying firm value F_t and the asset return volatility σ_t. Although the original Merton (1974) model assumes a geometric Brownian motion with constant volatility, we choose an asset return volatility input σ_t to the pricing equation in (1) to match the observed market capitalization of the company at each date and for each company, much like the BMS implied volatility used in the options market to match the price of a particular option contract.

In reality, a firm can issue multiple bonds with different maturities. To apply the value representation in (1), we set the debt principal D to the total book value of debt and set the option maturity to the average maturity of the company's outstanding debt. We solve for the firm value F_t and the asset return volatility σ_t to match the company's market capitalization as the equity value S_t and the company's equity return volatility estimate σ_t^S.

Given the stock value representation in (1), we can represent the value of the company's bond as the difference between the firm value and the stock value,

$$B(t, F_t, \sigma_t) = F_t - S(t, F_t, \sigma_t) = F_t(1 - N(d_t)) - De^{-r\tau}N(d_t - \sigma_t\sqrt{\tau}). \tag{3}$$

The BMS option value representation involves two standardized moneyness measures, d_t and $(d_t - \sigma_t\sqrt{\tau})$. Under the original model dynamics assumptions, they measure the number of standard deviations by which the mean of the natural logarithm of the firm value, $\ln(F_T)$, exceeds the natural logarithm of the debt principal, $\ln(D)$, under the share measure and the risk-neutral measure, respectively. The literature has used both measures to define the distance to default. In this paper, we choose d_t as the standardized distance to default measure because it has a more direct linkage to the risk exposures of the stocks and bonds. From this distance to default measure, we can construct a default probability measure as,

$$p_t = N(-d_t). \tag{4}$$

The default probability becomes larger as the distance to default shortens.

1.1.1 The delta exposure to firm value variation

The delta of an option captures the contract's linear exposure to the underlying security's price movement. Under the BMS value representations in (1) and (3), the delta exposures of the stock and the bond can be written as,

$$S_F = N(d_t) = \delta_t, \quad B_F = 1 - N(d_t) = 1 - \delta_t, \tag{5}$$

where S_F and B_F denote the partial derivatives of the stock and bond value functions against the underlying firm value, respectively. To highlight their connections, we use $\delta_t \equiv N(d_t)$ to denote the delta of the stock as a call option. By being long the firm value and short the call option, the bond has a delta exposure of $1 - \delta_t$, complementing the delta of the stock.

The delta exposures of the stock and the bond of a company are both positive. As the sum of the stock and the bond reconstructs the firm value, the deltas of the stock and bond sum to 100%. When the firm value increases, both the stock value and the bond value increase with it. For a company with little debt, its distance to default is large, its stock delta δ_t is close to one whereas its bond delta is close zero. The firm's stock value is much more sensitive to the firm value change than is the bond value. As a result, the stock is as risky as the firm whereas the bond has little risk. On the other hand, when a company has a large amount of debt and its distance to default is close to zero, the company's stock and bond can become equally sensitive to the firm value variation. Both securities become equally risky and tend to share strongly positive co-movements driven by their equally positive delta exposures to the firm value variation.

1.1.2 The vega exposure to optionality

As a call option has a convex payoff structure, its value not only increases with the underlying security price, but also increases with the return volatility of the underlying security. The vega exposure, or the value sensitivity of a contract to the underlying return volatility movement, can be used to quantify the optionality embedded in the contract. Under the BMS value representations in (1) and (3), the vega exposures of the stock and the bond can be written as,

$$S_\sigma = F_t \sqrt{\tau} n(d_t) = F_t \nu_t, \quad B_\sigma = -F_t \sqrt{\tau} n(d_1) = -F_t \nu_t, \tag{6}$$

where S_σ and B_σ denote the partial derivatives of the stock and bond value functions against the underlying asset return volatility σ_t and $n(\cdot)$ denotes the probability density function of the standard normal distribution. We use $\nu_t \equiv \sqrt{\tau} n(d_t)$ to denote the vega exposure per unit firm value for the call option. As the stock is long the call option and the bond is short the call option, the vega exposure of the stock and the bond are exactly opposite of each other, positive for the stock and negative for the bond.

A linear contract has linear (delta) exposure to the underlying security price movement but does not have any vega exposure to the underlying return volatility. Aggregating the stock and bond together recreates the firm asset, which has 100% delta exposure to the firm value by construction, but zero vega exposure to the volatility movement. The opposite optionality exposures of the stock and the bond cancel out each other.

Similar to the case for the delta exposure, the relative magnitude of the optionality exposure is directly determined by the distance to default measure d_t of the company. For companies with little debt and hence long distance to default, the optionality exposure ν_t is small. As a company increases its debt and as its distance to default approaches zero, the option vega reaches its maximum value. In the rare case where a company's distance to default d_t becomes negative, the vega exposure starts to decline again with further increasing default risk. For most companies, however, the distances to default are positive, and both the option delta and the option vega increase with declining distance to default.

1.2 Return attribution on the stock and the bond of a company

Based on the value representations in (1) and (3) and the risk exposure constructions in (5) and (6), we can perform the following return attribution on the stock and the bond of a company,

$$\begin{bmatrix} \frac{dS_t}{F_t} \\ \frac{dB_t}{F_t} \end{bmatrix} = \begin{bmatrix} \delta_t & \nu_t \\ 1-\delta_t & -\nu_t \end{bmatrix} \begin{bmatrix} \frac{dF_t}{F_t} \\ d\sigma_t \end{bmatrix} + O(dt), \tag{7}$$

where we scale the instantaneous change in the stock and the bond value (dS_t, dB_t) by the firm value of the company to define the unlevered return on the stock and the bond, and we use the term $O(dt)$ to collect the deterministic drift terms generated by the option's time decay and second-order exposures to the firm value and the asset return volatility. The attribution highlights the impacts of the two random sources of shocks $(dF_t/F_t, d\sigma_t)$ on the stock and bond value variation. The stock return increases with the firm's asset return (dF_t/F_t) through its delta exposure δ_t and increases with the volatility change $(d\sigma_t)$ through its vega exposure (ν_t). By comparison, the bond return increases with the asset return with its positive delta exposure $(1-\delta_t)$ but declines with the volatility change due to its negative vega exposure ($-\nu_t$).

Equation (7) defines the stock and bond returns in unlevered form, with the firm value as the common denominator. This definition highlights the symmetric nature of the stock and bond risk exposures. In the equity market, Doshi, Jacobs, Kumar, and Rabinovitch (2019) show how leverage induces heteroskedasticity in stock returns and complicates risk-return relations; Choi and Richardson (2015) find that leverage induces asymmetry in the stock volatility dynamics. In the option pricing literature, researchers have also been realizing the potential instability of levered option returns and are shifting to unlevered option return constructions for both theoretical risk attribution and empirical analysis (e.g., Tian and Wu 2023; Fournier, Jacobs, and Orlowski 2024; Wu and Zhang 2025; Wu and Xu 2023).

We can also readily convert the unlevered returns in (7) to levered returns via the scaling of F_t/S_t and F_t/B_t, respectively,

$$\begin{bmatrix} \frac{dS_t}{S_t} \\ \frac{dB_t}{B_t} \end{bmatrix} = \begin{bmatrix} \frac{F_t}{S_t} & 0 \\ 0 & \frac{F_t}{B_t} \end{bmatrix} \begin{bmatrix} \delta_t & \nu_t \\ 1-\delta_t & -\nu_t \end{bmatrix} \begin{bmatrix} \frac{dF_t}{F_t} \\ d\sigma_t \end{bmatrix} + O(dt). \tag{8}$$

As financial leverage increases, the distance to default becomes shorter and the delta exposure δ_t becomes smaller; nevertheless, the levered delta exposure of the stock return, $\frac{F_t}{S_t}\delta_t$, increases. Furthermore, the financial leverage amplifies the increasing absolute vega exposure with increasing leverage for both stock and bond returns. As a result, holding asset return volatility fixed, levered returns on both the stock and the bond become riskier as financial leverage increases.

1.3 Decomposing bond and stock return risk premium

Based on the structural return attribution in (8), we can decompose the stock and bond return risk premiums into risk premiums on the firm's asset return risk and the volatility risk through their respective delta and vega exposures.

Proposition 1 *The stock risk premium (SRP) and bond risk premium (BRP) of a company can be decomposed into two structural components: (1) their delta exposures to the underlying firm value variation and the firm's asset return risk premium (RRP) and (2) their vega exposures to the underlying asset volatility variation and the optionality risk premium (ORP),*

$$\begin{bmatrix} \mathrm{SRP}_t \\ \mathrm{BRP}_t \end{bmatrix} = \begin{bmatrix} \frac{F_t}{S_t} & 0 \\ 0 & \frac{F_t}{B_t} \end{bmatrix} \begin{bmatrix} \delta_t & \nu_t \\ 1-\delta_t & -\nu_t \end{bmatrix} \begin{bmatrix} \mathrm{RRP}_t \\ \mathrm{ORP}_t \end{bmatrix}, \tag{9}$$

*where the risk premiums (*SRP, BRP*) and (*RRP, ORP*) are defined as annualized expectation differences under the two probability measures ($\mathbb{P}$ and $\mathbb{Q}$) on the corresponding risk sources,*

$$\begin{bmatrix} \mathrm{SRP}_t \\ \mathrm{BRP}_t \end{bmatrix} \equiv \frac{1}{dt}\left(\mathbb{E}_t^{\mathbb{P}} - \mathbb{E}_t^{\mathbb{Q}}\right) \begin{bmatrix} \frac{dS_t}{S_t} \\ \frac{dB_t}{B_t} \end{bmatrix}, \quad \begin{bmatrix} \mathrm{RRP}_t \\ \mathrm{ORP}_t \end{bmatrix} \equiv \frac{1}{dt}\left(\mathbb{E}_t^{\mathbb{P}} - \mathbb{E}_t^{\mathbb{Q}}\right) \begin{bmatrix} \frac{dF_t}{F_t} \\ d\sigma_t \end{bmatrix}. \tag{10}$$

Proof. Take expectation on the stock and bond return attribution in (8) under the statistical measure ($\mathbb{P}$) and the risk-neutral measure ($\mathbb{Q}$), respectively. Perform annualization by $(1/dt)$. Taking the difference between the two annualized expectations leads to equation (9), with the risk premiums defined in (10). ∎

Based on our general premise that investors are averse to risk but love optionality, we form the following empirically testable hypothesis on the two types of risk premiums.

Hypothesis 1 *We hypothesize that the presence of investor risk aversion implies a positive average return risk premium whereas investor preference for optionality implies a negative average optionality risk premium.*

Under our hypothesis, the stock return risk premium would increase with its positive delta exposure but declines with its positive vega optionality exposure. By contrast, due to the bond's negative vega exposure, the bond risk premium would increase with both its positive delta exposure and the absolute magnitude of its negative vega exposure.

2. Data Processing and Summary Behaviors

We perform a joint analysis of the stocks and bonds on US publicly traded companies based on the following data sources: the Center for Research in Security Prices (CRSP) for stock return information, the WRDS Bond database for bond return information, and Compustat for quarterly financial information.

2.1 Sample construction

We start with the CRSP database for the stock pricing information. We include all the stocks listed on the NYSE, AMEX, and NASDAQ stock exchanges.

We use the WRDS Bond CRSP Link to merge all the corporate bonds from the WRDS Bond Dataset to the CRSP dataset. The WRDS Bond Database sources data from TRACE Standard dataset, TRACE Enhanced dataset, and the Mergent Fixed Income Securities Database (FISD). The Database provides monthly price, return, coupon, and maturity information for all corporate bonds traded since July 2002. We filter the bond data using criteria similar to that in Bessembinder, Kahle, Maxwell, and Xu (2009): (1) The bonds are listed/traded in the US public market. We exclude bonds issued through private placement, under the 144A rule, Yankee Bonds, bonds that are not denominated in US dollar, and bonds from issuers outside the jurisdiction of the United States. (2) The bonds belong to one of the three types: US Corporate Debentures ('CDEB'), US Corporate MTN ('CMTN') or US Corporate MTN Zero ('CMTZ'). (3) The bonds have either fixed or zero coupon type. (4) Bond prices are above \$1 and below \$1000. (5) Bond maturities are no less than one year. (6) Bond prices have shown variations in the last three months. A company can have several outstanding bond issuances. At each month, we aggregate the bond-level monthly return and maturity to company level using the bond outstanding as weights.

We merge the company financial information from Compustat to the CRSP dataset using the CRSP/Compustat Link Table. The financial statement for a company is released quarterly. We assume that the financial statement information becomes available two months after the end of the company's financial quarter.

To be included in our final sample, we require that the company has (1) valid current stock and bond pricing information and return calculation over the next month, (2) 1-year stock daily return history to construct the stock volatility estimator, and (3) valid firm financial report information to implement the structural model.

The final sample consists of 120,840 company-month observations for 1,392 issuing companies spanning the period from July 2002 to November 2020 for a total of 221 months. The selected number of firms per month ranges from 348 to 658, and averages at 547.

2.2 Estimating stock and bond risk exposures

To estimate the stock and bond risk exposures, at each month t and for each company i, we perform a simple implementation of the Merton (1974) model. The procedure is similar to standard literature practices (e.g., Vassalou and Xing 2004; Bharath and Shumway 2008; Bai and Wu 2016). We take the company's market capitalization as the equity value $S_{t,i}$, the company's total book value of debt (short-term debt plus long-term debt) as the strike of the call option $D_{t,i}$, and the weighted average maturity of the company's outstanding bonds as the maturity of the option $\tau_{t,i}$.

We use the past 1-year daily stock return history to construct a stock return volatility estimator $\sigma^S_{t,i}$. We also construct a bond return volatility estimator $\sigma^B_{t,i}$ using the past 3 years of bond monthly excess return history, when a minimum of 12 monthly observations are available. When both the stock and bond return volatility estimators are available, we link them to the asset return volatility via a value-weighted average,

$$\sigma_{t,i} = \frac{S_{t,i}}{F_{t,i}}\sigma^S_{t,i} + \frac{F_{t,i} - S_{t,i}}{F_{t,i}}\sigma^B_{t,i}. \tag{11}$$

When we do not have enough bond return history to construct the bond return volatility estimator, we link the asset return volatility $\sigma_{t,i}$ to the stock return volatility estimator $\sigma_{t,i}$ via the delta exposure,

$$\sigma_{t,i} = \frac{S_{t,i}}{F_{t,i}N(d_{t,i})}\sigma^S_{t,i}. \tag{12}$$

We use the zero-coupon Treasury yield matching the average bond maturity as the riskfree rate r_t. We obtain fixed-term zero-coupon Treasury yields from the Federal Reserve Economic Data (FRED) database and interpolate the yield curve to generate the rate at the average bond maturity. The maximum maturity of the FRED yield curve is 30 years. For bond average maturities longer

Table 1
Summary behaviors of risk exposures

	Mean	SD	Percentile values					Tails	
			1	25	50	75	99	Skewness	Kurtosis
A. Model inputs									
Debt-equity (D/S)	0.87	1.81	0.03	0.21	0.43	0.88	7.71	8.60	108.27
Bond maturity (τ)	8.53	4.79	1.86	5.32	7.28	10.82	24.13	1.88	7.16
Stock volatility (σ^S)	0.35	0.14	0.16	0.25	0.31	0.41	0.83	1.83	6.94
B. Model outputs									
Debt ratio (B/F)	0.28	0.19	0.02	0.13	0.23	0.38	0.83	1.00	0.62
Asset volatility (σ)	0.26	0.11	0.10	0.19	0.24	0.31	0.64	2.05	12.32
Distance to default (d)	2.84	1.53	0.31	1.87	2.60	3.51	7.93	1.63	7.68
Default probability (%)	3.93	7.54	0.00	0.24	1.07	3.96	37.98	3.95	21.04
Delta (δ)	0.96	0.08	0.62	0.96	0.99	1.00	1.00	-3.95	21.04
Vega (ν)	0.16	0.22	0.00	0.02	0.07	0.22	0.92	2.07	5.09

Entries report time-serial averages of the cross-sectional statistics, including the cross-sectional average (Mean), standard deviation (SD), percentile values, skewness, and excess kurtosis. Panel A reports the statistics on the inputs to the structural model. Panel B reports the statistics on the model outputs.

than 30 years, we perform flat extrapolation and set the interpolated rate to the rate at 30 years.

With these inputs, we solve for the company's firm value $F_{t,i}$ and asset return volatility $\sigma_{t,i}$ from the option pricing representation in (1) and the volatility linkage equation in (11) or (12). We then compute the company's distance to default $d_{t,i}$, the delta $\delta_{t,i}$ and the vega $\nu_{t,i}$ risk exposure.

Table 1 reports the time-series averages of the cross-sectional statistics on the inputs and the outputs of our structural model implementation. The statistics include the cross-sectional average (Mean), standard deviation (SD), percentile values at 1, 25, 50, 75, and 99th percentiles, and the cross-sectional skewness and excess kurtosis of the distribution. Panel A reports the statistics on the inputs of the structural model, including the ratio of total book value of debt to market capitalization (D/S), the average bond maturity (τ), and the stock return volatility estimator (σ^S).

For our selected universe, the companies have an average debt-equity ratio of 0.87, but with a large cross-sectional variation, from an average of 0.03 at the first percentile to an average of 7.71 at the 99th percentile. The cross-sectional distributions show high positive skewness and large excess kurtosis. The bond maturity averages at 8.53 years, and ranges from 1.86 at the first percentile to 24.13 at the 99th percentile. The stock return volatility estimators average at 35%, but can vary cross-sectionally from 16% at the first percentile to 83% at the 99th percentile.

Panel B reports the statistics on the model outputs. The market value of debt to firm value (B/F), or the debt ratio, averages at 28%, and varies from 2% to 83% within the 1-99th percentile range. The asset return volatility estimates average at 26%, varying from 10% to 64% within the 1-99th percentile range.

The average distance to default (d) is large at 2.84. As a result, the average default probability (p) is low at merely 3.93%. Nevertheless, the cross-sectional distribution of default probability is highly asymmetric with a heavy right tail. The average default probability can reach as high as 37.98% at the 99th percentile.

Corresponding to the large average distance to default and low default probability, the average stock delta (δ) exposure is very high at 96%. In fact, for over 25% of the companies, the delta exposures are close to 100% for their stocks. The vega (ν) exposure averages at 0.16, and varies from 0 to 0.92 within the 1-99th percentile.

To visualize the cross-sectional distributional behaviors of the risk exposures, we estimate the density of the cross-sectional distribution each month using a Gaussian kernel and a bandwidth twice the default choice (Simonoff 1996). Figure 1 plots the time-series averages of the density estimates. Panel A plots the average cross-sectional distribution of the distance to default measure (d), which has a reasonably symmetric bell shape. When we convert the distance to default into

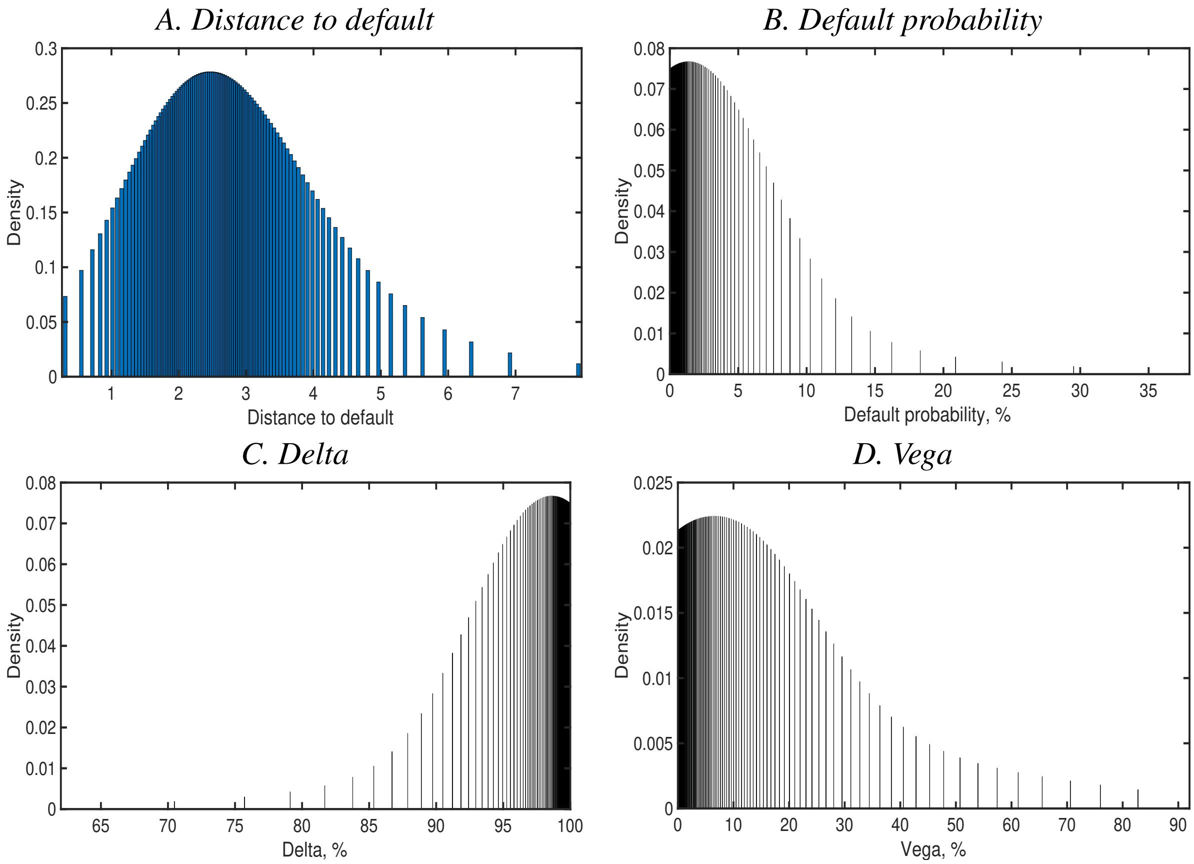

Figure 1
Distributions of delta and vega exposures.
The bars in each panel represent the time-series average of the cross-sectional probability density estimates. The four panels are for four variables: (A) distance to default (d); (B) default probability (p); (C) delta (δ); (D) vega (ν). The densities are estimated with a Gaussian kernel with twice the default bandwidth choice.

a default probability measure, $p = N(-d)$, and estimate the density of the default probability, panel B shows that the average cross-sectional distribution of the probability density is highly asymmetric. A large proportion of the companies have very low default probabilities, as shown by the heavy dark mass on the left side. Meanwhile, the plot highlights the wide dispersion of the default probabilities on the right tail.

Panel C plots the average cross-sectional density of the stock delta exposure (δ). As the delta is simply one minus the default probability $\delta = 1 - p$, the density function shape of the delta exposure represents a mirror image of the density function shape of the default probability, where the wide dispersion is on the side of high default probability and low delta exposure.

Panel D plots the average density of the vega exposure (ν). At a given maturity, the vega exposure reaches its maximum as the distance to default approaches zero. The distance to default estimates are predominantly positive. Out of the 120,840 company-month observation, only 215 observations have negative distance to default estimates. Therefore, for the majority of firms, the vega exposure estimates largely increase with declining distance to default and increasing default probability.

The distance to default measure (d) simultaneously determines the company's default probability ($p = N(-d)$) and the company's stock and bond risk exposures. The delta δ is simply one minus the default probability. The vega ν depends both on the distance to default and the bond maturity, $\nu = \sqrt{\tau} n(d)$. Since the average bond maturity varies within a narrow range, the cross-sectional variation of the vega exposure is also mainly dictated by the variation of distance to default.

To highlight the linkage between the distance to default and default probability on the one side and the delta and vega exposures on the other, we perform local linear regression at each date on the delta and vega exposures against the distance to default measure and the default probability measure, respectively. The local linear regression applies a Gaussian kernel with twice the default bandwidth choice. Figure 2 plot the time-series averages of the estimated relations, with the circles denoting the average percentile values of the distance to default in panel A and default probability in panel B, at 5, 10, 25, 50, 75, 90, and 90th percentiles, respectively. Panel A shows the average dependence structure of the delta (solid line) and vega (dash-dotted line) exposures on distance to default. The bond's delta exposure is simply one minus the stock delta exposure, and the bond's negative vega exposure has the same absolute value as the stock's positive vega exposure. As

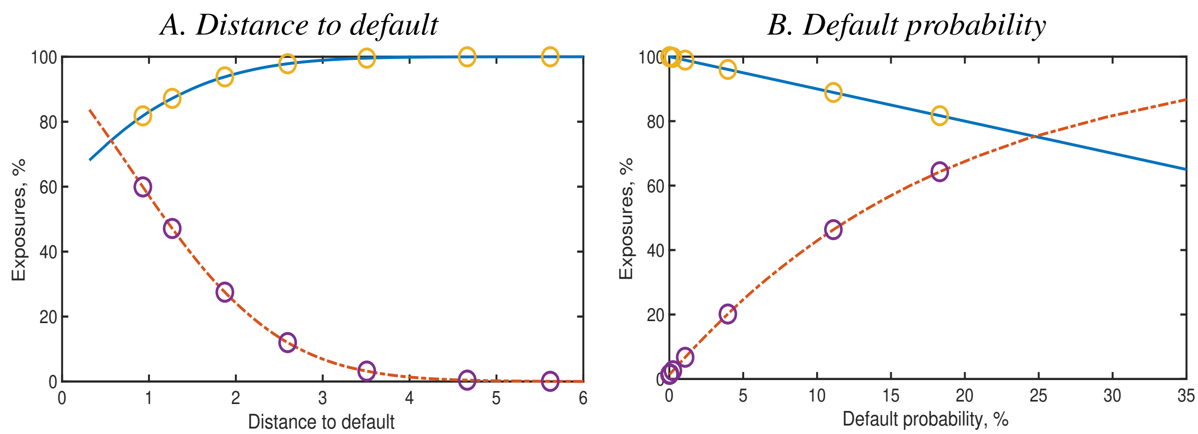

Figure 2
Delta and vega exposures at different default risk levels.
Lines plot the time-series averages of the delta (solid lines) and vega (dash-dotted lines) exposure at different distance to default (panel A) and the default probability levels (panel B). The circles denote the values of the distance to default and default probability at 5, 10, 25, 50, 75, 90, and 95th percentiles, respectively.

distance to default increases, the stock delta exposure increases while the vega exposure declines.

When we plot the average relations of the exposures against the default probability measure in panel B, about half of the companies are clustered to the left with very small default probabilities and the lines highlight the exposure variations of the companies with significant default risks. As the default probability increases, the stock's delta exposure declines, but the embedded optionality, as measured by the vega exposure, increases.

2.3 Stock and bond returns

For each selected company at each month, we construct the stock and bond returns over the next month. The stock returns are directly from CRSP. The bond returns for each company are weighted average of the monthly returns on all the selected bonds for that company. We deduct the one-month Treasury bill rate from the monthly returns to construct the excess returns. The monthly Treasury bill rate data are obtained from French's online data library.

Table 2
Summary behaviors of stock and bond returns

	Mean	SD	Default risk quintile				
			1	2	3	4	5
A. Company default risk							
Default probability	3.93	7.54	0.05	0.36	1.11	3.18	14.96
B. Stock return							
Return (%)	11.41	103.13	10.52	11.42	11.59	11.99	11.55
Volatility (%)	34.51	14.33	25.37	29.03	32.01	36.82	49.30
Beta	1.09	0.43	0.87	0.97	1.04	1.17	1.40
Delta exposure	1.47	0.67	1.13	1.24	1.34	1.51	2.13
Vega exposure	0.41	1.07	0.01	0.03	0.11	0.29	1.62
C. Bond return							
Return (%)	6.21	27.56	4.95	6.08	5.89	6.81	7.33
Volatility (%)	7.81	5.03	5.64	6.65	7.33	8.39	11.08
Delta exposure	0.09	0.12	0.01	0.03	0.06	0.11	0.27
Vega exposure	-0.50	0.56	-0.12	-0.26	-0.43	-0.66	-1.03

Entries report the time-series averages of the cross-sectional statistics on the stock and bond monthly excess returns, including the cross-sectional average (Mean), standard deviation (SD), and averages at each default risk quintile constructed based on the default probability estimates. Panel A repeats the statistics on the default probability, based on which the quintile portfolios are constructed. Panels B and C report the statistics on the stock and bond excess returns, respectively, as well as the risk exposures. The mean excess return and return volatility are in annualized percentages.

Table 2 reports the time-series averages of the cross-sectional statistics of the stock and bond monthly excess returns. At each month, we compute the cross-sectional average (Mean) and standard deviation (SD) of the stock and bond monthly excess returns. We also sort the companies into default risk quintiles based on the default probability estimates and compute the average stock and bond monthly excess returns and their risk exposures for each quintile portfolio. Panel A repeats the mean and standard deviation statistics on the default probability, as well as the average default probability levels at the five quintiles. The average default probability increases from merely 5 basis points at the first quintile to 14.96% at the fifth quintile.

Panel B reports the stock excess return behavior, including the mean annualized excess return, the annualized return volatility, the stock return beta, and the stock's levered delta and vega exposures. The stock return volatility and beta estimators are directly constructed from the daily stock return histories at each date. The levered delta and vega exposures are constructed by multiplying the unlevered delta and vega exposure estimates ($\delta_{t,i}, \nu_{t,i}$) with the leverage multiple F_t/S_t. The annualized monthly stock excess return averages at 11.41%. The annualized stock return volatility averages at 34.51%. Across the five default risk quintiles, the mean excess return first increases with the default risk from 10.52% at the first quintile to 11.99% at the fourth quintile. As the default risk further increases, the average stock excess return becomes lower at 11.55% at the fifth quintile. The quintile portfolio return volatility and beta estimates both increase monotonically with default probability, driven by increasing levered delta and vega exposures.

Panel C reports the bond excess return behavior. The annualized mean bond excess return averages at 6.21%. The bond return volatility estimators computed from the monthly excess return histories have an average of 7.81%. The mean excess returns are similar for the first three quintiles, but increase strongly at the last two quintiles as the average default risk becomes significantly larger. The bond's levered delta exposure and its absolute vega exposure both increase monotonically with the default probability quintile. As a result, the bond return volatility also increase monotonically with the risk quintile.

The unlevered delta exposures of stocks and bonds sum to one and their unlevered vega exposures are exactly opposite of each other. The levered exposures lose this mirror image because different leverage multiples F/S and F/B are applied to the stock and bond returns, respectively. Compared to the levered exposures of the stock returns, the bond returns have much smaller levered delta exposures across all risk quintiles, but larger absolute vega exposures.

3. A Joint Stock and Bond Return Factor Model

We hypothesize that investors are averse to risk but love optionality; nevertheless, it is inherently difficult to disentangle the two effects because optionality tends to increase hand in hand with volatility and the two types of risk exposures tend to be highly correlated. We implement the Merton (1974) structural model for each company to separate the delta and vega exposures of the stock and the bond for that company. Under this structural setting, the cross-sectional variations of the delta exposure $\delta_{t,i} = N(d_{t,i})$ and the vega exposure $\nu_{t,i} = \sqrt{\tau_{t,i}}n(d_{t,i})$ are largely dictated by the variation of the distance to default measure ($d_{t,i}$). For much of the universe with strictly positive distance to default estimates, $N(d_{t,i})$ and $n(d_{t,i})$ have a monotonic relation. As a result, with the bond maturity varying within a narrow range, the two risk exposures are structurally linked by the distance to default measure and are hardly separable.

To better disentangle the two types of exposures, we propose the following joint cross-sectional stock and bond return factor model,

$$\begin{bmatrix} \mathbf{SUR}_{t+1} \\ \mathbf{BUR}_{t+1} \end{bmatrix} = \zeta_{t+1} + \begin{bmatrix} \delta_t & \nu_t \\ 1-\delta_t & -\nu_t \end{bmatrix} \begin{bmatrix} \eta_{\delta,t+1} \\ \eta_{\nu,t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{s,t+1} \\ \mathbf{e}_{b,t+1} \end{bmatrix}, \tag{13}$$

where ($\mathbf{SUR}_{t+1}, \mathbf{BUR}_{t+1}$) denote the $(N_t \times 1)$ vectors of stock and bond unlevered excess returns over the next month $(t+1)$, respectively, across the universe of N_t companies, (δ_t, ν_t) denote the two $(N_t \times 1)$ vectors of the stock unlevered delta and vega exposures, and $(1-\delta_t, -\nu_t)$ denote the two $(N_t \times 1)$ vectors of the bond unlevered delta and vega exposures. At each month t, Equation (13) stacks the two sets of excess returns into a $(2N_t \times 1)$ vector and links it to the corresponding $(2N_t \times 2)$ stacked delta and vega exposure matrix.

Performing the cross-sectional regression on the $(N_t \times 1)$ stock or bond excess returns alone against their respective delta and vega exposures would face a potential collinearity issue because the two exposures are highly correlated for a given stock or bond universe. Over our sample period, the cross-sectional correlations between the stock unlevered delta and vega exposures ($\delta_{t,i}$ and $\nu_{t,i}$) average at -89%. Since the bond's unlevered delta exposure is $(1-\delta_{t,i})$ and its unlevered vega exposure is $-\nu_{t,i}$, the cross-sectional correlations between the bond's unlevered delta and vega exposures are the same and equally negative. Stacking the exposures on stocks and bonds together helps break this collinearity. The stacked delta and vega exposures have an average cross-sectional correlation of 44%. The stacking also expands the range of the two regressors. Table 1 shows that the estimates for $\delta_{t,i}$ have a narrow average range of variation between 0.62 and 1 within the 1-99th percentiles, and the estimates for $\nu_{t,i}$ vary between 0 and 0.92 on average within the 1-99th percentiles. The stacking expands the delta exposure to a much wider range of [0,1] and also doubles the vega exposure range to an average of $(-0.92, 0.92)$ within the 1-99th percentiles. The expanded range and the broken collinearity between the two types of exposures for the stacked excess returns can help disentangle the two types of risk exposures and enhance the identification.

We perform the cross-sectional regression with value weighting. We use the firm value of each company as the weight for the unlevered stock and bond excess returns and their exposures. The factor model can be specified either in unlevered excess returns as in (13) with the firm value as value weights or in levered excess returns with the corresponding stock and bond value as value weights. The value weighted regressions would generate identical slope coefficient estimates for the two types of specifications.

The slope estimates from the factor return model represent the excess returns on the two risk factor portfolios, with $\eta_{\delta,t+1}$ capturing the excess return on the asset return risk portfolio that targets a unit asset return risk exposure but zero vega optionality exposure, and $\eta_{\nu,t+1}$ capturing the excess return on the optionality risk portfolio that targets a unit vega exposure but zero delta

Table 3
Summary behaviors of joint stock-bond return factor models

Model	*A. Unadjusted*			*B. Volatility-adjusted*			*C. Beta-adjusted*		
	ζ	η_δ	η_ν	ζ	η_δ	η_ν	ζ	η_δ	η_ν
Mean	0.23	7.72	-5.41	-0.25	41.08	-43.17	0.03	11.26	-12.21
Volatility	2.25	10.77	6.26	2.37	52.79	53.27	2.31	14.62	14.34
NW	0.39	2.98	-3.59	-0.42	3.35	-3.49	0.06	3.31	-3.66
IR	0.10	0.72	-0.86	-0.11	0.78	-0.81	0.01	0.77	-0.85
Adjusted-R^2		28.68			30.73			30.91	

Entries report the summary statistics on the coefficient estimates of the joint stock and bond return factor model. The statistics include the mean return (Mean) and return volatility (Volatility) in annualized percentages, Newey-West t-statistics (NW), and annualized information ratio (IR). The last row reports the time-series averages of the adjusted-R^2 estimates in percentages. The three panels represent three model specifications that differ in the adjustment of the risk exposures: (A) unadjusted, (B) adjusted by asset return volatility, and (C) adjusted by asset return beta.

risk exposure. The time-series averages of the excess returns on the two factor portfolios can be regarded as the average estimates of the return risk premium and optionality risk premium, respectively. The factor model in (13) also incorporates an intercept ζ_{t+1}, the estimate of which represents the excess return on a joint stock and bond portfolio with neutralized exposures to the delta and vega risk.

Panel A of Table 3 reports the summary time-series behaviors of the cross-sectional regression estimates for the joint return factor model in (13). The statistics include the annualized percentage mean return (Mean), annualized percentage return volatility (Volatility), Newey and West (1987) t-statistics (NW), and the annualized information ratio (IR). The last row reports the time-series averages of the percentage adjusted R^2 estimates of the cross-sectional regressions. On average, the factor model can explain 28.68% of the joint stock-bond cross-sectional return variation.

The return risk portfolio that targets a unit delta exposure but zero vega exposure has an annualized mean excess return of 7.72% and an annualized volatility of 10.77%. The mean excess return estimate is strongly positive and statistically significant with a Newey and West (1987) t-value of 2.98. Investing in this portfolio generates an annualized information ratio of 0.72.

The optionality risk portfolio that targets a unit vega exposure and zero delta exposure has an annualized mean excess return of -5.41%, and an annualized return volatility of 6.26%. The mean excess return is strongly negative and strongly statistically significant with a Newey and West (1987) t-value of -3.59. Shorting the optionality risk portfolio generates an annualized information ratio estimate at 0.86.

By construction, the intercept portfolio has zero delta and vega exposure. The excess returns on the zero-risk portfolio average at merely 23 basis points and are not statistically different from zero. The intercept portfolio with neutralized delta and vega exposures does not generate significantly positive or negative average excess returns.

3.1 Adjusting risk magnitude difference across firms

The return factor model in (13) assumes that the same delta and vega exposure are priced the same across different companies. In practice, even for the same delta and vega risk exposure, different companies can show different degrees of firm value variations, as reflected by the different magnitudes of the asset return volatility estimates. The variation of the asset return volatility can also differ across different companies, potentially in proportion to the asset return volatility level. To accommodate the risk magnitude differences across different companies, we propose an alternative return factor model specification that adjusts the risk exposures with the asset return volatility level differences across different companies,

$$\begin{bmatrix} \mathbf{SUR}_{t+1} \\ \mathbf{BUR}_{t+1} \end{bmatrix} = \zeta_{t+1} + \begin{bmatrix} \delta_t \sigma_t & \nu_t \sigma_t \\ (1-\delta_t)\sigma_t & -\nu_t \sigma_t \end{bmatrix} \begin{bmatrix} \eta_{\delta,t+1} \\ \eta_{\nu,t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{s,t+1} \\ \mathbf{e}_{b,t+1} \end{bmatrix}. \tag{14}$$

By adjusting the risk exposures with the asset return volatility σ_t, Equation (14) replaces the common market pricing per unit exposure assumption in (13) with common market pricing per unit of risk-adjusted exposure, where the risk adjustment is through the asset return volatility estimate of each company. Table 3 reports the the estimation results on this volatility-adjusted risk factor model in panel B. With the volatility adjustment on the risk exposures, the time-series average on the adjusted R^2 estimates increases to 30.73%.

The volatility adjustment changes the scale of the factor portfolios, making the magnitudes of the mean excess returns not directly comparable. More informative are the absolute t-values of the mean excess returns and the absolute annualized information ratios. With the volatility adjustment, the average excess return on the asset return risk portfolio remains strongly positive and the average excess return on the optionality risk portfolio remains strongly negative. In particular, long the asset return risk portfolio generates an annualized information ratio of 0.78, whereas shorting the optionality risk portfolio generates an annualized information ratio of 0.81. The time-series average of the intercept estimates remains small and insignificant.

Classic asset pricing theories often differentiate systematic risk from diversifiable risk and argue that only systematic risk is priced because idiosyncratic risk can be diversified away without cost. To examine this diversification effect, we propose another return factor model by replacing the asset total return volatility adjustment σ_t with the asset return beta β_t,

$$\begin{bmatrix} \mathbf{SUR}_{t+1} \\ \mathbf{BUR}_{t+1} \end{bmatrix} = \zeta_{t+1} + \begin{bmatrix} \delta_t \beta_t & \nu_t \beta_t \\ (1-\delta_t)\beta_t & -\nu_t \beta_t \end{bmatrix} \begin{bmatrix} \eta_{\delta,t+1} \\ \eta_{\nu,t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{s,t+1} \\ \mathbf{e}_{b,t+1} \end{bmatrix}. \tag{15}$$

Analogous to the asset return volatility computation in (12), we estimate the stock return beta ($\beta^S_{t,i}$) based on the past 1-year daily return history at each time t and for each company i, and convert the stock return beta to asset return beta by adjusting for the delta exposure,

$$\beta_{t,i} = \frac{S_{t,i}}{F_{t,i} N(d_{t,i})} \beta^S_{t,i}. \tag{16}$$

Table 3 summarizes the regression results of this beta-adjusted return factor model in panel C. Compared to the volatility adjustment, beta adjustment generates slightly higher average adjusted R^2 estimate at 30.91%. The information ratios and the t-values of the risk factor portfolio returns are very similar, with the information ratio slightly lower on the return risk portfolio and slightly higher on the optionality portfolio. The average intercept estimates remain small and insignificant.

Overall, the evidence lends support to the hypothesis of common market pricing per unit of risk-adjusted exposure. Furthermore, adjusting the risk exposures with asset return beta generates slightly higher average R^2 estimate than adjusting the risk exposures with asset return volatility.

3.2 Accommodating potential market segmentation

The intercepts in the return factor model specifications in (13)-(15) capture the excess returns on a joint stock-bond portfolio with zero delta and vega exposures. Estimation shows that on average, the stock and bond market together does not charge a risk premium once the delta and vega risk exposures are neutralized.

To accommodate potential market segmentation in the sense that the stocks and bonds are priced differently after controlling for their risk exposures, we replace the single intercept with two dummy variables to construct separate zero-risk portfolios on the stock and bond markets, respectively,

$$\begin{bmatrix} \mathbf{SUR}_{t+1} \\ \mathbf{BUR}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{1}_s & 0 \\ 0 & \mathbf{1}_b \end{bmatrix} \begin{bmatrix} \zeta_{s,t+1} \\ \zeta_{b,t+1} \end{bmatrix} + \begin{bmatrix} \delta_t a_t & \nu_t a_t \\ (1-\delta_t)a_t & -\nu_t a_t \end{bmatrix} \begin{bmatrix} \eta_{\delta,t+1} \\ \eta_{\nu,t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{s,t+1} \\ \mathbf{e}_{b,t+1} \end{bmatrix}, \tag{17}$$

where $(\mathbf{1}_s, \mathbf{1}_b)$ denote two dummy variable vectors separating out the stock and bond excess returns. The coefficient estimates $(\zeta_{s,t+1}, \zeta_{b,t+1})$ on the two dummy variables capture the excess returns on the zero-risk stock and bond portfolios, respectively. As before, we consider three types of risk exposure adjustments: (A) no adjustment ($a_t = 1$), (B) asset return volatility adjustment ($a_t = \sigma_t$), and (C) asset return beta adjustment ($a_t = \beta_t$).

Table 4 reports the summary behaviors of the coefficient estimates. The return behaviors for the delta and vega factor portfolios are similar to those reported in Table 3. The asset return risk factor portfolio generates significantly positive average excess returns whereas the optionality risk factor portfolio generates significantly negative average excess returns. The magnitudes and signs for the average intercept estimates vary with different risk adjustments, but none are statistically different from zero. Therefore, once the delta and vega risk exposures are neutralized, neither the stock market nor the bond market generates significant average excess returns.

Table 4
Summary behaviors of joint stock-bond return factor models with separate intercepts

Model	*A. Unadjusted*				*B. Volatility-adjusted*				*C. Beta-adjusted*			
	ζ_s	ζ_b	η_δ	η_ν	ζ_s	ζ_b	η_δ	η_ν	ζ_s	ζ_b	η_δ	η_ν
Mean	0.29	0.17	7.72	-5.32	-0.63	0.08	41.37	-40.15	-0.13	0.19	11.26	-11.74
Volatility	4.12	1.09	10.79	6.13	4.29	1.23	53.56	50.05	4.13	1.16	14.72	13.69
NW	0.29	0.58	2.97	-3.62	-0.61	0.23	3.32	-3.45	-0.14	0.64	3.29	-3.69
IR	0.07	0.16	0.72	-0.87	-0.15	0.07	0.77	-0.80	-0.03	0.16	0.77	-0.86
Adjusted-R^2	29.13				31.20				31.35			

Entries report the summary statistics on the coefficient estimates of the joint stock and bond return factor model with separate intercepts. The statistics include the mean return (Mean) and return volatility (Volatility) in annualized percentages, Newey-West t-statistics (NW), and annualized information ratio (IR). The last row reports the time-series averages of the adjusted R^2 estimates in percentages. The three panels represent three specifications that differ on the adjustment of the risk exposures: (A) unadjusted, (B) adjusted by asset return volatility, and (C) adjusted by asset return beta.

3.3 Evidence and discussion on the factor return behavior

Estimating the joint return factor model generates the excess return time series on the delta and vega factor portfolios. The two excess return series also show distinct time-series variations. The correlation estimates between the two excess return series from specifications (13)–(15) are at -44%, -26%, and -13%, respectively. The lowest absolute correlation estimate comes from the specification in (15), lending further support to beta risk adjustment on the risk exposures. Figure 3 plots the 1-year moving average of the excess return time series on the two factor portfolios, solid line for the asset return risk factor portfolio and dashed line for the optionality risk portfolio. The factor portfolios are constructed with asset return beta adjustment as in specification (15). The monthly excess returns are very noisy. Applying the moving average removes the short-term noise and reveals the longer-term variation patterns of the underlying risk premiums.

The solid line in Figure 3 shows that the asset return risk portfolio generates positive excess returns on average and moves largely in line with the business cycle. The excess returns are high

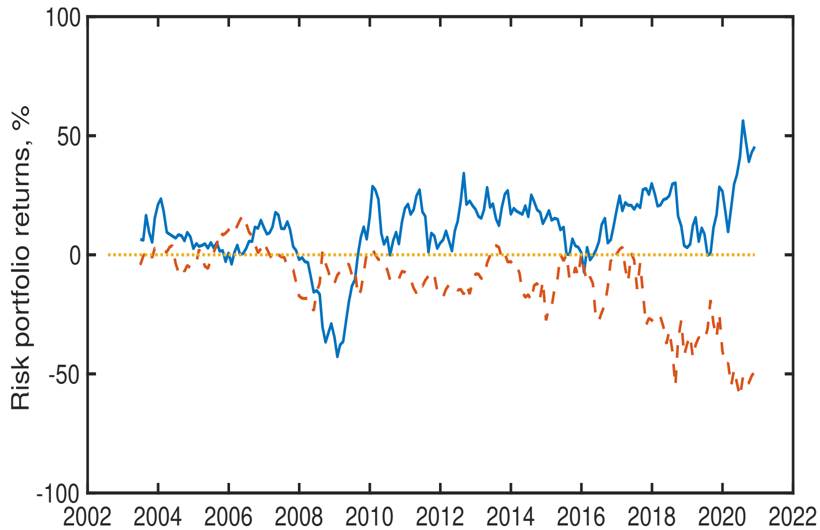

Figure 3
Time-series variation of factor portfolio returns
Lines plot the 1-year moving average of the excess return series on the asset return risk portfolio (solid line) and the optionality risk portfolio (dashed line), obtained from the cross-sectional regression of the joint return factor model in (15).

during booming market conditions, but become negative during the 2008–2009 financial crises and the ensuing recession. The strongly significant positive average excess return per unit beta exposure is in line with the implication of classic capital asset pricing theories (e.g., Sharpe 1964; Lintner 1965; Black 1972; Merton 1980; Campbell and Cochrane 1999; He and Krishnamurthy 2013). Investors are on average risk averse and require an average positive compensation for taking on risky investments.

By comparison, the dashed line in Figure 3 shows that the optionality risk portfolio generates strongly negative excess returns on average. Just as investors are risk averse and ask for a risk compensation, the highly negative average excess return per unit vega exposure highlights the market's strong preference for positive optionality exposure.

The literature has proposed various channels to generate the positive investor preference for optionality. In particular, the cumulative prospect theory of Tversky and Kahneman (1992) captures this optionality-loving behavior by generating risk aversion for gains of high probability and losses of low probability but risk seeking for losses of high probability and gains of low probability. In line with its theoretical prediction, several studies find that securities with lottery-like payoffs tend to generate lower returns on average (e.g., Barberis and Huang 2008; Kumar 2009; Boyer, Mitton, and Vorkink 2010; Conrad, Kapadia, and Xing 2014; Eraker and Ready 2015).

The most direct evidence on investor preference for optionality comes from the options market, where researchers have found that investors are willing to pay a high premium to buy options. Bakshi and Kapadia (2003) document that the delta-hedged stock option returns are on average negative. Carr and Wu (2009) use a portfolio of options across all strikes at a fixed maturity to create a synthetic variance swap contract and find that the synthetic variance swap contracts on stock index options bear a highly negative variance risk premium. The negative variance risk premium reflects the fact that investors are willing to pay a high premium to gain the positive convexity of the variance payoff structure.

Based on implementation of structural models, investors have designed capital structure arbitrage strategies. The strategy is to long bonds, or short credit default swap contracts, and short the stock of the same company to hedge the delta exposure when the credit spreads of the company are deemed too high relative to model valuation (e.g., Yu 2006; Duarte, Longstaff, and Yu 2007). Such a delta-neutral strategy is effectively short the embedded call option and loads negatively on the vega exposure. Hence, although the strategy is designed to capture bond mispricing, its investment performance is at least partially driven by the positive risk premium on the strategy's negative optionality exposure. In experimenting with different capital structure arbitrage implementations, Bajlum and Larsen (2007) find that using implied volatility from equity options results in a substantial gain in strategy execution and highly significant excess returns. We conjecture that such an implementation at least partially accounts for the volatility risk premium and enhances the identification of true mispricing opportunities. For future research, constructing long-short bond portfolios with explicit vega neutrality constraint can better highlight the statistical arbitrage opportunities from bond mispricing, while minimizing the contributions from the optionality exposure.

3.4 Statistical risk exposures on the factor return series

We have constructed the delta and vega exposures for the stock and the bond of a company based on an implementation of the Merton (1974) structural model. Alternatively, we can also estimate the statistical exposures of any security returns on the two factor return series. While the structural construction is more fundamental driven, the statistical approach is more generally applicable, even to securities with ambiguous structures and unknown optionality.

We estimate the statistical exposures of the stock and bond excess returns on the two factor excess return series with a 3-year rolling window. At each date t and for each company i, we generate the beta estimates on the two factor portfolios with a bivariate time-series regression on the stock and bond excess returns, respectively

$$\begin{aligned} \mathbf{r}_{s,t,i} &= a_{t,i} + \beta^{\delta}_{s,t,i}\eta_{\delta,t} + \beta^{\nu}_{s,t,i}\eta_{\nu,t} + \mathbf{e}_{s,t,i}, \\ \mathbf{r}_{b,t,i} &= a_{t,i} + \beta^{\delta}_{b,t,i}\eta_{\delta,t} + \beta^{\nu}_{b,t,i}\eta_{\nu,t} + \mathbf{e}_{b,t,i}, \end{aligned} \tag{18}$$

where $(\mathbf{r}_{s,t,i}, \mathbf{r}_{b,t,i})$ denote the vector of stock and bond excess returns for company i over the past 3-year history at month t and $(\eta_{\delta,t}, \eta_{\nu,t})$ denote the corresponding historical vector of the excess returns on the delta and vega factor portfolios. The slope coefficients $(\beta^{\delta}_{s,t,i}, \beta^{\nu}_{s,t,i}, \beta^{\delta}_{b,t,i}, \beta^{\nu}_{b,t,i})$ are the stock and bond's statistical risk exposure estimates on the two factor portfolios. We perform the regression estimation as long as we have 12 monthly observations during the past 3 years.

Table 5 reports in panel A the time-series averages of the cross-sectional summary statistics on

Table 5
Summary behaviors of stock and bond return exposures

Exposure	*A. Statistical*			*B. Structural*			*C. Correlation*
	Average	Median	SD	Average	Median	SD	Average
β_s^δ	0.94	0.96	0.69	0.96	1.05	0.43	0.93
β_s^ν	0.22	0.28	0.76	0.14	0.07	0.51	0.49
β_b^δ	0.10	0.07	0.22	0.09	0.04	0.13	0.75
β_b^ν	-0.32	-0.24	0.24	-0.31	-0.21	0.75	0.80

Entries report the time-series averages of the cross-sectional summary statistics on the stock and bond return risk exposure estimates. Panel A generates the risk exposure estimates statistically from a 3-year rolling-window regression of the stock and bond excess returns on the two factor excess return series. Panel B constructs the risk exposures through the implementation of the Merton structural model. The statistics include the value-weighted average, the median, and cross-sectional standard deviation (SD). Panel C reports the time-series averages of the value-weighted cross-sectional correlation estimates between the two sets of risk exposure estimates.

the four sets of statistical risk exposure estimates obtained from the above return regressions. The statistics include the value-weighted average, the median, and the cross-sectional standard deviation (SD). For value weighting, we use stock value on stock return exposures and bond value for bond return exposures. For comparison, panel B reports the summary statistics on the corresponding structural risk exposure estimates based on our implementation of the structural model,

$$\begin{aligned} \beta_{s,t,i}^{\delta} &= \frac{F_{t,i}}{S_{t,i}}\delta_{t,i}\,\beta_{t,i}, & \beta_{s,t,i}^{\nu} &= \frac{F_{t,i}}{S_{t,i}}\nu_{t,i}\,\beta_{t,i} \\ \beta_{b,t,i}^{\delta} &= \frac{F_{t,i}}{B_{t,i}}(1-\delta_{t,i})\,\beta_{t,i}, & \beta_{b,t,i}^{\nu} &= -\frac{F_{t,i}}{B_{t,i}}\nu_{t,i}\,\beta_{t,i} \end{aligned} \tag{19}$$

where we first lever up the unlevered delta and vega exposures of the stock and bond and then adjust the levered exposures by the company's asset return beta $\beta_{t,i}$.

The statistics in panels A and B compare the average magnitudes and variations on the two sets of exposure estimates. In panel C, we measure the value-weighted cross-sectional correlation between the two sets of risk exposure estimates and report the time-series average of the cross-sectional correlations on each exposure. The structural risk exposure estimates are available on the full sample of 120,840 company-month observations. The rolling-window beta estimation starts 1 year from the start of the sample in July 2003 and does not generate estimates for companies with fewer than 12 monthly historical observations. The procedure generates statistical beta estimates on 102,937 company-month observations. For comparability, the statistics in the table are computed over the common sample of the statistical and structural estimates.

The statistical and structural estimates on the stock return delta exposure β_s^δ have similar average magnitudes at 0.94–0.96, but the statistical estimates vary over a wider range with an average cross-sectional standard deviation estimate of 0.69, compared to a standard deviation estimate of 0.43 for the structural estimates. The larger variation of the statistical estimates can come from statistical estimation errors with the monthly returns; nevertheless, the two sets of estimates show very high cross-sectional correlations, averaging at 93%. The high average correlation estimate suggests that the statistical return regression can effectively identify the stock return's delta exposure, although with somewhat larger noise.

The statistical estimates on the stock return optionality exposure (β_s^ν) are larger and show wider dispersion than its structural counterparts. The statistical exposure estimates have a value-weighed average of 0.22 and a median of 0.28, compared to the value-weighted average of 0.14 and the median of 0.07 for the structural estimates. The statistical estimates also have larger standard deviation at 0.76, compared to the standard deviation estimate of 0.51 for the structural estimates. The two sets of estimates show positive cross-sectional correlation on average, but the average estimate of 49% is lower than that on the stock delta exposure. Compared to the structural vega exposure estimates, the larger statistical estimates may capture some other forms of optionality (such as operational optionality) not fully captured by the structural model.

Compared to the stock return's delta exposure, the bond return's delta exposure are much smaller, averaging at 0.1 for the statistical estimates and 0.09 for the structural estimates. Despite the small average magnitudes, the two sets of estimates have highly positive cross-sectional correlation estimates averaging at 75%.

As bonds are short the call option in the structural model, the structural estimates for the bond return optionality exposure are all negative, with a value-weighted average of -0.31. The average statistical bond optionality exposure estimate is also negative with a similar magnitude at -0.32. Interestingly, the statistical exposure estimates show much narrower cross-sectional variation, with an average standard deviation of 0.24, compared to an average standard deviation of 0.75 on the structural estimates. The value-weighted cross-sectional correlations between the two sets of estimates remain highly positive, with a sample average of 80%.

We further examine whether we can effectively extract the factor portfolios from the statistical risk exposure estimates via the following joint stock and bond return factor model,

$$\begin{bmatrix} \mathbf{SER}_{t+1} \\ \mathbf{BER}_{t+1} \end{bmatrix} = \zeta_{t+1} + \begin{bmatrix} \beta^{\delta}_{s,t} & \beta^{\nu}_{s,t} \\ \beta^{\delta}_{b,t} & \beta^{\nu}_{b,t} \end{bmatrix} \begin{bmatrix} \eta_{\delta,t+1} \\ \eta_{\nu,t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{s,t+1} \\ \mathbf{e}_{b,t+1} \end{bmatrix}, \tag{20}$$

where at each month t we regress the stacked one-month ahead levered excess returns on the stocks and bonds $(\mathbf{SER}_{t+1}, \mathbf{BER}_{t+1})$ against their corresponding stacked statistical delta and vega risk exposures. We use the stock and bond values as the weights for the stock and bond returns, respectively. Table 6 reports the summary statistics of the regression estimates. Leaving the first year of data for statistical beta estimation, we perform the cross-sectional regression monthly from July 2003 to November 2020 for 209 months. The statistics are computed on the regression estimates over the 209 months.

Table 6
Joint stock and bond return factor model on statistical risk exposure estimates

	ζ	η_δ	η_ν
Mean	0.84	10.27	-8.35
Volatility	2.08	12.59	15.65
NW	1.63	3.33	-2.00
IR	0.40	0.82	-0.53
Adjusted-R^2		33.50	

Entries report the summary statistics of the coefficient estimates from the cross-sectional regression of stacked stock and bond excess returns on their respective statistical beta estimates on the asset return risk portfolio and the optionality risk portfolio. The statistics include the mean return (Mean) and return volatility (Volatility) in annualized percentages, Newey-West t-statistics (NW), and the annualized information ratio (IR). The last row reports the time-series averages of the percentage adjusted R^2 estimates.

With the statistical risk exposure estimates, the average excess return on the asset return risk factor portfolio remains significantly positive and the average excess return on the optionality risk factor portfolio remains significantly negative. The absolute magnitudes of the average excess return estimates are smaller than those extracted from the structural risk exposure estimates. Furthermore, the average intercept estimate becomes larger and marginally significant. The smaller average slope coefficient estimates and the larger average intercept suggest that the statistical risk exposures are potentially noisier than the structural risk exposure estimates, leading to larger biases due to errors-in-variable issues. Nevertheless, for securities or portfolios with other sources of optionality that are hard to quantify structurally (such as the growth options discussed in Barinov and Chabakauri (2023)), regressing their excess returns on the excess return of the optionality risk portfolio provides an effective alternative in estimating the optionality exposures.

In related literature, Ang, Hodrick, Xing, and Zhang (2006) use changes in the VIX index to proxy the volatility risk variation and estimate stock return betas on the VIX change. They find that stocks with high exposures to the volatility index variation generate low average excess returns.

4. Embedded Optionality and Risk-Return Behaviors

For a security with a convex payoff structure, high volatility means high risk, but it also generates high optionality value. The entanglement of risk with optionality, combined with the opposite investor preference for the two sources of risk, can confound the return-return analysis on the security. Applying our structural decomposition of the two risk types, this section strives to shed light on some seemingly anomalous risk-return behaviors.

4.1 The distress puzzle on stocks and bonds

Researchers have found that for companies with high default risk, their stocks tend to earn abnormally low average excess returns whereas their bonds tend to bear abnormally high credit spreads. Our structural separation of the asset return risk from optionality can help explain these puzzling pricing behaviors.

Proposition 1 decomposes the stock risk premium (SRP) and bond risk premium (BRP) for a company at a given date t into contributions from (1) their delta exposures and the asset return risk premium (RRP) and (2) their vega exposures and the optionality risk premium (ORP):

$$\begin{aligned} \mathrm{SRP}_t &= \left[\frac{F_t}{S_t}\delta_t\right]\mathrm{RRP}_t + \left[\frac{F_t}{S_t}\nu_t\right]\mathrm{ORP}_t, \\ \mathrm{BRP}_t &= \left[\frac{F_t}{B_t}(1-\delta_t)\right]\mathrm{RRP}_t - \left[\frac{F_t}{B_t}\nu_t\right]\mathrm{ORP}_t. \end{aligned} \tag{21}$$

While investors are averse to risk and demand a positive average asset return risk premium, their positive preference for optionality exposures generates a negative average optionality risk premium. For companies with high default risk, the large positive vega exposure of their stocks combined with the negative optionality risk premium lowers the stock risk premium, whereas the highly negative vega exposure of their bonds further increases the bond risk premium.

To highlight the relative contributions of the two types of risk exposures to stock and bond risk premiums, we start with a set of prefixed levels of default probabilities (p) from 0 to 10%, and take the pooled sample average values on the asset return volatility ($\overline{\sigma} = 26.28\%$), the asset return beta ($\overline{\beta} = 0.81$), the riskfree rate ($\overline{r} = 2.74\%$), and the bond maturity ($\overline{\tau} = 8.53$). Based on these average values, we convert the set of default probabilities into delta (δ) and vega (ν) exposures and market debt and equity ratios ($B/F, S/F$) through the structural model. Furthermore, we set the average asset return risk premium and optionality risk premium based on the estimations results on the return factor model in (15),

$$\overline{\mathrm{RRP}} = \overline{\beta}\,\overline{\eta}_\delta, \quad \overline{\mathrm{ORP}} = \overline{\beta}\,\overline{\eta}_\nu, \tag{22}$$

with $(\overline{\eta}_\delta, \overline{\eta}_\nu)$ denoting the average annualized excess returns on the return risk portfolio and the optionality risk portfolio, respectively. Finally, we combine the average risk premium estimates with the stock and bond risk exposures to generate the stock and bond risk premium at each default probability level.

Figure 4 plots the schematic relations. Panels A and B plot the stock and bond risk exposures at different levels of default probabilities, with the dash line in each panel denoting the delta exposure and the dash-dotted line denoting the vega exposure. Panels C and D plot the corresponding stock and bond risk premium contributions from the two exposures, with the dashed line for asset return risk premium contribution and the dash-dotted line for the optionality risk premium contribution. The solid lines in panels C and D plot the total stock and bond return risk premium, respectively, as the summation of the contributions from the two risk exposures.

At zero debt and zero default probability, the stock's delta exposure starts at 100%. As the

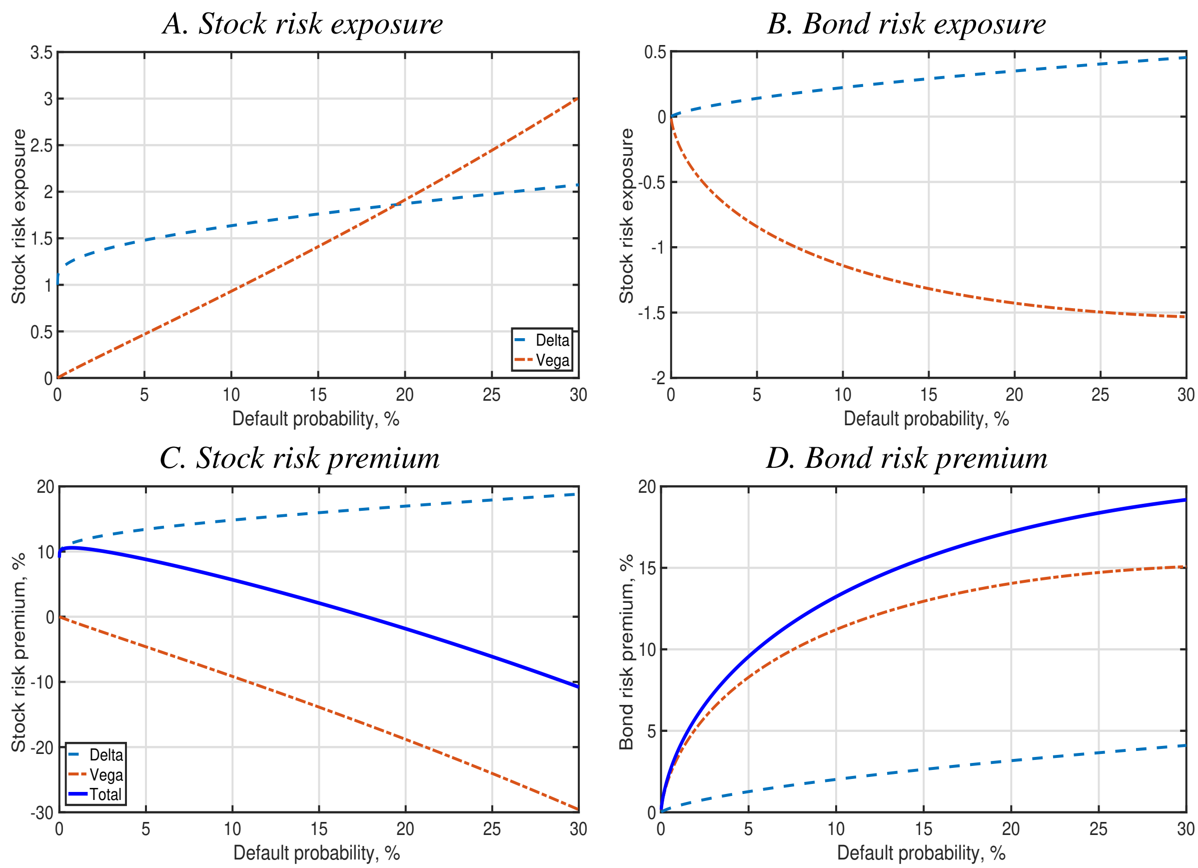

Figure 4
Decompose stock and bond risk premium variation with default risk
Panels A and B plot the stock and bond risk exposures at different levels of default probabilities. In each panel, the dash line denotes the delta risk exposure and the dash-dotted line denotes the vega risk exposure. Panels C and D plot the corresponding risk premium contributions from the two types of risk exposures, with the solid lines denoting the total stock and bond risk premium, respectively, as the summation of the contributions from the two risk sources.

leverage and accordingly the default probability increase, the distance to default shortens and the unlevered delta δ declines; nevertheless, the leverage multiple (F/S) also increases so that the stock's levered delta exposure $(F/S)\delta$ increases slowly with increasing default risk, as highlighted by the dashed line in panel A of Figure 4.

The stock's vega exposure also starts at zero when the company has zero debt and hence zero optionality. As the default probability increases, the dash-dotted line in panel A of Figure 4 shows that the stock's levered vega exposure increases almost linearly and much faster than the delta exposure increase. The vega exposure ultimately surpasses the delta exposure when the default probability is higher than 20%.

In Panel C, as the default probability increases, the dashed line shows that the increasing delta exposure on the stock generates increasingly positive stock risk premium, but the dash-dotted line shows that the increasing vega exposure generates increasingly negative risk premium. The solid line plots the total stock risk premium as the sum of the two contributions. The stock risk premium starts at 9.07% at zero default probability and according zero optionality contribution. As the default probability increases, the stock risk premium initially increases and reaches a maximum of 10.57% when the default probability reaches 0.74%. As the default probability increases further, the negative risk premium contribution from the optionality exposure starts to overtake the positive risk premium contribution from the delta exposure. The stock risk premium declines and ultimately becomes negative when the default probability exceeds 17.72%.

The low, or even negative, stock risk premium for financially distressed companies is in line with the literature findings (e.g., Dichev 1998; Campbell, Hilscher, and Szilagyi 2008). At the same time, the non-monotonic relation suggests that empirical analysis of stock return dependence on the default probability can lead to unstable results, and can become particularly sensitive to the inclusion or exclusion of a few observations with very default probabilities.

The relation between bond risk premium and the default probability is monotonic and much stronger. As shown in Panel B of Figure 4, the bond has zero delta exposure and zero vega exposure at zero default probability. As the default probability increases, the bond's delta exposure becomes increasingly positive while its vega exposure becomes increasingly negative. As highlighted in panel D of Figure 4, both exposures lead to increasingly positive bond risk premium contributions. In particular, the highly positive contributions from the negative optionality exposure make the expected excess returns on distress bonds seem abnormally high.

Earlier literature (e.g., Eom, Helwege, and Huang 2004; Huang and Huang 2012) finds that the average credit spreads on corporate bonds look too high relative to their actual level of default probabilities. The plots in Panel D of Figure 4 show that the embedded negative optionality and the negative optionality risk premium can be a significant contributor to the abnormally large bond risk premium. Classic structural models account for the asset return risk premium, but they often fail to account for the optionality risk premium. One exception is Cremers, Driessen, and Maenhout (2008), who incorporate jumps in the asset return dynamics and show that incorporating option-implied jump risk premium can bring the predicted credit spread levels much closer to observed levels. While the paper resorts to a different dynamics specification and different interpretation, calibrating their model to the index options market allows them to capture the well-documented volatility risk premium.

4.2 Hidden optionality exposure in high-beta stocks

Classic asset pricing theories often start with investor risk aversion and predict that the expected excess return on a stock increases with the stock's return risk, as measured by the stock's return beta on a market portfolio. The excess return behavior of our asset return risk factor portfolio is in line with this theoretical prediction. The average excess return on the asset return risk factor portfolio is highly positive and strongly statistically significant.

In contrast to our strong finding, the empirical support for a positive risk-return relation in the stock market has been weak. Many studies even find the relation to be negative.[1] In particular, Frazzini and Pedersen (2014) show that stocks with higher beta estimates generate lower average excess returns per unit risk. Based on this evidence, they construct a betting-against-beta (BAB) stock portfolio that is long stocks with low beta and short stocks with high beta while maintaining beta neutrality on the portfolio. The portfolio generates significantly positive average excess returns.

Since stocks with high risk also tend to have high optionality exposures, we conjecture that the embedded optionality exposure and investor preference for positive optionality may have also contributed to the observed anomalous risk-return relation. As a stock's risk increases, its return risk premium should increase with the risk level; nevertheless, if the embedded positive optionality exposure also increases with the risk, this optionality exposure can generate a negative optionality risk premium that partially cancels out the positive return risk premium contribution, weakening or even reversing the observed risk-return relation.

To examine our hypothesis and to highlight the entanglement of stock return risk and optionality, at each month t, we sort the stocks in our universe based on their 1-year historical return beta estimates ($\beta^S_{t,i}$) and construct five quintile portfolios. Table 7 reports the time-series averages of the quintile portfolio return beta estimates in the first row of statistics in panel A. The average return beta increases from 0.55 in the first quintile to 1.72 in the fifth quintile.

[1] See, for example, Campbell 1987; Breen, Glosten, and Jagannathan 1989; Turner, Startz, and Nelson 1989; Nelson 1991; Glosten, Jagannathan, and Runkle 1993; Whitelaw 1994; Harvey 2001.

Table 7
Risk and returns across stock beta quintiles

Beta quintile	1	2	3	4	5
A. Risk and return across beta quintiles					
Beta (β_s^δ)	0.55	0.84	1.04	1.27	1.72
Optionality (β_s^ν)	0.05	0.10	0.16	0.30	0.71
Excess return	9.57	11.18	12.63	11.33	12.39
Excess return per unit beta	16.55	12.92	11.56	8.37	6.76
B. Average risk premium decomposition					
Return risk premium (RRP)	6.24	9.46	11.75	14.33	19.40
Optionality risk premium (ORP)	-0.63	-1.16	-2.01	-3.65	-8.67
Stock risk premium (SRP)	5.61	8.29	9.74	10.68	10.73
Stock risk premium per unit beta	10.12	9.88	9.34	8.39	6.23

At each month, we sort the stocks based on the 1-year stock return beta estimates and form quintile stock portfolios. Entries report the summary statistics on the quintile portfolios. Panel A reports the time-series averages of each quintile portfolio's stock return beta (β_s^δ), optionality exposure (β_s^ν), excess return, and excess return per unit beta. Panel B computes the average risk premium decomposition for each portfolio based on the average portfolio risk exposure estimates, including the average asset return risk premium contribution (RRP), the average optionality risk premium contribution (ORP), the total stock risk premium (SRP) as the sum of return risk premium and optionality risk premium, and the stock risk premium per unit beta.

The second row of the panel reports the time series averages of the stock's optionality exposure (β_s^ν) for each quintile portfolio, which is constructed structurally as in (19) by adjusting the vega exposure (ν) with the leverage multiple (F/S) and the asset return beta. As the stock's return risk exposure increases from the first to the fifth quintile, the stock's optionality exposure also increases, from an average of 0.05 at the first quintile to 0.71 at the fifth quintile.

The third row of the panel reports the time-series averages of the quintile stock portfolio excess return, which no longer show a monotonic pattern across the beta quintiles. The average excess returns first increase with the beta quintile from 9.57% at the first quintile to 12.63% at the third quintile. As the return beta further increases, the quintile portfolio average excess returns start to decline to 11.33% and 12.39% at the fourth and fifth quintile, respectively.

When we scale each quintile portfolio excess return by the quintile portfolio beta, the last row of the panel shows that the average excess return per unit beta declines sharply with the beta quintile, from 16.55% at the first quintile to 6.76% at the fifth quintile, in line with Frazzini and Pedersen (2014)'s finding. The pattern allows them to form a beta-neutral betting-against-beta portfolio with significantly positively average excess returns.

In panel B of Table 7, we perform a schematic average risk premium decomposition on the beta quintile portfolios based on the average market pricing estimates from the joint return factor model in (15). The average market price of the asset return risk (η_δ) is estimated at 11.26% and the average market price of the optionality risk (η_ν) is at -12.21% (Panel C, Table 3). Multiplying the average market price of return risk of 11.26% with the return risk exposure (β_s^δ) in the first row of panel A generates the average return risk premium (RRP) for each quintile portfolio in the first row of panel B, which increases from 6.24% at the first quintile to 19.4% at the fifth quintile.

Meanwhile, multiplying the average market price of optionality risk of -12.21% with the optionality exposure (β^{ν}) in the second row of panel A generates the average optionality risk premium (ORP) for each quintile portfolio in the second row of panel B, which varies from -0.63% at the first quintile to -8.67% at the fifth quintile. Summing the two risk premium contributions generates the total average stock risk premium (SRP) for each quintile. As the return beta increases, the total risk premium first increases, but then reaches a plateau. Dividing the stock risk premium by the average return beta of each quintile in the last row of the panel shows a similarly declining pattern as the observed average excess return per unit beta in the last row of panel A.

Under our model structure, the average asset return risk premium per unit beta exposure is strongly positive and does not vary across the stock beta quintiles; nevertheless, as the stock return beta increases, the stock's optionality exposure increases. It is the increasing optionality exposure that drives the declining stock risk premium per unit beta across the stock beta quintile.

To quantify the contribution of the optionality exposure to the betting-against-beta (BAB) portfolio, we create the excess return series on a beta-against-beta portfolio ($r_{bab,t+1}$) by being short a unit beta exposure to the fifth high-beta quintile portfolio and long a unit beta exposure to the first low-beta quintile portfolio at each month. This portfolio is beta-neutral by construction. It generates an annualized average excess return of 9.79%, an annualized return volatility of 16.161%, an annualized information ratio of 0.59, and a Newey and West (1987) t-value of 2.46.

We regress this BAB excess return series against the excess return on the optionality factor portfolio ($\eta_{\nu,t+1}$) obtained from the cross-sectional regression on the joint return factor model in (15). The regression generates an R^2 estimate of 5.01%. The factor loading slope is estimated at -0.26, highly statistically significant with a Newey and West (1987) t-value of -3.27. The slope estimate shows that the BAB portfolio has a significantly negative exposure to the optionality risk.

The intercept of the regression represents the average excess return of the BAB portfolio with neutralized optionality exposure. The intercept has an annualized estimate of 6.63%, but the estimate is not statistically significant with a t-value of 1.64. With the optionality exposure neutralized, the average excess return on the BAB portfolio is no longer statistically significant over our sample period, highlighting the important contribution of the optionality exposure to the observed risk-return anomaly. Just as stocks on financially distressed companies have both high risk and high optionality, the BAB portfolio is by constructing beta neutral, but with strongly negative optionality exposure.

5. Conclusion

It is well recognized that investors should receive more compensation for bearing more risk. At the same time, investors are also willing to pay for optionality. When the embedded optionality of the securities increases with the risk level, the different preferences for risk and optionality can create seemingly puzzling risk-return behaviors.

In this paper, we propose to disentangle the asset return risk exposure from the optionality exposure in the stock and bond of a company through a structural model representation, and we propose to enhance the separate identification of the asset return risk premium and the optionality risk premium through a joint stock-bond return factor model. Performing cross-sectional regressions of the stock and bond excess returns jointly against their structurally constructed delta and vega exposures generates the excess returns on the two factor portfolios that target each of the two risk exposures. Estimation results confirm our starting hypothesis. The factor portfolio targeting a unit exposure to asset return risk but zero exposure to optionality generates a significantly positive average excess return, consistent with investor risk aversion. By contrast, the factor portfolio targeting a unit exposure to optionality but without directional exposure to firm value variation generates a significantly negative average excess return, reflecting positive investor preference for optionality exposure. The separation of risk from optionality, together with the separate risk premium estimates, sheds light on the distress puzzle in the stock and bond markets and helps explain the betting-against-beta and volatility risk premiums in the stock market.

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