What Drives Variance Swap Prices?

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Abstract

This paper studies time variation in variance risk premia and its role in S&P 500 variance swap price fluctuations. Using a present value identity for variance swap prices, I empirically document a term structure where short-term prices vary due to fluctuations in expected variance, while long-term prices vary due to fluctuations in variance risk premia. In contrast, prominent asset pricing models predict that this term structure is flat. Finally, my findings highlight that intermediary constraints are crucial in explaining the variation in variance risk premia across maturities, shedding light on the significance of intermediaries in variance markets.

Keywords: Asset pricing, derivatives, variance pricing, variance risk premia.

JEL codes: G12, G13.

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In this paper, I analyze time-series variation in risk premia from investing in instruments with direct exposure to stock market variance. I document economically sizable variation in these risk premia, and show that they explain a substantial fraction of the overall price variation in S&P 500 variance swaps. Afterward, I test whether prominent consumption-based asset pricing models that feature variance risk to capture time variation in the equity premium are also able to match the observed variation in variance risk premia. Furthermore, I analyze the variation in variance risk premia in detail, and find that intermediary constraints (Cheng, 2019) and variance beliefs of investors (Lochstoer and Muir, 2022) drive a significant fraction of the variation.

In theory, variation in variance risk premia is driven either by variation in preferences regarding variance risk or by variation in the quantity of variance risk. Analyzing variance risk premia therefore offers important insights into preferences regarding variance risk and into how this risk varies over time. Many prominent asset pricing models feature variance risk as the main mechanism to drive variation in the equity premium, and the variance market allows for a direct test of the pricing of variance risk. In asset pricing models, variance risk is driven by large and sudden movements (i.e., jumps) in cash flows, such as in consumption or dividend growth, or by jumps in the investment opportunity set, such as in the long-run mean or volatility of cash flows. However, the literature has not settled on the appropriate way of incorporating variance risk into an asset pricing model. Analyzing the variation of variance risk premia allows me to contrast the empirical findings on variance risk premia with the predictions of asset pricing models. This analysis therefore offers important insights into how variance risk should be incorporated into asset pricing models. In sum, my contribution to the literature is threefold.

First, I empirically document a term structure in how variance risk premia affect S&P 500 variance swap prices: fluctuations in short-term variance swap prices are predominantly driven by fluctuations in expected variance, while fluctuations in long-term prices are predominantly driven by fluctuations in variance risk premia. In the spirit of Campbell and Shiller (1988) for equity, I derive a present value identity for the logarithm of the variance swap price or the actual variance swap price. These identities show that price fluctuations of S&P 500 variance swaps are due to fluctuations in expected stock market variance or fluctuations in variance risk premia. Using predictive regressions, I show that 110.1% of the fluctuations in the one-month variance swap price is attributable to fluctuations in expected variance and this percentage decreases monotonically to 24.5% for the 18-month variance swap price. At the same time, price-fluctuations due to fluctuations in variance risk premia increase monotonically from -10.1% for one-month variance swaps to 72.8% for 18-month variance swaps.[1] The results are robust to using the present value identity for variance swap prices instead of the identity for the logarithm of variance swap prices.

Second, I show that prominent asset pricing models predict that this term structure documented in the data is flat. In the model of Gabaix (2012), variance swap prices fluctuate only due to variance risk premia fluctuations, but there is no differential effect for maturities (i.e., the term structure is flat). In the model of Wachter (2013), variance swap prices fluctuate only due to fluctuations in expected variance; because variance is highly persistent, there is again little differential effect across the term structure. The differential between these two consumption disaster models is due to the fact that, in the absence of disasters, stock market variance is constant in the model of Gabaix (2012). Finally, the model of Drechsler and Yaron (2011) matches the empirical decomposition of long-term variance swap prices relatively well. It is also able to match the empirical variation in variance risk premia, however the model predicts that the fluctuations in short-term variance swaps are also largely due to variance risk premia fluctuations—a prediction that is not supported by the data.

Third, I analyze the drivers of fluctuations in variance risk premia for different maturities. In particular, I show that investor beliefs over stock market variance (Lochstoer and Muir, 2022) explain fluctuations in short-term variance risk premia and intermediary constraints (Cheng, 2019) explain fluctuations in short- and long-term variance risk premia. Interestingly, the exposure of variance risk premia to intermediary constraints

[1]Notice that this percentage can be negative, because the coefficient is estimated using a regression and the sign of the coefficient depends on the correlation between realized variance risk premia and variance swap prices.

increase in the maturity of the risk premia, indicating that long-term variance risk premia are affected more by intermediary constraints than short-term variance risk premia. This result could explain the excess volatility documented in Giglio and Kelly (2018), a result that is often attributed to an overreaction in expectations of investors (Stein, 1989; Giglio and Kelly, 2018). Instead, my results show that also this form of excess volatility could be attributable to intermediary constraints. Finally, these results also show why the models analyzed in this paper struggle to the observed variation in variance risk premia in the data, because the models do not feature intermediary constraints or biased variance expectations.

Besides an assessment of asset pricing models with respect to the decomposition of variance swap prices, I also analyze their predictions with respect to the term structure of expected returns and of return volatilities on variance swaps. I show that all of the analyzed models severely overestimate the one-month return volatility on variance swaps, an indication that in their current calibrations one-month variance swaps are more risky than in the data. However, despite this large degree of risk of the one-month contract, the models of Wachter (2013) and Drechsler and Yaron (2011) do not match the empirical risk premium on one-month variance swaps. This indicates either that the short-term risks that are hedged using these contracts are not sufficiently severe or that the model needs more extreme preferences regarding these risks in order to match the empirical risk premium. All three models are able to match the expected returns on long-term variance swaps, but the models fail to capture its return volatility, those of Gabaix (2012) and Wachter (2013) understating the return volatility whereas that of Drechsler and Yaron (2011) overstates this volatility. This discrepancy between the models' predictions and the data is, however, much smaller than the discrepancy for the return volatility on short-term variance swaps. In sum, these models incorporate variance risk to capture empirical features of the equity premium, but fail to directly match empirically documented features with respect to the pricing of variance risk. The reason for this failure is likely due to fact that the analyzed models do not incorporate effects of constrained intermediaries or investor beliefs which I show are important to describe empirical features of variance risk premia.

The main results of the decomposition of variance swap prices are established using predictive regressions, however I show that the results are robust to specifying a vector autoregression (VAR) to obtain expectations. Using the VAR, I show that variance risk premia expectations are obtained effectively and drive economically sizable variation in returns on variance swaps. Predictive regressions show that a one standard deviation increase in expected variance risk premia predicts an increase of 12.3% in the average realized monthly returns of a one-month variance swap, and that this monotonically decreases to an increase of 4.1% in average realized monthly returns of an 18-month variance swap. Results indicate that hedging variance risk was expensive in the period prior to the crash in 2000, and in the period post the crash of 2008, whereas in the period between these crashes it was relatively cheap to hedge variance risk. Finally, the time-series properties of one-month variance risk premia differ from those of the other maturities. In particular, one-month variance risk premia tend to decrease during periods rapidly increasing stock market variance, a result in line with Cheng (2019) and Lochstoer and Muir (2022).[2] I show that this negative correlation is a unique feature of one-month variance risk premia, and that it weakens for long-term variance risk premia.

This paper contributes to several strands of the literature. First, I contribute to the recent literature that tests the implications of asset pricing models in variance markets. Bollerslev et al. (2009) and Drechsler and Yaron (2011) show that long-run risk models are able to explain the sizable variance risk premium in the data, Gabaix (2012) and Seo and Wachter (2019) show that rare consumption disaster models are able to explain the implied volatility slope on S&P 500 options, and Bekaert and Engstrom (2017) and Bekaert et al. (2020) show that habit models are able to explain the relation between variance premium and consumption growth uncertainty. A paper by Dew-Becker et al. (2017) shows that the model of Gabaix (2012) is able to explain the term structure of

[2]Lochstoer and Muir (2022) provide additional evidence of the finding of Cheng (2019). They show, moreover, that a model in which an agent has slow-moving beliefs regarding stock market volatility can reconcile the evidence of the negative correlation of stock market variance and the risk premium. Slow-moving volatility expectations lead the agent to initially underreact to volatility news, followed by a delayed overreaction.

variance premia, and is the closest to the present paper. Distinctly though, I focus on variance price fluctuations, and test which models align with the data on the basis of a Campbell–Shiller decomposition of the term structure of variance swap prices.

Second, I contribute to the empirical literature on risk premia in variance markets. Bollerslev et al. (2009), Kozhan et al. (2013), Dew-Becker et al. (2017), Eraker and Wu (2017), Aït-Sahalia et al. (2020), and Eraker and Yang (2022) show that risk premia are, on average, sizable in variance markets. Moreover, Bollerslev et al. (2009) find that the variance premium predicts future stock market returns, Bollerslev and Todorov (2011) show that a substantial fraction of the equity premium is compensation for variance risk, and Martin (2017) shows that risk-neutral variance is a lower bound for the equity premium. Johnson (2017), Cheng (2019), and Lochstoer and Muir (2022) analyze time-series variation in variance risk premia. Distinct from these studies, I study the impact of variance risk premia fluctuations on variance swap prices, and obtain variance risk premia from a present value identity using a VAR.

Third, I contribute to the literature on the impact of intermediary constraints for risk premia in various financial markets. Building on the work of He and Krishnamurthy (2013), Adrian et al. (2014) show the effects of intermediaries in the cross-section of stocks, Goldberg and Nozawa (2021) and He et al. (2022) show the effects in the cross-section of corporate bonds, He et al. (2017) and Chen et al. (2019) show the effects in option markets, and Cheng (2019) shows the effects in the market for VIX-futures. Distinct from these studies, I focus on the impact of intermediaries on the term structure of variance premia, and document that long-term variance premia are more exposed to intermediary constraints than short-term variance premia. Furthermore, my findings suggest that intermediary constraints play a role in the excess volatility puzzle documented in Giglio and Kelly (2018).

The remainder of the present paper is organized as follows. In Section 1, I derive the present value identity for variance swap prices. Section 2 describes the data on variance swaps. The results of the decomposition are presented in Section 3, and afterward compared to the predictions of asset pricing models. I show, in Section 4, that the results are robust to specifying a VAR to obtain variance expectations, and Section 5 analyzes the drivers of fluctuations in variance risk premia. Section 6 concludes.

1 Methodology

In this section, I present the methodology for decomposing S&P 500 variance swap prices into expected S&P 500 return variance and variance risk premia. A variance swap is a derivative security that pays the holder of the contract the realized variance of the underlying up to maturity. Variance swaps are used to manage market variance risk, and variance risk premia correspond to expected returns or expected payoffs from holding a variance swap.

In the subsection that follows, I formalize the cash flows of a variance swap before deriving several closely related pricing identities for variance swaps.

1.1 Variance swap contract

A variance swap pays its holder the realized variance of the underlying from the inception of the contract up to maturity. At maturity, the realized variance and the agreed upon variance swap price are exchanged. The payoff of a variance swap at maturity T-periods from origination time t is defined as follows:

$$\text{payoff}_{t+T} = \sum_{i=1}^{T} RV_{t+i} - VS_t^{(T)}, \tag{1}$$

where RV_{t+i} is the realized variance over period $t+i$ and $VS_t^{(T)}$ is the variance swap price at time t for a variance swap with T-periods to maturity. A variance swap can last for several periods, and the total realized variance at the end of the contract therefore equals the sum of the realized variance over each period. The holder of the variance swap receives the realized variance in exchange for a fixed rate at the end of the contract and therefore hedges variance risk until the contract reaches maturity. Given a risk-neutral pricing measure $\mathbb{Q}$, the variance swap price with T-periods to maturity at time t is given by the following:

$$VS_t^{(T)} = \sum_{i=1}^{T} \mathbb{E}_t^{\mathbb{Q}}\big(RV_{t+i}\big), \tag{2}$$

where $\mathbb{E}_t^{\mathbb{Q}}$ denotes the expectation under the risk-neutral measure conditional on information available at time t. Therefore, $VS_t^{(T)}$ corresponds to the risk-neutral expectation of the sum of realized variances from period $t+1$ until period $t+T$. The main analyses of this paper will focus on variance swap prices, however, in some cases I use variance forward prices. These are defined as follows:

$$F_t^{(T)} = \mathbb{E}_t^{\mathbb{Q}}\big(RV_{t+i}\big), \tag{3}$$

such that $F_t^{(T)} = VS_t^{(T)} - VS_t^{(T-1)}$ and $F_t^{(1)} = VS_t^{(1)}$.

Next, I define the gross return or gross payoff over period t to $t+1$ on a variance swap with T-periods to maturity, as follows:

$$R_{t+1}^{(T)} = \frac{VS_{t+1}^{(T-1)} + RV_{t+1}}{VS_t^{(T)}}, \tag{4}$$

$$VP_{t+1}^{(T)} = VS_{t+1}^{(T-1)} + RV_{t+1} - VS_t^{(T)}. \tag{5}$$

These definitions are the two main ways the literature has assessed risk premia on variance swaps, the former is used by Dew-Becker et al. (2017) and the latter by Carr and Wu (2009) and Bollerslev et al. (2009). The gross return of Equation (4) is defined as if the variance swap is bought for the current variance swap rate $VS_t^{(T)}$, and held for one period after which the one period realized variance RV_{t+1} and next period's variance swap price $VS_{t+1}^{(T-1)}$ are received. In the following period, the variance swap has $T-1$-periods remaining to maturity.[3] The realized variance risk premium of Equation (5) is defined

[3] The definition of the return in equation (4) corresponds to the return on a variance asset, which pays the realized variance at the end of each period rather than at the end of the contract. Under the assumption of no arbitrage, the price of such an asset equals the variance swap price discounted with the T-period risk-free rate to time t, and the realized variance payment of such an asset is discounted in a similar way. It is possible to show that the logarithm of the return defined in (4) equals the log-return on this

as the realized payoff on a variance swap which is held for one period, at the end of this period the investor receives $VS_{t+1}^{(T-1)}$ and RV_{t+1} and the difference is computed with the initial investment $VS_t^{(T)}$. It follows from the equations that the realized variance for returns on variance swaps is equivalent to dividend payments for returns on the stock market; that is, realized variance is the cash flow component of a variance swap.

1.2 Present value identities for variance swap (forward) prices

In this subsection, I discuss four closely related identities for variance swap (forward) prices. Similar to Fama and Bliss (1987) and Campbell and Shiller (1988), the identities have in common that the current price of the variance instrument is written as the difference between cashflows (stock market variance) and risk premia (variance risk premia). I briefly discuss each of the identities used in this paper and refer to Appendix A.1 for the details of the derivations. In the following, unless stated otherwise, variance risk premia are referred to as risk premia.

log-VS. The first (approximate) present value identity for variance swap prices is given in the following equation:

$$vs_t^{(T)} \approx E_{\mathrm{rv},t}^{(T)} - E_{\mathrm{rp},t}^{(T)}, \tag{6}$$

where

$$E_{\mathrm{rv},t}^{(T)} = \mathbb{E}_t \sum_{i=1}^{T} \left[1 - \rho(T-i+1)\right] \Big(\prod_{j=1}^{i-1} \rho(T-j+1) \Big) \cdot rv_{t+i} \quad \text{and} \tag{7}$$

$$E_{\mathrm{vdr},t}^{(T)} = \mathbb{E}_t \sum_{i=1}^{T} \Big(\prod_{j=1}^{i-1} \rho(T-j+1) \Big) \cdot r_{t+i}^{(T-i+1)}. \tag{8}$$

The present value identity of Equation (6) relates the current logarithm of the variance swap price ($vs_t^{(T)} := \log(VS_t^{(T)})$) to the logarithm of stock market variance ($rv_{t+i} := \log(RV_{t+i})$) and risk premia ($r^{(T-i+1)_{t+i}} := \log(R_{t+i}^{(T-i+1)})$. The identity has the standard

variance asset in excess of the risk-free rate.

intuition for a present value identity: Today's price can be high due to high expected stock market variance, low expected risk premia, or both. Equation (6) represents an approximate identity, because it relies on the standard Taylor expansion of log-return of Equation (4) and $\rho(T)$ is the log-linearization constant for a variance swap with maturity T. In Appendix A.1, I show how to estimate $\rho(T)$ using an OLS regression, and show that the approximation works really well for the linear approximation of logarithm of variance swap returns.

log-F. The second present value identity relates the current logarithm of the price of a variance forward to expected stock market variance and risk premia, as follows:

$$f_t^{(T)} = \mathbb{E}_t rv_{t+T} - \mathbb{E}_t r_{f,t+T}^{(T)}, \tag{9}$$

where $r_{f,t+T}^{(T)}$ is the log-return for holding a variance forward to maturity. Equation (9) is not an approximation as it directly follows from the definition of the gross-return on a variance forward. This present value identity is used by Fama and Bliss (1987) for treasury bonds and by van Binsbergen et al. (2013) for dividend strips.

level-VS. The third present value identity relates the current variance swap price to stock market variance and risk premia, as follows:

$$VS_t^{(T)} = E_{\mathrm{RV},t}^{(T)} - E_{\mathrm{RP},t}^{(T)}, \tag{10}$$

where

$$E_{\mathrm{RV},t}^{(T)} = \mathbb{E}_t \sum_{i=1}^{T} RV_{t+i},$$
$$E_{\mathrm{RP},t}^{(T)} = \mathbb{E}_t \sum_{i=1}^{T} VP_{t+i}^{(T-i+1)}.$$

In particular, Equation (10) relates the current variance swap price (in levels rather than a log-transformation) to expected stock market variance over the lifetime of the contract and risk premia. Notice that this present value identity also constitutes an exact relationship, because the definition of the variance risk premium (Equation 5) is a linear function of the variance swap price, and therefore does not depend on any Taylor approximations.

level-F. The fourth present value identity relates the current price of a variance forward to stock market variance and risk premia, as follows:

$$F_t^{(T)} = \mathbb{E}_t RV_{t+T} - \mathbb{E}_t VP_{F,t+T}^{(T)}, \tag{11}$$

where $VP_{F,t+T}^{(T)}$ is the realized variance risk premium for holding a T-period variance forward to maturity. Furthermore, the identity is an exact relation, because it follows directly from the definition of the realized variance risk premium definition for a variance forward with maturity T.

In Section 3, I use Equations (6), (9)–(11) to decompose variance swap (forward) prices into variance expectations and risk premia for several maturities. This allows me to study the drivers of the variation in variance swap prices for various maturities.

2 Data

In this paper, I use data on S&P 500 options from January 1996 until June 2019 from OptionMetrics. Using the methodology described in Kozhan et al. (2013) and discussed in greater detail in Appendix A.2, I construct variance swaps with maturities ranging from 1 to 18 months. The maturity of 18 months to maturity is the longest for which I can calculate a variance swap price every month. I calculate variance swap prices at the end of each month in the sample and interpolate the variance swap prices linearly, such that the maturity equals T months. Note that interpolating variance swap prices linearly is equivalent to taking long positions in two variance swaps with maturities T_1 and T_2, such that the weighted average of the maturities equals T.

I use the methodology of Kozhan et al. (2013) because these variance swaps are most closely related to the variance swaps that are traded over the counter (OTC). In

Appendix A.3, I show that my data on variance swaps is highly similar to the data from the OTC market that is analyzed in Dew-Becker et al. (2017). I show in Figure 6 of Appendix A.3 that the synthetic variance swap prices are highly similar to the variance swap prices in the OTC market, and simple regressions indicate R^2s above 97%. Table 4 in Appendix A.3 shows that the returns on the synthetic variance swaps are also highly similar to the returns on the OTC swaps as shown in Table 2 of Dew-Becker et al. (2017).

In addition to the pricing information on variance swaps, I obtain data that is used in the VAR. From the panel of variance swap prices, the first two principal components, $pc_t^{(1)}$ and $pc_t^{(2)}$, are calculated. The first of these components, $pc_t^{(1)}$, captures the level in the term structure of variance swap prices, while $pc_t^{(2)}$ captures the slope of the term structure of variance swap prices. Realized variance is defined as in Kozhan et al. (2013) and approximately equals the sum of daily squared returns within a month. Finally, the default spread is obtained from the Federal Reserve Bank of St. Louis and defined as the difference between the yields on BAA and on AAA credit-rated corporate bonds.

3 Results

In this section, I present the empirical term structure of the decomposition of variance swap prices and relate these findings to the predictions of leading asset pricing models. First, I present the results of a variance decomposition on the basis of predictive regressions, and these serve as my benchmark results for the models. In Section 4, I show that the results are robust to specifying a VAR, and decompose the variation in variance swap prices. Second, I consider the asset pricing models of Gabaix (2012), Wachter (2013) and Drechsler and Yaron (2011), and repeat the empirical exercise with each of these models in order to analyze their predictions regarding the term structure of the decomposition.

3.1 Decomposition of variance swap prices in the data

In this subsection, I decompose the variation in variance swap prices using predictive regressions. These regressions derive from the intuition of pricing identities presented in

Section 1.2: the current variance swap (forward) price is high due to high future stock market variance, due to low future returns on the variance swap (forward), or both. Therefore, the current variance swap (forward) price should predict future stock market variance, future risk premia, or both. This analysis is the equivalent of the analyses employed by Fama and Bliss (1987) for treasury bonds, van Binsbergen et al. (2013) for dividend strips and Cochrane (2008, 2011) for the stock market. Relative to the stock market, my analysis has the advantage that variance swaps have a finite maturity, and thus that future prices do not play a role.

In the following, I explain the intuition for the predictive regressions based on identity (6) for the logarithm of the variance swap price. However, the intuition carries over to other present value identities derived in Section 1.2. Based on the identity for the logarithm of variance swap prices, Equation (6), the following predictive regressions decompose the price variation:

$$y_{\mathrm{rv},t+T} = a_{\mathrm{rv}} + b_{\mathrm{rv}} \cdot vs_t^{(T)} + \epsilon_{t+T}^{\mathrm{rv}}, \tag{12}$$

$$y_{\mathrm{rp},t+T} = a_{\mathrm{rp}} + b_{\mathrm{rp}} \cdot vs_t^{(T)} + \epsilon_{t+T}^{\mathrm{rp}}, \tag{13}$$

$$1 \approx b_{\mathrm{rv}} - b_{\mathrm{rp}} \tag{14}$$

where

$$y_{\mathrm{rv},t+T} = \sum_{i=1}^{T} \left[1 - \rho(T-i+1)\right] \left(\prod_{j=1}^{i-1} \rho(T-j+1) \right) \cdot rv_{t+i},$$

$$y_{\mathrm{rp},t+T} = \sum_{i=1}^{T} \left(\prod_{j=1}^{i-1} \rho(T-j+1) \right) \cdot r_{t+i}^{(T-i+1)}.$$

Equation (14) follows directly from the present value identity (6), and indicates whether variation in variance swap prices is driven by future realized variance or by returns. The regression coefficients indicate whether a high current variance swap price predicts high future stock market variance or low future returns. Therefore, economic intuition suggests that $b_{\mathrm{rv}} > 0$ and $b_{\mathrm{rp}} < 0$. Moreover, if $b_{\mathrm{rv}} \approx 1$, it indicates that variation in the variance swap price is exclusively driven by expected stock market variance, whereas if $b_{\mathrm{rp}} \approx -1$, it indicates that variation in the variance swap price is exclusively driven by risk premia. Due to the fact that a variance swap is a finite cash flow, these regressions provide a powerful test of whether the present value identity holds. This identity holds if the differences between the regression coefficients are, indeed, close to one. The dependent variables of predictive regressions (12) and (13) depend on realized stock market variance (rv_t), realized returns on variance swaps ($r_t^{(T)}$) and the log-linearization constants ($\rho(T)$). I show in Appendix A.1 how to estimate these using OLS, and report the estimates in Table 6 of that appendix.

The next step is to estimate the predictive regressions (12) and (13), which decompose the variance swap price into expected variance and risk premia. I show the results for each of the derived present value identities in Section 1.2, to do so I run the same regressions only the dependent and independent variables are adjusted accordingly.[4] The results of these predictive regressions are presented in Table 1.

[Table 1 here]

Table 1 presents the primary finding of the paper, namely the existence of a strong term structure in the decomposition of variance swap prices. In particular, variation in prices attributable to expected stock market variance decreases strongly with the maturity, whereas the variation in prices due to risk premia strongly increases with the maturity. Focusing on the variation in the logarithm of variance swap (forward) prices, Columns (1)–(4) indicate that one-month variance swaps (forwards) are explained by 110.1% (110.1%) due to expected variance, with this percentage declining monotonically to 24.5% (-22.9%) for 18-month variance swap (forward) prices. At the same time, the

[4] To decompose the logarithm of the variance forward (Equation 9), $y_{\mathrm{rv},t+T}$ is replaced with rv_{t+T}, $y_{\mathrm{rp},t+T}$ is replaced with $r_{f,t+T}$ and the dependent variable is $f_t^{(T)}$. To decompose the variance swap in levels (Equation 10), $y_{\mathrm{rv},t+T}$ is replaced with $y_{\mathrm{RV},t+T} := \sum_{i=1}^{T} RV_{t+i}$, $y_{\mathrm{rp},t+T}$ is replaced with $y_{\mathrm{RP},t+T} := \sum_{i=1}^{T} VP_{t+i}^{(T-i+1)}$ and the dependent variable is $VS_t^{(T)}$. To decompose the variance forward in levels (Equation 11), $y_{\mathrm{rv},t+T}$ is replaced with RV_{t+T}, $y_{\mathrm{rp},t+T}$ is replaced with $VP_{F,t+T}^{(T)}$ and the dependent variable is $F_t^{(T)}$.

variation in prices attributable to risk premia increases monotonically from -10.1% (-10.1%) for one-month variance swaps (forwards) to 72.8% (122.9%) for swaps (forwards) with 18 months to maturity.[5] Furthermore, these conclusions are supported by the R^2 strongly decreasing (increasing) with the maturity when the logarithm of prices are used to predict future stock market variance (future returns).

Next, I discuss the results based on the present value identities for actual variance swap (forward) prices as shown in Columns (5)–(8) of Table 1. Importantly, the results of this decomposition are highly similar to the results of the decomposition in logarithmic terms. In particular, the variation in prices due to stock market variance decreases monotonically from 92.2% (92.2%) for one-month variance swap (forward) prices to 19.6% (-19.7%) for 18-month variance swap (forward) prices. Similarly, the variation in prices attributable to risk premia increases monotonically from 7.8% (7.8%) for one-month variance swaps (forwards) to 80.4% (119.7%) for swaps (forwards) with 18 months to maturity. A notable difference between the specifications in logarithmic terms and in levels is seen for one-month variance swaps (Panel A), the results indicate that a high variance swap price in logarithmic terms predict future returns positively whereas the variance risk premium is predicted negatively by a high variance swap price in levels. This difference is likely driven by how a logarithmic-transformation makes a right-skewed distribution (such as the distribution of variance swap prices) more symmetric and thus makes this right tail less important in an OLS regression.

In Table 1, the t-statistics based on standard errors adjusted for heteroskedasticity and auto-correlation using Newey and West (1987) and Hansen and Hodrick (1980) with T-lags are presented in parentheses and brackets below the estimated coefficients. Given the small differences between the corresponding t-statistics, the conclusions regarding statistical significance are the same. The results of the predictive regressions show that short-term variance swap (forward) prices significantly predict future stock

[5]Because the relation between the logarithm of variance swap prices, stock market variance and returns constitutes an approximation, the difference between the coefficients in columns (1) and (2) does not necessarily equals 100%. However, given that the difference is relatively close to 100% it shows that the approximation is a close one. In fact, the difference between the coefficients lies between 97.3% and 100.1%.

market variance, whereas long-term variance swap (forward) prices significantly predict future returns (variance risk premia). However, in Section 4 of the present paper, where I show that the results are robust to specifying a VAR for the decomposition, I find that short-term risk premia too vary significantly over time.

In sum, the results in Table 1 indicate a strong term structure in the decomposition of variance swap (forward) prices: long-term variance swap (forward) prices are mostly driven by risk premia whereas short-term variance swap prices are mostly driven by expected stock market variance. In Internet Appendix C, I show that the results are robust to repeating the analysis at a quarterly rather than a monthly frequency. In Appendix A.4, I show that most of the variation in variance swap prices comes from variation in downside variance swap prices rather than upside variance swap prices.[6] The variance decomposition of Table 8 in Appendix A.4 shows that the variance swap prices are driven by downside variance swap prices for 63.4% to 74.2%, depending on the maturity of the contract. Finally, in Appendix A.5, I show that the results continue to hold on the subsample from September 2005 to June 2019. Furthermore, over this subsample it is possible to compute the price of a variance swap with 24 months to maturity every month, and find that variation in the 24-month variance swap price is for 123.9% attributable to risk premia.[7]

In the following, I obtain the predictions of several prominent asset pricing models regarding this decomposition of variation in variance swap prices. I focus for this exercise on variance swap prices rather than variance forward prices, because the latter are by its definition more subject to noise which could potentially overstate the variation attributable to risk premia.

[6] My definition of up- and downside variance swap prices closely follows the definitions of Andersen and Bondarenko (2009), Dew-Becker et al. (2017), Baele et al. (2019), and Kilic and Shaliastovich (2019) such that $VS_t^{(T)} = VS_{u,t}^{(T)} + VS_{d,t}^{(T)}$, where $VS_{u,t}^{(T)}$ and $VS_{d,t}^{(T)}$ are the up- and downside variance swap, respectively. The formal definitions of these swaps are given in Appendix A.4.

[7] In September 2005, Cboe introduced Long-term Equity Anticipation Secturities (Leaps) for the S&P 500, and as a result options with three years to maturity were listed on an annual basis. For that reason, it is possible to obtain a 24-month variance swap price every month using interpolation.

3.2 Decomposition of variance swap prices in asset pricing models

In this subsection I discuss the predictions of several prominent asset pricing models with respect to the pricing of variance risk. The ability of asset pricing models to match stylized facts of variance markets has been an important topic in the literature.[8] This is because variance risk is a central component of many asset pricing models, and variance markets allow researchers to study variation in variance risk in combination with the preferences of investors regarding this risk. I discuss the implications of the following three models: the variable rare disaster model of Gabaix (2012), the time-varying rare disaster model of Wachter (2013), and the long-run risk model of Drechsler and Yaron (2011). These models are able to match some empirical results on the pricing of variance risk. Dew-Becker et al. (2017) show that the model of Gabaix (2012) matches the empirical results on the term structure of the risk premia of variance risk, Seo and Wachter (2019) show that the model of Wachter (2013) matches the implied volatility slope on S&P 500 options, and the model by Drechsler and Yaron (2011) is designed to match the empirical results on the variance risk premium of Bollerslev et al. (2009).

For each of the considered models, I decompose the variation in variance swap prices using predictive regressions similar to the empirical exercise in Subsection 3.1 for the logarithm of variance swap prices. Afterward, I calculate the following moments in each model: the expected log-returns on variance swaps, the standard deviation of log-returns on variance swaps, and the variance of the logarithm of variance swap prices. These results from the models are obtained from a simulation study.[9] Finally, in Internet Appendix D.1–Internet Appendix D.3 I discuss the models in more detail, and show that the analyzed moments are relatively stable across the simulation sets.

[8] Bollerslev et al. (2009), Drechsler and Yaron (2011), Gabaix (2012), Bekaert and Engstrom (2017), and Lochstoer and Muir (2022) present consumption-based asset pricing models that match various moments of the variance premium. Seo and Wachter (2019) show that an asset pricing model that features time-varying disaster risk matches the implied volatility slope on S&P 500 options. Dew-Becker et al. (2017) analyze the ability of asset pricing models to match the term structure of variance risk premia.

[9] For each model, 1,000 independent simulation sets of a time series with 1,000 data points are obtained. On the basis of these simulation sets, each of the considered statistics is calculated.

[Figure 1 here]

In Figure 1, I plot the results for the decomposition of variance swap prices in each of the considered models. Figure 1 plots how much of the variation in variance swap prices is attributable to expected variance (risk premia) in the left (right) panel. I show the results from the decomposition of the logarithm of variance swap prices in the data (solid line) as well as the predictions of the models. Overall, the considered models struggle to produce the steep term structure in the decomposition of variance swap prices that is documented in the data. In the models of Wachter (2013) (dash-dotted line) and Drechsler and Yaron (2011) (dotted line), expected stock market variance drives a too large fraction of the variation in long-term variance swap prices. This is due to the strong persistence of state variables in these models, which makes stock market variance more persistent than in the data. A simple solution to this would be to decrease the persistence of the variables that drive stock market variance, that is, the persistence of the disaster intensity in Wachter (2013) and the persistence of consumption volatility in Drechsler and Yaron (2011). However, the persistence of these state variables is the key mechanism of the models to match the equity and the variance premium. Neither is the model of Gabaix (2012) (dashed line) able to match the observed term structure of the variance swap price decomposition. Instead, all of the variation in variance swap prices, both short- and long-term, is driven by risk premia.

The main reason why the model of Gabaix (2012) does not match the term structure of the variance price decomposition is that the time variation in the disaster size only affects the realized variance, conditional on a disaster hitting the economy. Given that the probability of this occurring is relatively low (1% per year), expected stock market variance only increases marginally when the disaster size increases. However, given that this consumption disaster is highly undesirable for the investor, risk premia adjust accordingly when the disaster size increases. As I will show in Figure 2, the model of Gabaix (2012) is not able to match the observed variation in variance swap prices, and the overall variation in risk premia is therefore still relatively small. In order to match the observed variation and empirical decomposition, the model has to incorporate an additional source of stochastic volatility.

The model of Wachter (2013) (dash-dotted line) predicts a large variation in stock market variance because of the heteroskedastic nature of the disaster intensity process; that is, high levels of the disaster intensity scale the future variance of the disaster intensity upward. Stock market variance therefore varies—even in the absence of disasters—over time. At the same time, high levels of the disaster intensity correspond to low risk premia. However, variation in risk premia drives a much smaller fraction of the total variation in variance swap prices, as seen in the right panel of Figure 1. Even if realizations in which the consumption disaster hits the economy are excluded, the variation in 18-month variance swap prices due to risk premia only increases to 6%. Therefore, even in the absence of consumption disasters the model of Wachter (2013) is not able to match the data.

The model of Drechsler and Yaron (2011) (dotted line) matches the data most closely, as short-term variance swap prices are mostly driven by expected stock market variance and long-term variance swap prices are mostly driven by risk premia. The term structure of the variance price decomposition is, however, considerably less steep than that of the data. For example, in the data the variation due to risk premia of the one-month variance swap price is close to zero (even negative), whereas in the model of Drechsler and Yaron (2011) variance discount rate variation accounts for 36% of this variation. The variation in short-term risk premia is considerably larger in the model of Drechsler and Yaron (2011), because variation in variance risk is sizable. At the same time, the model is not able to capture the empirical result that the variation in variance swap prices due to expected stock market variance strongly decreases in maturity. Again, this result is an indication that the state variables governing stock market variance are too persistent.

[Figure 2 here]

Figure 2 plots the term structure of the variance of variance swap prices in the data (solid line) and in each of the asset pricing models. The models are not able to produce the strong downward slope in the term structure, in particular the slope up to six months to maturity. However, in line with the data, the models of Wachter (2013) (dash-dotted line) and Drechsler and Yaron (2011) (dotted line) do produce a downward sloping term structure, whereas that of Gabaix (2012) (dashed line) predicts a flat term structure of the variance of variance swap prices. The model of Drechsler and Yaron (2011) matches the observed variation in variance swap prices well, and in particular the variation in long-term variance swap prices. This is interesting, because the model was calibrated to match the dynamics of the one-month variance risk premium. The models of Wachter (2013) and Gabaix (2012) severely over- and understate, respectively, the observed variation in variance swap prices. This variation is overstated in Wachter (2013) because the high persistence of the disaster intensity makes variation in expected stock market variance high. The variation is understated in Gabaix (2012) because the model produces relatively little variation in expected stock market variance.

[**Figure 3 here**]

Figure 3 shows the term structure of expected returns (return volatility) on variance swaps in the left (right) panel. I show the results from the data (solid line) as well as the predictions of the models. As seen in the left panel, the model of Gabaix (2012) (dashed line) is able to match the strongly upward sloping term structure of expected returns in the data. However, it overstates the return volatility of the one-month variance swap and understates the return volatility of the variance swap beyond three months to maturity. The models of Wachter (2013) (dash-dotted line) and Drechsler and Yaron (2011) (dotted line) are not able to match the strong upward sloping term structure of expected returns on variance swaps. In particular the expected return on the one-month variance swap is far off, and this indicates that this short-term risk is not sufficiently severe such that the investor is willing to pay a risk premium that is similar in magnitude to that in the data. The right panel of Figure 3 shows, however, that the one-month variance swap is sufficiently more risky in the models of Wachter (2013) and Drechsler and Yaron (2011) than in the data, but that despite this large risk the models are not able to match the one-month risk premium. Moreover, the model of Wachter (2013) slightly understates the return volatility of long-term variance swaps, whereas that of Drechsler and Yaron (2011) slightly overstates it.

In sum, I show in the Figure 1 that the models I considered struggle to match the empirical term structure on the decomposition of variance swap prices. The data shows a very strong term structure, whereas the models all predict a more or less flat term structure on the decomposition of variance swap prices. Moreover, these results, in combination with the pattern of Figure 2, show that only the model of Drechsler and Yaron (2011) is able to match the empirical magnitude of the variation in risk premia. Finally, the results presented in Figure 3 indicate that the model of Gabaix (2012) is best able to match the empirical term structure of the expected returns and return volatility of variance swaps. The models of Wachter (2013) and Drechsler and Yaron (2011), meanwhile, severely understate the risk premium on a one-month variance swap, despite the fact that the one-month variance swap is substantially more risky in the models compared to in the data.

In the following section, I show that the empirical results of Subsection 3.1 are robust to specifying a VAR to obtain expected stock market variance and risk premia. Afterward, I show that risk premia are related to many important macroeconomic variables.

4 Decomposition of variance swap prices using a VAR

In this section, I show that the variance decomposition of Subsection 3.1 is robust to specifying a VAR to obtain expectations. Using the VAR it is possible to estimate variance expectations and risk premia and analyze their time-series variation. Furthermore, I provide evidence that my VAR specification estimates variance expectations and risk premia effectively.

The VAR is used to model expected stock market variance and obtain risk premia as a latent variable from the present value identity (6) for the logarithm of the variance swap price. It is convenient to model stock market variance with a VAR because this allows one to obtain variance expectations for each period by iterating forward.

In the benchmark exercise, I focus on the following VAR with four state variables to model the logarithm of market variance:[10]

$$z_{t+1} = Bz_t + \epsilon_{t+1} \quad \text{and} \tag{15}$$

$$z_t = \begin{pmatrix} rv_t & pc_t^{(1)} & pc_t^{(2)} & DEF_t \end{pmatrix}', \tag{16}$$

where $B \in \mathbb{R}^{4\times4}$ is a matrix with regressor coefficients and $\epsilon_{t+1} \in \mathbb{R}^{4\times1}$ is a vector with errors. The vector z_t consists of the following variables: rv_t is the log of realized variance, $pc_t^{(1)}$ is the first principal component of the panel of log variance swap prices, $pc_t^{(2)}$ is the second principal component of the panel of log variance swap prices, and DEF_t is the default spread defined as the yield difference between BAA and AAA credits. For simplicity, all the variables included in z_t are demeaned such that the intercepts in the VAR are zero.

The first row in the VAR of equation (15) represents the predictive model for stock market variance. In order to decompose the variance swap prices, I calculate expected variance, and the remaining variables in the VAR are therefore included based on their ability to predict stock market variance. The first principal component $pc_t^{(1)}$ captures the level of the term structure of variance swap prices, and predicts future stock market variance well. The level of term structure of variance swap prices is highly correlated ($\approx$ 0.93) with the VIX index, which Drechsler and Yaron (2011) show to be a good predictor of stock market variance. The second principal component $pc_t^{(2)}$ relates to the slope of the term structure of variance swap prices, which rises during episodes of low stock market variance and falls when the converse is true. Finally, the default spread DEF_t, which Campbell et al. (2018) show predicts variation in long-term variance, is thus included in the VAR, also because it is a well-known business cycle indicator.

[10] In Appendix A.6, I discuss the VAR-specification to model stock market variance in levels rather than in logarithmic terms, and I will show in Table 3 that the results are highly similar across the two specifications.

Based on the VAR model, monthly variance expectations for a variance swap with T-months to maturity equals

$$E_{\mathrm{rv},t}^{(T)} = e_1' \Bigg(\big(1-\rho(T)\big)B + \cdots + \big(1-\rho(1)\big)\rho(T) \times \cdots \times \rho(2)B^T \Bigg) z_t, \tag{17}$$

where $e_1 \in \mathbb{R}^{4\times 1}$ is a unit vector with the first element equal to one and the remaining elements equal to zero. By the pricing identity (6), risk premia are a function of variance expectations from equation (17) and the current variance swap price, as follows:

$$E_{\mathrm{rp},t}^{(T)} = E_{\mathrm{rv},t}^{(T)} - vs_t^{(T)}. \tag{18}$$

In this way, I obtain an ex ante estimate of expected variance over the lifetime of the variance swap and an estimate of the risk premia that price the variance swap. I use these estimates to decompose variation in the variance swap price into either the expected variance of Equation (17) or the risk premia of Equation (18). Before I show the results of this decomposition, I present the estimation results of the VAR, in Table 2.

[Table 2 here]

The first row in Panel A of Table 2 presents the model for log realized variance each month. In line with expectation, the level of the term structure of variance swap prices $pc_t^{(1)}$ predicts next month's realized variance positively. Variance swap prices rise during episodes of elevated stock market variance. The slope of the term structure of variance swap prices, $pc_t^{(2)}$, predicts next month's realized variance negatively—a result that is also expected because the slope of the term structure rises (falls) during periods of low (high) stock market variance. Finally, DEF_t predicts future realized variance positively and is in line with Campbell et al. (2018). The R^2 of 58.9% to predict next month's variance indicates that most variation is captured. Furthermore, the impulse response functions in online Appendix Internet Appendix A show that $pc_t^{(1)}$ and $pc_t^{(2)}$ mainly capture variation in short- to mid-term variance, whereas DEF_t captures variation in long-term variance.

The remaining rows in Panel A of Table 2 summarize the dynamics of the explanatory variables in the VAR. The level of the term structure of variance swap prices, $pc_t^{(1)}$, is approximately an AR(1) process, with an autoregressive coefficient of 0.86. The slope of the term structure of variance swap prices, $pc_t^{(2)}$, has a similar persistence, of 0.85, but is also predicted with a positive coefficient by the level of the term structure of variance swap prices. Finally, the default spread, DEF_t, is more persistent, with an autoregressive coefficient of 0.96. The persistence of the variables indicates whether they capture variation in short- or long-term variance and the implications are similar to the results of the impulse response functions, as shown in Internet Appendix A.

The estimates in Panel A of Table 2 are used to calculate expected variance, using Equation (17), and risk premia, using Equation (18). Variation in the variance swap price with T-months to maturity is attributable to variation in either $E_{\mathrm{rv},t}^{(T)}$ or $E_{\mathrm{rp},t}^{(T)}$ or to correlation between $E_{\mathrm{rv},t}^{(T)}$ and $E_{\mathrm{rp},t}^{(T)}$. This intuition follows from the following equation and is obtained if I calculate the variance of the pricing identity (6) for the variance swap price, as follows:

$$\mathrm{var}\big(vs_t^{(T)}\big) \approx \mathrm{var}\big(E_{\mathrm{rv},t}^{(T)}\big) + \mathrm{var}\big(E_{\mathrm{rp},t}^{(T)}\big) \underbrace{-2 \cdot \mathrm{cov}\big(E_{\mathrm{rv},t}^{(T)}, E_{\mathrm{rp},t}^{(T)}\big)}_{=:C_{\mathrm{rv,rp}}^{(T)}}. \tag{19}$$

Thus far, this section focused on the identity for the logarithm of variance swap prices for which the logarithm of stock market variance has to be modeled. However, in Appendix A.6 I show how to decompose the actual variance swap price using a VAR, and present the estimates when modeling stock market variance instead. In Table 3, I show the results for the decomposition of the logarithm of variance swap prices and variance swap prices in levels based on VAR of this section in the former case and the VAR of Appendix A.6 in the latter case.

[Table 3 here]

Table 3 illustrates how the variation in variance swap prices is attributable to expected stock market variance, risk premia, or the correlation between these two factors. The decomposition in Table 3 shows that the main result is robust to specifying a VAR: Short-term variance swap prices mainly vary due to expected stock market variance, whereas long-term variance swap prices mainly vary due to risk premia. This result holds across both specifications, i.e. the results of the decomposition are very similar for the logarithm of variance swap prices and variance swap prices in levels.

Focusing on the variation in the logarithm of variance swap prices, Columns $E^{(T)}_{\mathrm{rv}}$, $E^{(T)}_{\mathrm{rp}}$ and $C^{(T)}_{\mathrm{rv,rp}}$ indicate that one-month variance swaps are driven by 123.5% by expected variance and this number decreases monotonically to 28.5% for 18-month variance swap prices. At the same time, the variation in prices attributable to risk premia increases monotonically from 6.9% for one-month variance swaps to 76.0% for swaps with 18 months to maturity. The results indicate that risk premia for variance swaps vary significantly over time, because for all of the analyzed maturities is the variation attributable to risk premia significantly different from zero. Interestingly, the price-variation attributable to correlation between expected variance and risk premia are negative indicating a positive correlation between these two variables. This means that during periods of increased expected variance, risk premia increase which indicates that it becomes less expensive to hedge variance risk and this result is not in line with economic intuition. The positive correlation between expected variance and risk premia is in line with Cheng (2019) who first documented a similar result for VIX-futures. My results indicate that this relation is strongest at (relatively) short horizons, because at a maturity beyond one month the correlation is not significantly different from zero.

Next, I discuss the results based on the present value identity for variance swap prices in levels as shown in Columns $E^{(T)}_{\mathrm{RV}}$, $E^{(T)}_{\mathrm{RP}}$ and $C^{(T)}_{\mathrm{RV,RP}}$ of Table 3. The price-variation attributable to expected stock market variance decreases monotonically from 105.2% for one-month variance swaps to 43.5% for variance swaps with 18 months to maturity. Similarly, the price-variation due to risk premia increases monotonically from 25.4% for one-month variance swaps to 83.3% for 18-month variance swaps. Finally, also in this specification are risk premia and expected variance generally positively correlated, and in particular at the one-month horizon.

Overall, the results from the decomposition using the VAR of Table 3 are remarkably close to the results of the decomposition using predictive regressions of Table 1. The results from the decomposition using predictive regressions are model-free and, therefore, the similarity suggests that the VAR is correctly specified. In Table 11 of Appendix A.6, I show that expected stock market variance and risk premia obtained using the VAR are able to predict its realized counterparts well. Moreoever, I show in Internet Appendix C that if I perform the decomposition using the VAR at a quarterly frequency, the results are highly similar. Finally, in Internet Appendix B, I show that the results of the decomposition are robust to different specifications of the current VAR, and to the inclusion of additional variables such as: the S&P 500 price-earnings ratio and long-term variance.

In the following section, I analyze the time-series variation in risk premia obtained via the VAR in detail. In particular, I show that there is substantial common variation in the term structure, and show that a significant fraction of the variation is explained by variance beliefs of investors and capital constraints of intermediaries. These latter results also explain why the models analyzed in Section 3.2 fail to match the data, because the models do not model investor variance beliefs or constraints by intermediaries.

5 Time-series variation of risk premia

In this section, I analyze the time-series variation in risk premia in detail. Throughout this section, risk premia obtained from the VAR are multiplied by minus one, such that an increase in the variable implies that risk premia increase, or equivalently, investor have to pay a higher premium to hold a variance swap. First, I show the decomposition of a short-term (one month) and long-term (18 months) variance swap contract graphically. Second, I show that variation in risk premia yields economically sizable variation in expected returns for variance swaps. Third, I analyze the variation in the term structure of risk premia, and show how it relates to measures of variance beliefs of investors (Lochstoer and Muir, 2022) and capital constraints of intermediaries (Cheng, 2019).

In Figure 4, I plot the decomposition of the one-month variance swap price and that of the 18-month variance swap price.

[**Figure 4 here**]

Figure 4 plots the decomposition of the logarithm of the variance swap price (black line) into expected variance (dotted line) and risk premia (dashed line). The left panel shows the demeaned variance swap price with one month to maturity, and it is clearly visible that the monthly variance swap price closely follows expected stock market variance. Moreover, in line with Cheng (2019), there is a negative correlation between expected variance and (the negative of) risk premia, which indicates that risk premia decrease when expected variance increases.

The right panel of Figure 4 plots the decomposition of the 18-month variance swap price (black line) into variance expectations (dotted line) and risk premia (dashed line). Risk premia are a more important determinant of the 18-month variance swap price than they are of the one-month variance swap price. It follows from the graph that risk premia were relatively high during the period following the financial crisis in 2008, whereas risk premia were relatively low during the period leading up to that crisis. Moreover, the variation in the short-term risk premia (left panel) and the variation in long-term risk premia (right panel) are correlated. In order to analyze this correlation further, I plot, in Figure 5, the risk premia obtained from the logarithm of variance swap prices for the benchmark maturities.

[**Figure 5 here**]

Figure 5 plots the risk premia obtained from the logarithm of variance swap prices ranging from 1 to 18 months to maturity. The main result from Figure 5 is that the time variation in the term structure of risk premia is strongly correlated, such that short-term risk premia move in the same direction as long-term risk premia—a finding that suggests that short- and long-term risk premia are driven by similar state variables. Overall, the variation in long-term risk premia is larger than the variation in short-term risk premia. Interestingly, during the financial crisis of 2008 short-term variance risk premia decreased whereas long-term risk premia increased. Therefore, only the price of hedging short-term variance risk decreased during the financial crisis.

Before analyzing the relation of risk premia with other macroeconomic variables, I want to establish that time variation in risk premia implies economically sizable differences in the average returns from investing in variance swaps. To establish this result, I run the following regressions:

$$R^{(T)}_{t+1} - 1 = \mu_1 + \mu_2 \cdot \Big(- E^{(T)}_{\mathrm{rp},t}\Big) + \epsilon_{t+1}, \tag{20}$$

where $R^{(T)}_{t+1} - 1$ corresponds to a monthly simple return on a variance swap with T-months to maturity and $E^{(T)}_{\mathrm{rp},t}$ are the risk premia obtained from the VAR discussed in the previous section. Because $-E^{(T)}_{\mathrm{rp},t}$ has a mean of zero, μ_1 equals the average return on a variance swap, and μ_2 equals by how much the average return increases in $-E^{(T)}_{\mathrm{rp},t}$. Table 4 shows the results of regression (20).

[**Table 5 here**]

The first result from Table 4 is that the average simple return on variance swaps strongly increases in the maturity. The second result, and the main takeaway of Table 4, is that the variation in risk premia is economically sizable. A one standard deviation increase in $-E^{(T)}_{\mathrm{rp},t}$ results in a decrease in the average return of 4.1% for the 18-month variance decreasing to 12.3% for the one-month variance swap. These results suggest that the average simple returns of a variance swap with maturities beyond one month are positive rather than negative during periods in which $-E^{(T)}_{\mathrm{rp},t}$ is below its standard deviation. This puzzling result has already been noted by Johnson (2017), and indicates that during some periods investors want to receive a risk premium for holding a variance swap. Furthermore, it is interesting to note that long-term variance swaps do not have an average risk premium significantly different from zero, while at the same time exhibiting sizable time-series variation.

In the following, I analyze how the estimates of risk premia relate to several variables brought forward by the literature. In particular, Cheng (2019) shows how positions of intermediaries in VIX-futures affect risk premia on these futures. I will use the measure by He et al. (2017) that relates to average intermediary leverage to proxy for constraints of intermediaries. Lochstoer and Muir (2022) document using surveys that investor beliefs regarding stock market variance are slow moving, and they find that their proxy for slow-moving variance beliefs affect risk premia on variance instruments. The focus of the following analyses will be on the explanatory power of these variables for the full term structure of risk premia.

To analyze the impact of these variables for risk premia, I run the following regression:

$$-E_{\mathrm{rp},t}^{(T)} = \beta_0 + \beta_1 \hat{\sigma}_t^2 + \beta_2 VB_t + \beta_3 ILS_t + \theta' X_t + \epsilon_t, \tag{21}$$

where $\hat{\sigma}_t^2$ and $VB_t := \sum_{i=1}^{6} \phi^j \hat{\sigma}_{t-i+1}^2$ are expected next month variance and slow-moving variance beliefs taken directly from Lochstoer and Muir (2022).[11] In Lochstoer and Muir (2022) these two variables are used to predict variance risk premia, and they find that risk premia initially underreact to increases in expected variance and there is a delayed overreaction due to slow-moving variance beliefs. These findings support the empirical results documented in Cheng (2019), only Lochstoer and Muir (2022) attribute it to slow-moving variance beliefs, whereas Cheng (2019) attributes it to demand and supply effects in the market. Furthermore, ILS_t is intermediary leverage (squared) by He et al. (2017) which should capture these intermediary pricing effects, and X_t specifies a vector of control variables to illustrate the robustness of the relationship. Finally, the estimates from regression Equation (21) establish how both channels affect the term structure of

[11] Following Lochstoer and Muir (2022), I use $\phi = 0.5$, which is the estimate for slow-moving variance beliefs that is estimated in their paper.

risk premia.

The estimates of regression (21) are presented in Table 5. All variables in the regressions are scaled to have a mean of zero and standard deviation of one. In this way, the estimated coefficients can be compared along the term structure because the variation in long-term risk premia is larger than variation in short-term risk premia, as shown in Section 4.

[Table 5 here]

The main result of Table 5 is that a sizable fraction of the variation in risk premia is explained by variance beliefs and intermediary constraints, and that the exposure differs along the term structure. Overall, the results suggest that short-term risk premia are driven to a larger extent by variance beliefs, whereas a more important driver of long-term risk premia are intermediary constraints.

The results for risk premia obtained via the logarithm of variance swap prices are presented in Panel A, and I find that, in general, higher variance beliefs relate to lower risk premia. Focusing on the results without controls, Columns (1)–(5) show that short-term risk premia are driven to a larger extent by variance beliefs than long-term risk premia and the coefficients are insignificant for maturities beyond three months. On the other hand, the exposure of risk premia to intermediary constraints increases monotonically with the maturity and the coefficients indicate, as expected, that when intermediaries are more constraint risk premia increase. In case the regression includes additional controls, Columns (6)–(10), these patterns in the exposure of risk premia to these variables are robust. Furthermore, these regressions show that a sizable fraction of the risk premia is driven by these macroeconomic variables as indicated by the high R^2's.

Next, I repeat the same analysis but for risk premia obtained via variance swap prices in levels and the results are presented in Panel B of Table 5. Different form the results in Panel A, variance beliefs now relate positively to risk premia which is in line with the results in Lochstoer and Muir (2022).[12] Similar to the results in Panel A, however, Columns (1)–(5) show that variance beliefs mostly matter for short-term maturities as the statistical significance disappears beyond a maturity of six months. Intermediary constraints relate positively to risk premia, and the coefficient increases monotonically with the maturity indicating that risk premia on long-term contracts are affected more by intermediary constraints than risk premia on short-term contracts. This monotonic increase in the coefficients for intermediary constraints is robust to the inclusion of controls, in fact Columns (6)–(10) show that the explanatory power and statistical significance increase in these specifications. Finally, the statistical significance for the relation between variance beliefs and risk premia is not robust to the inclusion of controls and this is likely driven by the relation between variance beliefs and other macroeconomic variables.

The results in Table 5 support the findings in the literature that variance beliefs and intermediary constraints drive a significant fraction of the variation in risk premia. Furthermore, it can explain why the models analyzed in Section 3.2 fail to capture the variation in risk premia in the variance market, because the models analyzed in this paper do not contain investors with biased variance expectations or a constrained intermediary sector. The results show that in order to describe dynamics of risk premia in variance markets, the models should include biased variance beliefs and/or intermediary constraints.

One of the main takeaways of Table 5 is that intermediary constraints have a differential effect on the term structure of risk premia. In particular, short-term risk premia are to a lesser extent driven by intermediary constraints than long-term risk premia. Because long-term risk premia are more affected by intermediary constraints, it could explain the excess volatility documented in Giglio and Kelly (2018) for variance swaps, a result that is often attributed to an overreaction in expectations of investors (Stein, 1989; Giglio and Kelly, 2018). Instead, my results show that also this form of excess volatility could be

[12] The result that the sign of variance beliefs depends on whether risk premia are obtained from the logarithm of variance swap prices or the actual variance swap prices is probably due to the logarithmic transformation. In principle, both are acceptable ways to analyze risk premia, however given that variance swap prices are right-skewed the logarithmic transformation makes the distribution more symmetric, and thus the right tail less extreme.

attributable to intermediary constraints.

In sum, I show that the ex ante obtained risk premia relate to macroeconomic variables and, in particular, to variance beliefs (Lochstoer and Muir, 2022) and intermediary constraints (Cheng, 2019). Furthermore, my findings suggest that variance beliefs are mostly important for short-term risk premia, whereas intermediary constraints are important for short- and long-term risk premia and the magnitude of the effect increases in the maturity. Finally, these results explain why the models analyzed in this paper fail to capture the variation in risk premia of the data, and provide intermediary constraints as a novel channel to explain the excess volatility documented in Giglio and Kelly (2018).

6 Conclusion

I show that variance risk premia vary over time, and that it drives a significant fraction of variance in S&P 500 variance swap prices. I document a strong term structure in the decomposition of variance swap price variation. Short-term variance swap prices are driven by variation in variance expectations, whereas long-term variance swap prices are mostly driven by variation in variance risk premia. This newly documented stylized fact provides a new challenge to existing asset pricing models, because the models considered in this paper predict a flat term structure. In particular, the disaster model of Gabaix (2012) predicts that all variation in variance swap prices is attributable to variation in variance risk premia. The disaster model of Wachter (2013), meanwhile, predicts that all variation is attributable to variance expectations. This is driven by the fact that this model incorporates a strong persistence in stock market variance, a feature that is not present in the model of Gabaix (2012). The long-run risk model of Drechsler and Yaron (2011) matches the decomposition of long-term variance swaps relatively well, and is able to match the overall variation in variance risk premia. However, due to the large variation in short-term disaster risk, short-term variance risk premia move more of the variation in short-term variance swap prices than empirically observed.

The documented variation in variance risk premia is attributable to several variables. In particular, intermediary constraints (Cheng, 2019) and variance beliefs of investors (Lochstoer and Muir, 2022) play an important role for describing time-series variation in the term structure of variance risk premia. I show that variance beliefs mostly matter for short-term variance risk premia, whereas intermediary constraints matter for short- and long-term variance risk premia. The exposure of risk premia to intermediary constraints increases in the maturity, and therefore intermediary constraints are likely to play a role in the excess volatility documented by Giglio and Kelly (2018) for variance swaps.

In sum, the present paper presents new key stylized facts about the market for variance risk. I show that these stylized facts pose a challenge for state-of-the-art asset pricing models, and augmenting asset pricing models to better describe the pricing of variance risk is thus an interesting avenue for future research.

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Table 1: The results of the decomposition using predictive regressions of Equations (12) and (13). The results are shown for the different present value identities, and for a maturity of 1 month (Panel A) up to a maturity of 18 months (Panel E). The coefficients in columns (1), (3), (5) and (7) indicate how much of the variation in prices is attributable to stock market variance, whereas the coefficients in columns (2), (4), (6) and (8) indicate the variation due to risk premia. All coefficients are presented in percentages. Standard errors are adjusted for auto-correlation using Newey and West (1987) (Hansen and Hodrick 1980) with T-lags, and the corresponding t-statistics are presented below in parentheses (brackets).

	log-VS		**log-F**		**level-VS**		**level-F**	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: $T = 1$.								
Coef.	110.07	10.07	110.07	10.07	92.16	-7.84	92.16	-7.84
(t_{nw})	(18.89)	(1.73)	(18.89)	(1.73)	(5.15)	(-0.44)	(5.15)	(-0.44)
$[t_{hh}]$	[17.99]	[1.65]	[17.99]	[1.65]	[4.68]	[-0.40]	[4.68]	[-0.40]
R^2	57.74	1.13	57.74	1.13	42.42	0.53	42.42	0.53
Panel B: $T = 3$.								
Coef.	95.73	-4.02	86.36	-13.64	66.71	-33.29	43.79	-56.21
(t_{nw})	(10.88)	(-0.44)	(7.41)	(-1.17)	(5.66)	(-2.83)	(4.08)	(-5.23)
$[t_{hh}]$	[9.42]	[-0.38]	[6.48]	[-1.02]	[5.05]	[-2.52]	[3.73]	[-4.78]
R^2	43.20	0.13	22.45	0.72	22.96	6.91	5.99	9.51
Panel C: $T = 6$.								
Coef.	83.34	-16.77	60.49	-39.51	49.29	-50.71	19.83	-80.17
(t_{nw})	(6.82)	(-1.35)	(3.69)	(-2.41)	(4.16)	(-4.28)	(1.98)	(-8.00)
$[t_{hh}]$	[5.88]	[-1.17]	[3.27]	[-2.13]	[4.37]	[-4.49]	[1.79]	[-7.23]
R^2	31.53	1.72	8.88	3.99	13.73	14.41	1.00	14.15
Panel D: $T = 12$.								
Coef.	55.83	-41.85	12.81	-87.19	33.83	-66.17	16.29	-83.71
(t_{nw})	(3.69)	(-2.65)	(0.75)	(-5.08)	(3.45)	(-6.75)	(1.49)	(-7.66)
$[t_{hh}]$	[3.44]	[-2.48]	[0.69]	[-4.68]	[3.25]	[-6.36]	[1.63]	[-8.39]
R^2	14.21	8.04	0.48	18.15	7.59	23.91	0.76	16.83
Panel E: $T = 18$.								
Coef.	24.52	-72.80	-22.93	-122.93	19.56	-80.44	-19.65	-119.65
(t_{nw})	(1.33)	(-3.89)	(-1.58)	(-8.47)	(2.11)	(-8.69)	(-0.99)	(-6.03)
$[t_{hh}]$	[1.34]	[-3.92]	[-1.49]	[-7.98]	[2.62]	[-10.78]	[-0.97]	[-5.90]
R^2	3.03	20.54	2.24	39.66	3.14	35.42	1.03	27.77

Table 2: The estimated coefficients of the VAR of equation (15) with t-values in parentheses in Panel A. All variables are normalized to have a mean equal to zero, and $pc_t^{(1)}$ and $pc_t^{(1)}$ are additionally standardized to have a standard deviation equal to one. Panel B shows the correlation matrix of the residual vector ϵ_t with the standard deviations on the diagonal. The sample period for the dependent variables is January 1996 to June 2019, with 282 monthly data points.

Panel A: Coefficients, VAR model					
	rv_t	$pc_t^{(1)}$	$pc_t^{(2)}$	DEF_t	R^2
rv_{t+1}	0.056	0.514	-0.344	0.514	0.589
(t-stat.)	(0.73)	(7.43)	(-7.13)	(3.38)	
$pc_{t+1}^{(1)}$	0.049	0.856	0.003	0.105	0.847
(t-stat.)	(1.02)	(20.00)	(0.09)	(1.11)	
$pc_{t+1}^{(2)}$	-0.022	0.151	0.845	-0.022	0.745
(t-stat.)	(-0.35)	(2.70)	(21.77)	(-0.18)	
DEF_{t+1}	0.013	-0.010	-0.004	0.964	0.936
(t-stat.)	(1.34)	(-1.11)	(-0.66)	(48.81)	

Panel B: Correlation/Std matrix of residuals				
corr/std	rv_{t+1}	$pc_{t+1}^{(1)}$	$pc_{t+1}^{(2)}$	DEF_{t+1}
rv_{t+1}	0.627	0.627	-0.484	0.354
$pc_{t+1}^{(1)}$	0.627	0.388	-0.598	0.372
$pc_{t+1}^{(2)}$	-0.484	-0.598	0.505	-0.154
DEF_{t+1}	0.354	0.372	-0.154	0.082

Table 3: The results of the variance decomposition of variance swap prices using the VAR. The columns indicated by log-VS rely on identity (6) for log prices, and the columns indicated by level-VS rely on identity (10) for prices in levels. Note that reported coefficient represent the percentage of variance that is attributable to each component such that the sum equals one. Standard errors are computed using the Delta method and presented in parentheses.

	log-VS			**level-VS**		
	$E_{\text{rv}}^{(T)}$	$E_{\text{rp}}^{(T)}$	$C_{\text{rv,rp}}^{(T)}$	$E_{\text{RV},t}^{(T)}$	$E_{\text{RP},t}^{(T)}$	$C_{\text{RV,RP}}^{(T)}$
$T=1$	123.51	6.87	-30.38	105.18	25.35	-30.53
	(12.24)	(2.71)	(13.96)	(11.91)	(4.44)	(14.20)
$T=3$	112.80	14.66	-27.47	92.12	30.16	-22.28
	(18.74)	(6.96)	(21.89)	(17.57)	(8.86)	(21.34)
$T=6$	87.00	31.24	-18.24	77.11	47.67	-24.78
	(24.05)	(15.39)	(29.76)	(21.93)	(16.38)	(29.99)
$T=12$	48.56	60.59	-9.15	58.58	72.10	-30.68
	(24.35)	(27.18)	(35.15)	(25.21)	(25.32)	(38.92)
$T=18$	28.50	75.99	-4.49	43.54	83.25	-26.79
	(20.35)	(29.39)	(32.95)	(23.62)	(27.18)	(38.13)

Table 4: The estimates of regression equation (20). The first row corresponds to the average simple return on a variance swap with maturity T, and the second row corresponds to the coefficient for $-E_{\text{rp},t}^{(T)}$. Note that $E_{\text{rp},t}^{(T)}$ is scaled to have a standard deviation of one. t-statistics are represented in parentheses.

Maturity	1	3	6	12	18
Mean return	-0.285	-0.098	-0.050	-0.013	-0.006
	(-7.16)	(-3.77)	(-2.72)	(-1.01)	(-0.52)
$-E_{\text{rp},t}^{(T)}$	-0.123	-0.110	-0.082	-0.056	-0.041
	(3.09)	(4.24)	(4.46)	(4.24)	(3.77)
R^2	0.033	0.061	0.067	0.061	0.049

Table 5: This table reports the estimation results of regression Equation (21). In Panel A the risk premia derived from the log-VS identity are the dependent variable, and in Panel B the risk premia derived from the level-VS identity are the dependent variable. All variables in the regression are standardized to facilitate interpretation, and to make the coefficients comparable along the term structure. The coefficients reported in columns (1)–(5) are obtained from regressions without additional control variables, whereas the coefficients reported in column (6)-(10) include control variables. The included control variables: Risk-neutral skewness from CBOE, macroeconomic risk perceptions (PVS) from Pflueger et al. (2020), term spread, TED spread and market earnings yield. Standard errors are adjusted for auto-correlation using Newey and West (1987) with six lags, and the corresponding t-statistics are presented below in parentheses.

Panel A: log-VS

	$-E^{(1)}_{\mathrm{rp},t}$	$-E^{(3)}_{\mathrm{rp},t}$	$-E^{(6)}_{\mathrm{rp},t}$	$-E^{(12)}_{\mathrm{rp},t}$	$-E^{(18)}_{\mathrm{rp},t}$	$-E^{(1)}_{\mathrm{rp},t}$	$-E^{(3)}_{\mathrm{rp},t}$	$-E^{(6)}_{\mathrm{rp},t}$	$-E^{(12)}_{\mathrm{rp},t}$	$-E^{(18)}_{\mathrm{rp},t}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\hat{\sigma}^2_t$	0.21	-0.03	-0.06	-0.04	0.01	0.28	0.13	0.18	0.25	0.29
	(1.71)	(-0.27)	(-0.45)	(-0.28)	(0.05)	(3.09)	(1.21)	(1.62)	(2.13)	(2.46)
VB_t	-0.82	-0.38	-0.17	-0.05	-0.03	-0.70	-0.46	-0.41	-0.37	-0.37
	(-3.20)	(-1.88)	(-0.78)	(-0.23)	(-0.13)	(-3.30)	(-2.46)	(-2.16)	(-1.94)	(-2.07)
ILS_t	0.24	0.30	0.32	0.39	0.44	0.52	0.60	0.68	0.71	0.68
	(1.44)	(1.72)	(1.81)	(2.26)	(2.58)	(2.62)	(2.38)	(2.42)	(2.42)	(2.37)
Controls	N	N	N	N	N	Y	Y	Y	Y	Y
R^2	0.24	0.10	0.06	0.12	0.18	0.40	0.28	0.24	0.28	0.34

Panel B: level-VS

	$-E^{(1)}_{\mathrm{RP},t}$	$-E^{(3)}_{\mathrm{RP},t}$	$-E^{(6)}_{\mathrm{RP},t}$	$-E^{(12)}_{\mathrm{RP},t}$	$-E^{(18)}_{\mathrm{RP},t}$	$-E^{(1)}_{\mathrm{RP},t}$	$-E^{(3)}_{\mathrm{RP},t}$	$-E^{(6)}_{\mathrm{RP},t}$	$-E^{(12)}_{\mathrm{RP},t}$	$-E^{(18)}_{\mathrm{RP},t}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\hat{\sigma}^2_t$	-0.75	-0.68	-0.46	-0.30	-0.18	-0.45	-0.38	-0.18	-0.03	0.07
	(-3.08)	(-3.43)	(-2.60)	(-1.96)	(-1.22)	(-1.98)	(-2.30)	(-1.31)	(-0.31)	(0.61)
VB_t	0.52	0.60	0.46	0.32	0.22	0.22	0.34	0.20	0.08	-0.02
	(1.73)	(2.15)	(1.73)	(1.36)	(0.93)	(0.74)	(1.45)	(0.99)	(0.45)	(-0.14)
ILS_t	0.06	0.13	0.20	0.32	0.40	0.42	0.57	0.68	0.75	0.75
	(0.32)	(0.73)	(1.15)	(1.96)	(2.43)	(2.58)	(4.20)	(4.18)	(3.64)	(3.34)
Controls	N	N	N	N	N	Y	Y	Y	Y	Y
R^2	0.12	0.11	0.11	0.16	0.21	0.22	0.25	0.27	0.31	0.34

Figure 1: The left (right) panel plots how much of the variation in log variance swap prices is driven by expected variance (risk premia) in the data (solid line), the model of Gabaix (2012) (dashed line), the model of Wachter (2013) (dash-dotted line), and the model of Drechsler and Yaron (2011) (dotted line). The grey area corresponds to a 95% confidence interval. The results are plotted for variance swap prices with 1, 3, 6, 12, and 18 months to maturity. The y-axis corresponds to how much of the variation is attributable to expected variance or risk, respectively, in percentage terms.

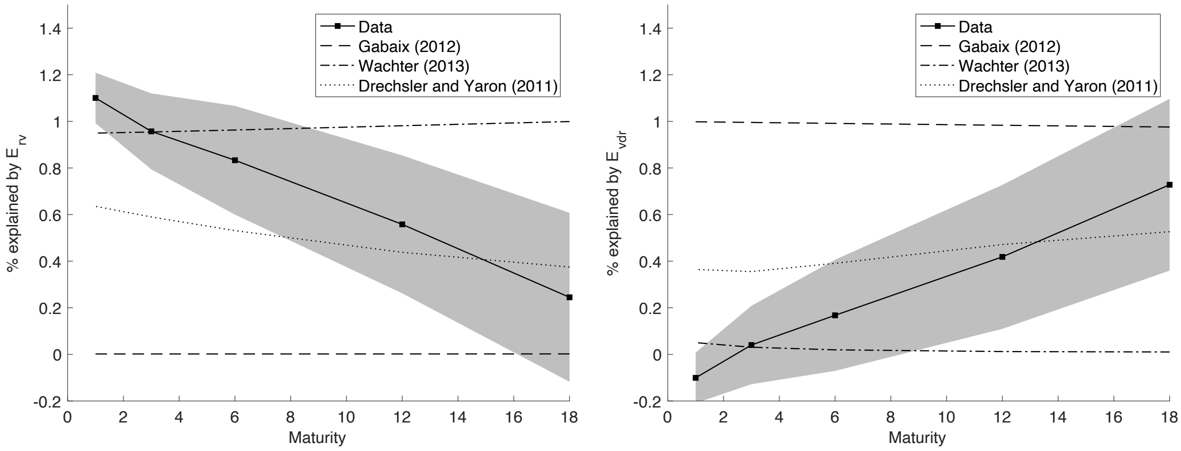


Figure 2: The term structure of the variance of variance swap prices in the data (solid line), the model by Gabaix (2012) (dashed line), the model by Wachter (2013) (dash-dotted line), and the model by Drechsler and Yaron (2011) (dotted line). The grey area corresponds to a 95% confidence interval. The results are plotted for variance swap prices with 1, 3, 6, 12, and 18 months to maturity. The y-axis corresponds to monthly volatility.

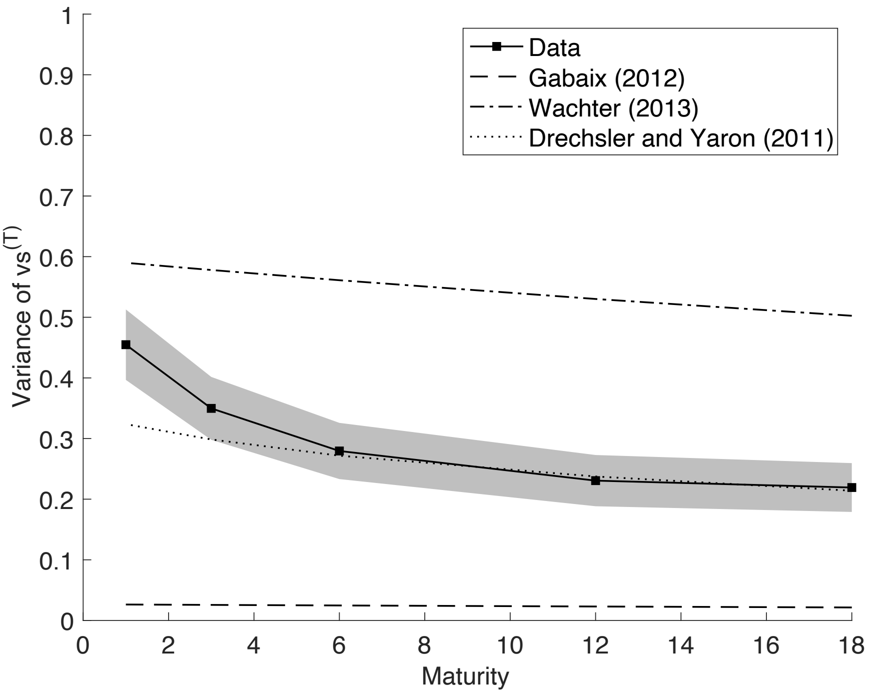


Figure 3: The left (right) panel plots the term structure of expected returns (return volatility) on variance swaps in the data (solid line), the model of Gabaix (2012) (dashed line), the model of Wachter (2013) (dash-dotted line), and the model of Drechsler and Yaron (2011) (dotted line). The grey area corresponds to a 95% confidence interval. The results are shown for variance swaps with 1, 3, 6, 12, and 18 months to maturity. The y-axis corresponds to monthly returns.

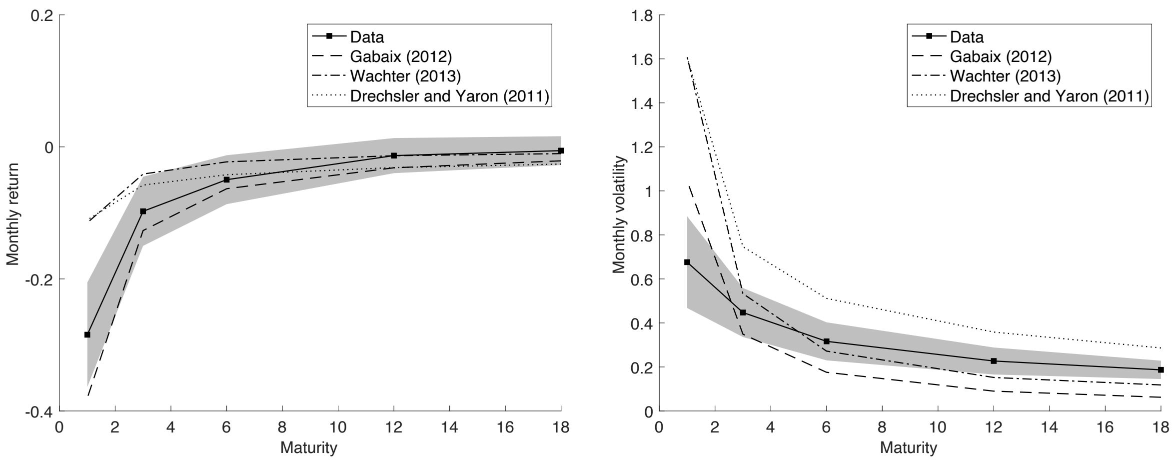


Figure 4: The decomposition of the variance swap price (black line) into variance expectations (dotted line) and risk premia (dashed line) over the sample period. The left panel shows the decomposition of the one-month variance swap price, and the right panel the decomposition of the 18-month variance swap price. The variables are represented as six-month moving averages. The shaded area corresponds to the NBER recessions.

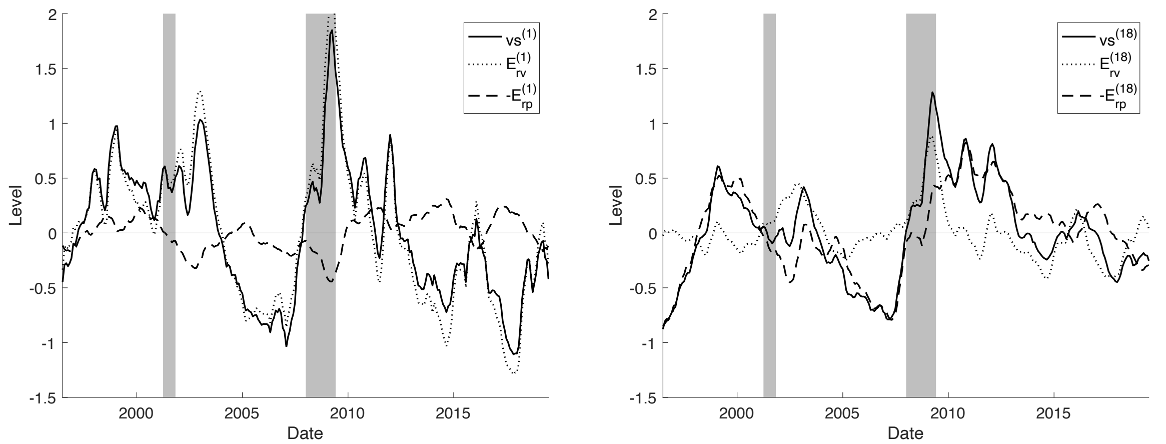
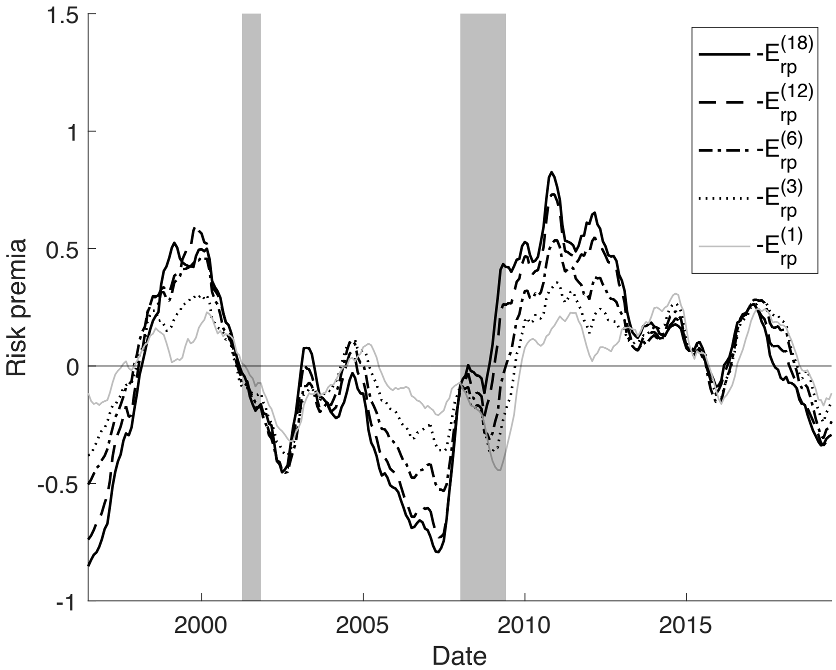


Figure 5: Six-month moving averages of the risk obtained from the log variance swap price with one month to maturity (solid grey line), three months to maturity (dotted black line), six months to maturity (dash-dotted black line), 12 months to maturity (dashed black line), and 18 months to maturity (solid black line). The grey area corresponds to NBER recessions



A Appendix: Additional empirical results

A.1 Derivation of the present value identities.

In this section, I discuss the derivation of each of the identities discussed in Section 1.2.

log-VS. The first step in deriving this identity is to Taylor-expand the log-return on the variance swap:

$$r_{t+1}^{(T)} \approx k(T) + \rho(T) \cdot vs_{t+1}^{(T-1)} + \big[1 - \rho(T)\big] rv_{t+1} - vs_t^{(T)}, \tag{22}$$

where $r_{t+1}^{(T)} = \log\big(R_{t+1}^{(T)}\big), vs_{t+1}^{(T-1)} = \log\big(VS_{t+1}^{(T-1)}\big), rv_{t+1} = \log\big(RV_{t+1}\big)$, and $vs_t^{(T)} = \log\big(VS_t^{(T)}\big)$. In equation (22), $k(T)$ represents the approximation constant and $\rho(T)$ governs the relative importance of the next period's price and the next period's realized variance in the calculation of the return of the variance swap. These constants depend on the maturity, T, of the variance swap because the one-period return on a short-term variance swap is mostly driven by the next period's realized variance whereas the one-period return on a long-term variance swap is mostly driven by the next period's price. Intuitively, $\rho(T)$ increases in the maturity of the variance swap, and thus gets closer to one. Below, I show that $\rho(T)$ can be estimated using a simple regression and, importantly, that equation (22) approximates the return on a variance swap really well.

The next step is to substitute the next period's variance swap price with the following approximation:

$$vs_{t+i}^{(T-i)} \approx k(T-i) + \rho(T-i) \cdot vs_{t+i+1}^{(T-i-1)} + \big[1 - \rho(T-i)\big] rv_{t+i+1} - r_{t+i+1}^{(T-i)},$$

where $k(T-i)$ and $\rho(T-i)$ are the log-linearization coefficients of a variance swap with $T-i$-periods to maturity. This equation allows me to substitute future variance swap prices up to the point that the current variance swap price depends on the one-month variance swap price, $vs_{t+T-1}^{(1)}$. The following holds regarding the one-month variance swap price: $vs_{t+T-1}^{(1)} = rv_{t+T} - r_{t+T}^{(1)}$, that is, $k(1) = \rho(1) = 0$. After this substitution, I obtain the following:

$$vs_t^{(T)} \approx K + \sum_{i=1}^{T} \big[1 - \rho(T-i+1)\big] \Big(\prod_{j=1}^{i-1} \rho(T-j+1) \Big) \cdot rv_{t+i} - \sum_{i=1}^{T} \Big(\prod_{j=1}^{i-1} \rho(T-j+1) \Big) \cdot r_{t+i}^{(T-i+1)}, \tag{23}$$

where K is a constant and a function of the constants $k(T-i)$ and $\rho(T-i)$ from the individual log-linearizations. I discard the constant K from the pricing identity because the focus of the paper is on time-series variation in variance swap prices. Equation (23) is an accounting identity; therefore it also holds in expectation conditional on the information at time t, and equations (6)–(8) of Section 1.2 follow.

Campbell and Shiller (1988) show that the current price–dividend ratio increases in dividend growth expectations and decreases in stock market discount rates. Therefore, identity (6) for the variance swap price is similar to the pricing identity of the price–dividend ratio, where variance expectations take the role of expected cash flows and variance risk premia replace stock market discount rates. There are two main differences between the identity of equation (6) and the identity in Campbell and Shiller (1988), and these are due to the fact that a variance swap is a finite cash flow, whereas equity is a perpetual cash flow. First, the variance risk premia in equation (8) depend on the maturity of the variance swap, which is important because Dew-Becker et al. (2017) show that there is a strong term structure in risk premia on variance swaps. Second, in the derivation of the pricing identity for equity, Campbell and Shiller (1988) assume the so-called no-bubble condition. This assumption is not needed in the case of a variance swap.

In order run the predictive regressions of Section 3.1, I need the log-linear approximation coefficients $\rho(T)$. I estimate $\rho(T)$ using a simple regression, and the results of this exercise are presented in Table 6. As expected, the log-linearization coefficient $\rho(T)$

Table 6: The regression results of $r_{t+1}^{(T)} - rv_{t+1} + vs_t^{(T)} = k(T) + \rho(T) \cdot \left(vs_{t+1}^{(T-1)} - rv_{t+1}\right) + \epsilon_{t+1}^{(T)}$, for different maturities T. For each maturity, the log-linearization coefficient $\rho(T)$ is given as is the R^2 of the regression.

Maturity	1	2	3	4	5	6	7	8	9
$\rho(T)$	0	0.634	0.777	0.841	0.876	0.898	0.914	0.925	0.933
R^2	100%	98.24%	99.28%	99.57%	99.70%	99.78%	99.82%	99.85%	99.88%
Maturity	10	11	12	13	14	15	16	17	18
$\rho(T)$	0.940	0.945	0.950	0.954	0.958	0.961	0.963	0.965	0.967
R^2	99.90%	99.91%	99.92%	99.94%	99.94%	99.95%	99.96%	99.96%	99.96%

depends on the maturity of the variance swap, and increases in the maturity, which indicates that the next period's variance swap price is relatively more important than the one-period realized variance for long-term variance swaps. The second row in Table 6 shows that the log-linear approximation of the variance swap returns is in fact very good, as indicated by the large R^2s, which range from 98.24% to 99.96%, with an average of 99.7%.

log-F. In the following, I derive the identity for the logarithm of the variance forward price. To derive the identity, I first have to define the gross return on a variance forward for holding it to maturity, as follows:

$$R_{F,t+T}^{(T)} = \frac{RV_{t+T}}{F_t^{(T)}}.$$

In case the investor holds the variance forward with T-periods to maturity until maturity, the investor receives only the realized variance in period $t+T$. Taking the logarithm of the gross return on the forward and rearranging yields the pricing identity for the logarithm of the variance forward:

$$f_t^{(T)} = rv_{t+T} - r_{f,t+T}^{(T)}.$$

It constitutes an exact relation between the logarithm of the variance forward, realized variance and risk premia because it follows directly from the defintion of the variance forward return to maturity. Again, because it is an accounting identity, the relation also holds in expectation, and Equation (9) follows.

level-VS. In the following, I derive the present value identity for the variance swap price. First, I rearrange the definition of the realized variance premium of Equation (5), as follows:

$$VS_t^{(T)} = VS_{t+1}^{(T-1)} + RV_{t+1} - VP_{t+1}^{(T)}.$$

The second step is to iterate this equation forward by substituting an equivalent equation for $VS_{t+i}^{(T-i)}$ up to maturity, as follows:

$$VS_t^{(T)} = \sum_{i=1}^{T} RV_{t+i} - \sum_{i=1}^{(T)} VP_{t+i}^{(T-i+1)}.$$

Again computing the expectation on both sides, yields Equation (10).

level-F. Finally, I derive the present value identity for the variance forward price. To derive the identity, I first define the payoff for holing a variance forward to maturity, as follows:

$$VP^{(T)}_{F,t+T} = RV_{t+T} - F^{(T)}_t.$$

In case the investor holds the variance forward to maturity, the investor is entitled to the realized variance in period $t+T$. Rearranging and computing the expectation, yields the pricing identity of Equation (11).

A.2 Variance swaps as in Kozhan et al. (2013)

The realized variance of a variance swap entered at time t with maturity T is calculated in the following way:

$$RV^{(T)}_t = \sum_{j=1}^{T} \Big[2\big(e^{r_{t+j}} - 1 - r_{t+j}\big) \Big], \tag{24}$$

where r_{t+j} is daily log return realized on day $t+j$. Note that equation (24) is similar to the sum of squared daily returns as $r^2 \approx 2\big(e^r - 1 - r\big)$. The variance swap rate is defined as the risk-neutral expectation of the realized variance specified in equation (24). Kozhan et al. (2013) show how to calculate the variance swap rate with maturity T at time t from option prices, as follows:

$$VS^{(T)}_t = \frac{2}{B^{(T)}_t} \left[\int_0^{F^{(T)}_t} \frac{P^{(T)}_t(K)}{K^2} dK + \int_{F^{(T)}_t}^{\infty} \frac{C^{(T)}_t(K)}{K^2} dK \right], \tag{25}$$

where $B^{(T)}_t$ is the risk-free bond price at time t with maturity T, $F^{(T)}_t$ is the forward price at time t with maturity T and $P^{(T)}_t(K)$ and $C^{(T)}_t(K)$ are prices of European put and call options at time t with maturity T and strike price K.

Kozhan et al. (2013) show how to approximate equation (25) using a finite number of available put and call options. Given the set of available option prices $P^{(T)}_t(K_i)$ and $C^{(T)}_t(K_i)$ for $0 \leq i \leq N$ where prices are mid points from bid and ask quotes, Kozhan et al. (2013) compute variance swap rates as follows. Define the following function:

$$\Delta I(K_i) = \begin{cases} \frac{K_{i+1}-K_{i-1}}{2}, & \text{for } 0 \leq i \leq N \text{ (with } K_{-1} := 2K_0 - K_1,\ K_{N+1} := 2K_N - K_{N-1}) \\ 0, & \text{otherwise.} \end{cases}$$

Then the variance swap rate is computed as follows:

$$VS_t^{(T)} \approx 2 \sum_{K_i \leq F_t^{(T)}} \frac{P_t^{(T)}(K_i)}{B_t^{(T)} K_i^2} \Delta I(K_i) + 2 \sum_{K_i > F_t^{(T)}} \frac{C_t^{(T)}(K_i)}{B_t^{(T)} K_i^2} \Delta I(K_i). \tag{26}$$

Options on the S&P 500 expire every month on the third Friday. Using linear interpolation, I calculate variance swap rates that expire on the last trading day of each month. The linear interpolation works in the following way: Variance swap rates with maturity $T_1 < T$ and $T_2 > T$ are calculated by equation (26); then the variance swap rate with maturity T is constructed as follows:

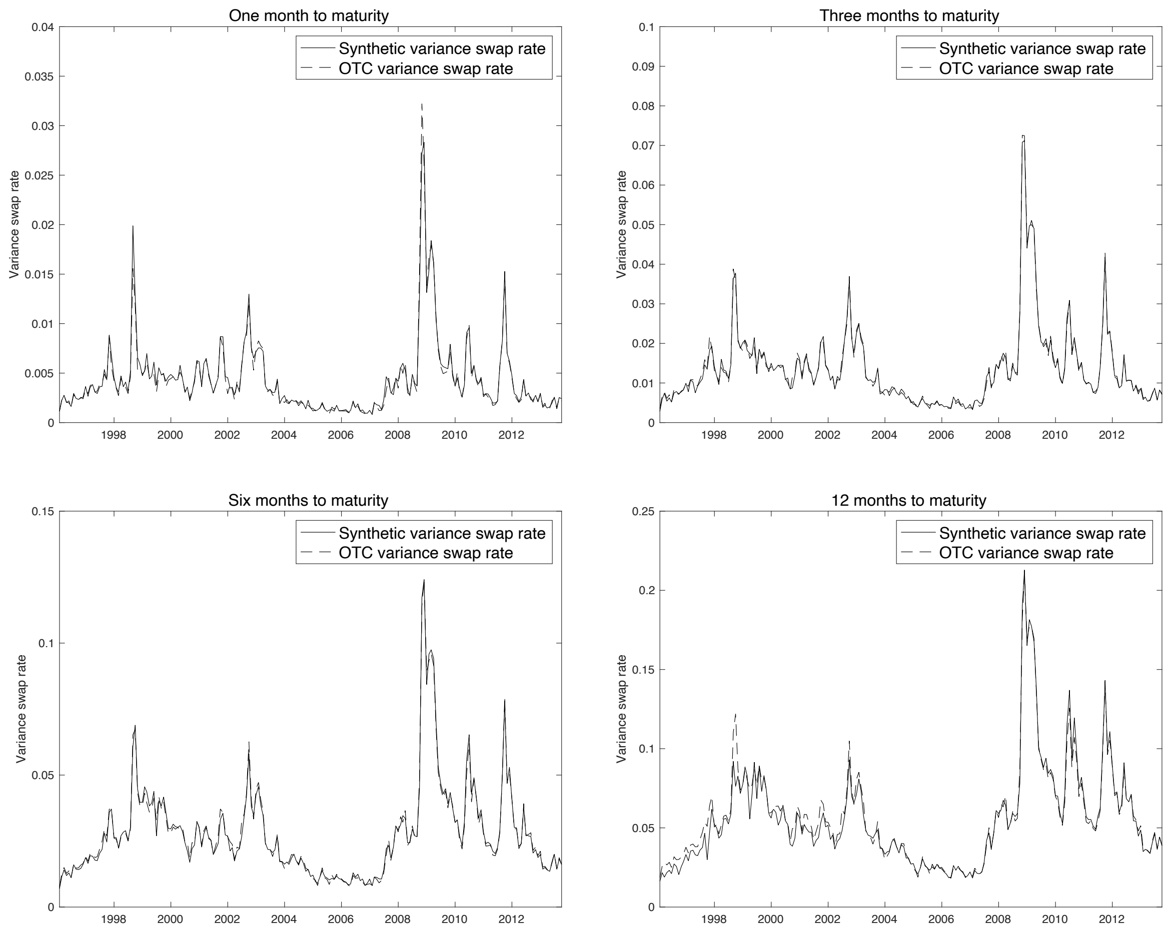
$$VS_t^{(T)} = \alpha VS_t^{(T_1)} + (1-\alpha) VS_t^{(T_2)},$$

where $T = \alpha T_1 + (1-\alpha)T_2$. With the data from OptionMetrics I calculate a panel of variance swap rates with one month up to 18 months to maturity.

A.3 Compare synthetic variance swaps to OTC variance swaps

In this section, I compare the data on synthetic variance swaps that are obtained from option pricing to the data on variance swaps from the OTC market. The data on variance swaps from the OTC market is from Dew-Becker et al. (2017). Their sample covers the period from December 1995 to September 2013 and variance swap rates up to a maturity of 12 months. During the period from January 1996 to September 2013, I observe a synthetic variance swap rate obtained using my methodology and a variance swap rate from the actual OTC data. I plot these rates in following graphs for one, three, six, and 12 months to maturity, which are the maturities of my benchmark analysis.

Figure 6: This figure plots the synthetic variance swap rate and OTC swap rate from Dew-Becker et al. (2017) for four maturities. The top-left graph plots the one-month swap rate, the top-right graph plots the three-month swap rate, the bottom-left graph plots the six-month swap rate, and the bottom-right graph plots the 12-month swap rate.



Overall, Figure 6 provides strong evidence that the synthetic variance swap rate is very similar to the swap rate in the OTC market. This indicates that the option market and the variance swap market are integrated markets and contain the same information regarding the pricing of variance risk. Notable differences include the difference in the one-month swap rate during the financial crisis and the difference in the 12-month swap rates during the first years of my sample. Furthermore, the average correlations between synthetic and OTC swap rate of the four maturities equals 0.991.

In Table 7, I present sample statistics from the panel of variance swap returns. Note that I also present sample statistics on the realized variance of the S&P 500, because realized variance plays the role of the dividend payment in the calculation of the return. Moreover, I calculate simple returns and log-returns in order to quantify its differences in the case of variance swaps. The sample statistics of the realized variance and variance swap returns are shown in Table 7. In Table 7, sample statistics of monthly realized

Table 7: The table shows sample statistics of realized variance in Panel A, simple variance swap returns in Panel B and log variance swap returns in Panel C. The sample statistics of monthly realized variance in Panel A are scaled to represent the yearly standard deviation. The mean, standard deviation, yearly Sharpe ratio and the 5%, 25%, 50%, 75% and 95% quantiles are presented.

Panel A: Realized variance								
Maturity	Mean	SD	SR	5%	25%	Median	75%	95%
-	0.162	0.095	-	0.068	0.101	0.142	0.188	0.323
Panel B: Simple returns on variance swaps								
18	-0.006	0.187	-0.106	-0.231	-0.123	-0.037	0.071	0.317
12	-0.013	0.227	-0.202	-0.269	-0.151	-0.056	0.081	0.346
6	-0.050	0.316	-0.544	-0.363	-0.232	-0.110	0.058	0.480
3	-0.098	0.447	-0.756	-0.477	-0.353	-0.204	0.002	0.581
1	-0.285	0.676	-1.458	-0.779	-0.630	-0.456	-0.193	0.838
Panel C: Log-returns on variance swaps								
18	-0.021	0.167	-	-0.262	-0.131	-0.037	0.069	0.275
12	-0.034	0.195	-	-0.313	-0.164	-0.057	0.078	0.297
6	-0.090	0.264	-	-0.451	-0.265	-0.117	0.057	0.392
3	-0.181	0.365	-	-0.649	-0.435	-0.228	0.002	0.458
1	-0.572	0.640	-	-1.508	-0.995	-0.608	-0.214	0.609

variance on the S&P 500 and returns on variance swaps with different maturities ranging from one to 18 months are presented. Panel A presents sample statistics of realized variance scaled to yearly standard deviation, Panel B presents simple returns on variance swaps, and Panel C presents log-returns on variance swaps. The mean monthly realized variance over the sample is equal to 16.2% p.a., with a standard deviation of 9.5%. Furthermore, the distribution of monthly realized variance is right-skewed, indicated by the quantiles of the distribution.

The first observation from Panel B of Table 7 is that, on average, returns on variance swap returns are negative. This result is in line with a positive variance risk premium for the market portfolio as in Bollerslev et al. (2009) and Drechsler and Yaron (2011). Economically, a negative expected return on a variance swap indicates that if an investor wants to hedge variance risk, she pays a risk premium. The premium the investor pays for holding a variance swap is decreasing in the maturity of the variance swap. A variance swap with maturity longer than one month has exposure toward realized variance in the next month and toward expected variance for the remainder of the contract. Dew-Becker et al. (2017) show that the premium for hedging realized variance the next month is much larger than for hedging expected variance, and, therefore, the premium for holding a variance swap is decreasing in the maturity. Moreover, returns on variance swaps are volatile, as seen in the third column of Panel B in Table 7 and the volatility is decreasing in the maturity of the variance swap. However, the yearly Sharpe ratio is strongly increasing in maturity, and the yearly Sharpe ratio for investing in variance swaps with one month to maturity is very low (≈ -1.46), and similar to what Dew-Becker et al. (2017) find. Furthermore, the distributions of variance swap returns are right-skewed, indicated by the quantiles of the return distribution.

The sample average of the log-returns on variance swaps in Panel C of Table 7 are lower than the sample average for simple returns in Panel B of Table 7. This is driven by the fact that log-return distributions are less right-skewed than the simple return distributions and, therefore, is the sample average lower. The distribution of one-month returns is affected the most by the log-transformation. This result derives from the fact that the approximation of log-returns is equal to simple returns is close if the volatility of the return is low.

A.4 Variation in upside and downside variance swap prices

In this section, I show how the price of a variance swap can be decomposed as the sum of an upside and downside variance swap. In addition, I show that most of the variation in variance swap prices is attributable to variation in downside variance swap prices.

I start by showing how to decompose the variance swap as in Kozhan et al. (2013), into the sum of an upside and downside variance swap. As shown in Kozhan et al. (2013), the payoff of the variance swap is given by:

$$g(r_{t,T}) = 2(e^{r_{t,T}} - 1 - r_{t,T}),$$

where $r_{t,T}$ is the log-return on a forward contract from time t to maturity T. The payoff of the upside and downside variance swap are defined as follows:

$$g_u(r_{t,T}) = 2(e^{r_{t,T}} - 1 - r_{t,T}) \cdot \mathbb{1}(r_{t,T} \geq 0) \text{ and} \tag{27}$$

$$g_d(r_{t,T}) = 2(e^{r_{t,T}} - 1 - r_{t,T}) \cdot \mathbb{1}(r_{t,T} < 0), \tag{28}$$

where $\mathbb{1}(\cdot)$ is an indicator function. Using the formula from Bakshi and Madan (2000), I show that the prices of the payoff functions (27) and (28) are given by:

$$VS_{u,t}^{(T)} = \frac{2}{B_t^{(T)}} \left[\int_{F_t^{(T)}}^{\infty} \frac{C_t^{(T)}(K)}{K^2} dK \right] \text{ and}$$

$$VS_{d,t}^{(T)} = \frac{2}{B_t^{(T)}} \left[\int_{0}^{F_t^{(T)}} \frac{P_t^{(T)}(K)}{K^2} dK \right],$$

such that $VS_t^{(T)} = VS_{u,t}^{(T)} + VS_{d,t}^{(T)}$. Note, by Proposition 1 of Kozhan et al. (2013) there exists a unique trading strategy that perfectly hedges the payoff functions (28) and (27). However, the inclusion of the indicator function in the payoff functions makes this trading strategy infeasible to implement without strong assumptions on the process of the underlying. Therefore, I will only focus in this analysis on the prices of the upside and downside variance swap which is a combination of the cash flow and the discount rate.

My definition of the upside and downside variance swap is similar to Andersen and Bondarenko (2009) and Baele et al. (2019). While these papers focused on the average upside and downside variance premium for a maturity of one month, my analysis focuses on time-series variation in upside and downside variance swap prices and I include maturities from one-month up to 18 months. Moreover, in some other studies, the conditioning is different in the pricing equation than in the calculation of the realized payoff. For instance, Kilic and Shaliastovich (2019) and Dew-Becker et al. (2017) use a similar specification for the upside and downside variance swap price, but in order to calculate realized payoff they condition on intraday or daily returns. Hence, their realized payoff conditions on a different set of events compared to the pricing equation, for which the condition is whether the stock price at maturity is above or below the current forward price. This blurs the comparison of the realized payoff and price of the upside and downside variance swap, and, therefore, does not identify the upside and downside variance discount rate well.

I derive an identity to decompose variation in the variance swap price into variation due to the upside and downside variance swap price, in the following way (I suppressed the constants):

$$\begin{aligned} vs_t^{(T)} &= \log\left(VS_{\mathrm{u},t}^{(T)} + VS_{\mathrm{d},t}^{(T)}\right) \approx \rho_1(T) vs_{\mathrm{u},t}^{(T)} + (1-\rho_1(T)) vs_{\mathrm{d},t}^{(T)}, \\ \iff 1 &\approx \frac{\mathrm{cov}(vs_t^{(T)}, \rho_1(T) vs_{\mathrm{u},t}^{(T)})}{\mathrm{var}(vs_t^{(T)})} + \frac{\mathrm{cov}(vs_t^{(T)}, (1-\rho_1(T)) vs_{\mathrm{d},t}^{(T)})}{\mathrm{var}(vs_t^{(T)})} \\ &=: b_u + b_d. \end{aligned} \tag{29}$$

The coefficients b_u and b_d are estimated in two stages, in the first stage I estimate $\rho_1(T)$ using a simple regression and in the second stage b_u and b_d are estimated with a simple regression using $\rho_1(T)$ from the first stage. Similar to before, the sum of the coefficients should be close to one, and if this is the case it indicates that this log-linear approximation is, in fact, a good approximation. The results these second-stage regressions are presented in Table 8. The main result from Table 8 is that the main driver of variation in the variance swap price is the downside variance swap price. This result makes sense, because an important determinant of the variance swap price is crash risk. Overall, the importance of the downside variance swap price increases in the maturity of the variance swap (except for the one-month variance swap). Finally, the sum of the coefficients b_u and b_d is very close to one, which indicates that the log-linear approximation is a good approximation

Table 8: In this table the results of the decomposition of the variance swap price into the upside and downside variance swap price are presented. The definition of the coefficients b_u and b_d are given in equation (29). The t-statistics of the coefficients are given in parentheses and are calculated using Newey-West standard errors with 50 lags.

Dependent variable:	$\rho_1(T)vs_{\mathrm{u},t}^{(T)}$		$(1-\rho_1(T))vs_{\mathrm{d},t}^{(T)}$	
Maturity	b_u (t-stat.)	R^2	b_d (t-stat.)	R^2
18	0.256	0.859	0.742	0.980
	(19.18)		(57.96)	
12	0.275	0.865	0.722	0.978
	(16.58)		(45.13)	
6	0.368	0.886	0.634	0.956
	(20.74)		(50.42)	
3	0.371	0.923	0.635	0.970
	(22.96)		(56.47)	
1	0.314	0.957	0.687	0.991
	(27.06)		(64.09)	

for the log-price of the variance swap.

A.5 Decomposition of variance swap prices in a subsample

In this section, I show that the results of Subsection 3.1 are robust to running the predictive regressions on a subsample. The analysis is done on the sample period starting in September 2005, and the reason is that Cboe introduced Long-Term Equity Anticipation Securities (Leaps) for the S&P 500 in this month. Leaps are option contracts with the same specifications as before, only with a maturity up to three years. Cboe lists these options once every year, and the option expire on the third Friday in January. For this reason, I can obtain a balanced panel of variance swap prices up to a maturity of 24 months using interpolation on the sample period from September 2005 up to June 2019. To decompose the variance swap prices on this subsample, I run the predictive regressions of equations (12) and (13).

Table 9 has two main takeaways. First, the results on this subsample are highly similar to the results over the total sample presented in 3.1. Second, the pattern documented in the term structure of the variance price decomposition continues to hold for longer

Table 9: This table shows the results of the predictive regressions of equations (12) and (13), in which the variance swap price is the independent variable. t-statistics are represented in brackets and are computed using Newey-West standard errors with number of lags equal to T.

Dependent variable:	$y_{\mathrm{rv},t+T}$		$y_{\mathrm{rp},t+T}$	
Maturity	b_{rv} (t-stat.)	R^2	b_{rp} (t-stat.)	R^2
24	-0.205	0.023	-1.239	0.454
	(-0.61)		(-3.88)	
18	0.161	0.014	-0.844	0.263
	(0.65)		(-3.68)	
12	0.502	0.120	-0.495	0.109
	(2.81)		(-2.81)	
6	0.772	0.274	-0.231	0.030
	(4.93)		(-1.42)	
3	0.951	0.414	-0.048	0.002
	(9.18)		(-0.44)	
1	1.130	0.565	0.130	0.017
	(14.55)		(1.67)	

maturities. That is, the variance swap price for contract with 24 months to maturity is solely driven by risk premia. Notably, the coefficient b_{rp} is less than -1 which indicates the variation in the variance swap price is larger than the variation in risk premia. However, the coefficient is not significantly different from -1.

A.6 VAR specification to obtain expected variance in levels.

In this section, I explain how to decompose the price variation in variance swap prices in levels (identity 10) rather than in logs (identity 6) using a VAR. I focus on the same VAR, only model stock market variance in levels RV_t rather than in logs rv_t, as follows:

$$\begin{aligned} Z_{t+1} &= LZ_t + \epsilon_{t+1} \quad \text{and} \\ Z_t &= \begin{pmatrix} RV_t & pc_t^{(1)} & pc_t^{(2)} & DEF_t \end{pmatrix}'. \end{aligned} \tag{30}$$

Equivalent to the definition of $E^{(T)}_{\mathrm{rv},t}$ and $E^{(T)}_{\mathrm{rp},t}$ for log variance swap prices, expected variance and risk premia for variance swap prices in levels are defined as follows:

$$E^{(T)}_{\mathrm{RV},t} = \mathbb{E}_t \sum_{i=1}^{T} RV_{t+i} \quad \text{and}$$
$$E^{(T)}_{\mathrm{RP},t} = \mathbb{E}_t \sum_{i=1}^{T} VP^{(T-i+1)}_{t+i}.$$

These expectations are obtained using the VAR of this section by:

$$E^{(T)}_{\mathrm{RV},t} = e_1'\Big(L + L^2 + \cdots L^T)z_t \quad \text{and}$$
$$E^{(T)}_{\mathrm{RP},t} = E^{(T)}_{\mathrm{RV},t} - VS^{(T)}_t,$$

where the second equality follows by the identity of variance swap prices in levels.

Table 10: This table presents the estimated coefficients of the VAR of Equation (30). t-statistics are presented below the estimates in parentheses. The sample period for the dependent variables is January 1996 to June 2019, with 282 monthly data points.

Coefficients VAR model					
	RV_t	$pc^{(1)}_t$	$pc^{(2)}_t$	DEF_t	R^2
RV_{t+1}	0.56	0.02	-0.06	0.28	0.53
	(9.58)	(0.62)	(-2.85)	(3.21)	
$pc^{(1)}_{t+1}$	0.14	0.86	0.00	0.05	0.85
	(2.19)	(27.50)	(0.16)	(0.49)	
$pc^{(2)}_{t+1}$	0.01	0.13	0.85	-0.04	0.75
	(0.11)	(3.28)	(25.96)	(-0.30)	
DEF_{t+1}	0.03	-0.01	-0.01	0.96	0.94
	(1.93)	(-0.93)	(-1.05)	(46.44)	

In the following, I show that the current specifications of the VAR are able to estimate variance expectations and risk premia effectively. I test this by estimating the same predictive regressions of the future variance and future returns on the variance swap as before, only in this case variance expectations and risk premia obtained from the VAR are used as the predictive variables. The regressions are specified as follows:

$$y_{\mathrm{rv},t+T} = \gamma_{0,\mathrm{rv}} + \gamma_{1,\mathrm{rv}} \cdot E^{(T)}_{\mathrm{rv},t} + \gamma_{2,\mathrm{rv}} \cdot \big(-E^{(T)}_{\mathrm{rp},t}\big) + u^{\mathrm{rv}}_{t+T} \quad \text{and} \tag{31}$$

$$-y_{\mathrm{rp},t+T} = \gamma_{0,\mathrm{rp}} + \gamma_{1,\mathrm{rp}} \cdot E^{(T)}_{\mathrm{rv},t} + \gamma_{2,\mathrm{rp}} \cdot \big(-E^{(T)}_{\mathrm{rp},t}\big) + u^{\mathrm{rp}}_{t+T}. \tag{32}$$

If the VAR is correctly specified, I should find the following: $\gamma_{1,\mathrm{rv}} = 1$ in the case of regression equation (31) and $\gamma_{2,\mathrm{rp}} = 1$ in the case of regression equation (32). In case expectations are obtained using the identity for variance swap prices in levels, the variables in regressions (31) and (32) have to be adjusted accordingly. Table 11 presents the results of both specifications.

Table 11: The results of the predictive regressions of equations (31) and (32) in which the expected realized variance and expected risk premia are the independent variables. The columns indicated by $y_{\mathrm{rv},t+T}$ and $-y_{\mathrm{rp},t+T}$ ($y_{\mathrm{RV},t+T}$ and $-y_{\mathrm{RP},t+T}$) use the VAR estimates of Table 2 (10) to obtain expected variance and risk premia. t-statistics are represented in parentheses and are computed using Newey–West standard errors with number of lags equal to T.

	$y_{\mathrm{rv},t+T}$		$-y_{\mathrm{rp},t+T}$		$y_{\mathrm{RV},t+T}$		$-y_{\mathrm{RP},t+T}$	
	$E^{(T)}_{\mathrm{rv},t}$	$-E^{(T)}_{\mathrm{rp},t}$	$E^{(T)}_{\mathrm{rv},t}$	$-E^{(T)}_{\mathbf{rp},t}$	$E^{(T)}_{\mathrm{RV},t}$	$-E^{(T)}_{\mathrm{RP},t}$	$E^{(T)}_{\mathrm{RV},t}$	$-E^{(T)}_{\mathrm{RP},t}$
$T = 1$	1.04	0.35	-0.04	0.65	1.01	0.10	-0.01	0.90
	(18.11)	(1.20)	(-0.74)	(2.27)	(6.69)	(0.36)	(-0.09)	(3.36)
$T = 3$	0.96	0.14	0.03	0.84	0.85	-0.12	0.15	1.12
	(13.24)	(0.52)	(0.44)	(2.97)	(7.55)	(-0.38)	(1.33)	(3.53)
$T = 6$	1.02	0.18	-0.00	0.79	0.78	-0.05	0.22	1.05
	(7.72)	(0.73)	(-0.03)	(3.08)	(4.74)	(-0.20)	(1.33)	(4.44)
$T = 12$	1.10	0.12	-0.09	0.82	0.71	0.04	0.29	0.96
	(4.54)	(0.58)	(-0.34)	(3.64)	(3.43)	(0.23)	(1.40)	(5.04)
$T = 18$	0.94	-0.01	0.09	0.96	0.59	0.02	0.41	0.98
	(3.67)	(-0.03)	(0.35)	(4.06)	(3.06)	(0.10)	(2.10)	(6.20)

The main result from Table 11 is that, indeed, expected stock market variance and risk premia are effectively estimated using the VAR. First, focusing on the regressions to predict future variance, Columns $y_{\mathrm{rv},t+T}$ and $y_{\mathrm{RV},t+T}$, the results show that future variance is solely predicted significantly by expected variance and not by risk premia. Moreover, the regression coefficient of variance expectations, Columns $E^{(T)}_{\text{rv},t}$ and $E^{(T)}_{\text{RV},t}$, show that the estimated coefficient is relatively close to one for each maturity, and this shows that the VAR is able to capture stock market variance beyond one month. Finally, the estimated coefficient $E^{(T)}_{\text{RV},t}$ decreases in the maturity which suggests that the VAR for log stock market variance is better able to recover long-term variance than the VAR for stock market variance in levels.

Second, focusing on the regressions to predict realized risk premia, Columns $-y_{\text{rp},t+T}$ and $y_{\text{RP},t+T}$, the results show that future realized premia are predicted significantly by expected risk premia. Furthermore, the regression coefficient of risk premia, Columns $-E^{(T)}_{\text{rp},t}$ and $E^{(T)}_{\text{RP},t}$, show that the coefficient is not significantly distinct from one. In sum, Table 11 shows that the utilized VARs are able to effectively recover short- and long-term expected variance and risk premia effectively.

Internet Appendix A

Impulse response functions of the VAR

In this section, I show the impulse response function of stock market variance in response to a change of each of the other variables in the VAR of equation (15) in the main paper. The impulse responses are presented in Figure IA1.

Figure IA1: This figure plots the monthly impulse response functions of stock market variance in response to a change of the variables in the VAR. The scale of the y-axis is in standard deviation of stock market variance, where each of the variables increases by one standard deviation at time 0. The top-left graph plots the responses to a change in rv, the top-right graph plots the responses to a change in $pc^{(1)}$, the bottom-left graph plots the responses to a change in $pc^{(2)}$, and the bottom-right graph plots the responses to a change in DEF.

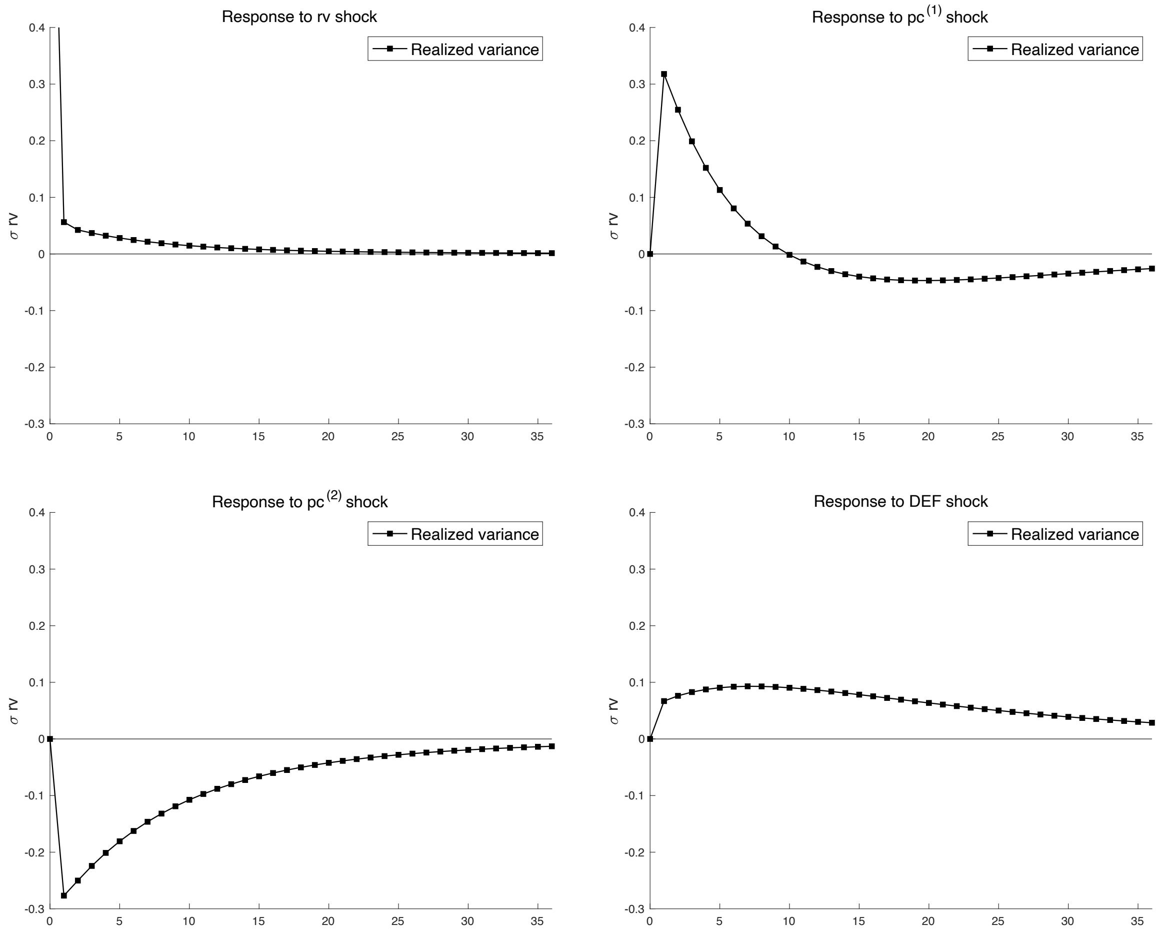


Figure IA1 plots the impulse response functions of stock market variance for a horizon up to 36 months. The top-left graph shows that there is very little persistence in stock market variance, if the current level increases. The top-right graph shows that an increase in $pc^{(1)}$ increases future stock market variance up to 10 months forward. The bottom-left graph shows that an increase in $pc^{(2)}$ decreases future stock market variance and the shock is more persistent than a shock in $pc^{(1)}$. Finally, the bottom-right graph shows that shocks towards DEF are the most persistent and, therefore, affect long-term stock market variance.

Internet Appendix B

Different specifications of the VAR

In this section, I show that the results of the decomposition of variance swap prices are robust to a variety of different specifications of the VAR. Many of the variables included have been shown by prior studies to be important state variables that predict stock market returns or stock market variance. In Table IA1, I show the results of the decomposition of variance swap prices for the following specifications:

1. $z_t = \begin{pmatrix} rv_t & pc_t^{(1)} & pc_t^{(2)} \end{pmatrix}'$. In this specification the default spread is excluded in order to alleviate any concerns induced by the persistence of the default spread.

2. $z_t = \begin{pmatrix} rv_t & pc_t^{(1)} & pc_t^{(2)} & rv_{t-13,t-1} \end{pmatrix}'$. In this specification lagged long-term stock market variance is included. $rv_{t-13,t-1}$ is defined as the log of realized variance in the past year, and lagged one month additionally such that the variable does not overlap with rv_t.

3. $z_t = \begin{pmatrix} rv_t & pc_t^{(1)} & pc_t^{(2)} & pe_t \end{pmatrix}'$. In this specification the log of smoothed price-earnings ratio is included, because Campbell et al. (2018) show that it predicts

future stock market variance.

4. $z_t = \begin{pmatrix} rv_t & pc_t^{(1)} & pc_t^{(2)} & def_t & pe_t & rv_{t-13,t-1} \end{pmatrix}'$. In this specification all state variables used in the previous analyses are included.

Table IA1: This table shows the results of the variance decomposition of variance swap prices using equation (19). Note that the (co)variances of the third, fourth, and fifth columns are scaled with the variance of the second column such that the sum of the three (co)variances equals one. Standard errors are computed using the Delta method.

T	$\frac{\sigma^2_{\text{rv}}}{\sigma^2_{\text{vs}}}$ (s.e.)	$\frac{\sigma^2_{\text{vdr}}}{\sigma^2_{\text{vs}}}$ (s.e.)	$\frac{-2\sigma_{\text{rv,vdr}}}{\sigma^2_{\text{vs}}}$ (s.e.)	$\frac{\sigma^2_{\text{rv}}}{\sigma^2_{\text{vs}}}$ (s.e.)	$\frac{\sigma^2_{\text{vdr}}}{\sigma^2_{\text{vs}}}$ (s.e.)	$\frac{-2\sigma_{\text{rv,vdr}}}{\sigma^2_{\text{vs}}}$ (s.e.)
	Specification 1			Specification 2		
18	0.148	0.678	0.174	0.423	0.604	-0.027
	(0.102)	(0.297)	(0.229)	(0.291)	(0.233)	(0.335)
6	0.750	0.201	0.049	0.855	0.223	-0.078
	(0.217)	(0.126)	(0.213)	(0.258)	(0.126)	(0.286)
1	1.200	0.035	-0.235	1.200	0.035	-0.236
	(0.124)	(0.018)	(0.136)	(0.124)	(0.018)	(0.136)
	Specification 3			Specification 4		
18	0.227	0.800	-0.028	0.564	1.016	-0.580
	(0.176)	(0.360)	(0.429)	(0.306)	(0.353)	(0.540)
6	0.774	0.219	0.007	1.040	0.443	-0.482
	(0.223)	(0.135)	(0.239)	(0.272)	(0.175)	(0.369)
1	1.200	0.036	-0.236	1.200	0.035	-0.236
	(0.124)	(0.019)	(0.136)	(0.124)	(0.018)	(0.136)

The results in Table IA2 show that the main result of the paper holds in each of the specifications; that is, short-term variance swap prices mainly driven by variance expectations, whereas long-term variance swap prices are mainly driven by risk premia. Moreover, the decomposition of the one-month variance swap price shows that the variables other than $pc_t^{(1)}$ and $pc_t^{(2)}$ add little in terms of predicting the one-month variance. The variables mostly add something for long-term stock market variance, due to the persistence of each of the variables. To test how well each of the specifications estimates variance expectations and risk premia, I run the following regressions(the same as equations (31) and (32) in the main paper):

$$\begin{aligned} y_{\mathrm{rv},t+T} &= \gamma_{0,\mathrm{rv}} + \gamma_{1,\mathrm{rv}} \cdot E^{(T)}_{\mathrm{rv},t} + \gamma_{2,\mathrm{rv}} \cdot \left(-E^{(T)}_{\mathrm{vdr},t}\right) + u^{\mathrm{rv}}_{t+T} \quad \text{and} \\ -y_{\mathrm{vdr},t+T} &= \gamma_{0,\mathrm{vdr}} + \gamma_{1,\mathrm{vdr}} \cdot E^{(T)}_{\mathrm{rv},t} + \gamma_{2,\mathrm{vdr}} \cdot \left(-E^{(T)}_{\mathrm{vdr},t}\right) + u^{\mathrm{vdr}}_{t+T}. \end{aligned}$$

In case variance expectations and risk premia are estimated effectively, I should find $\gamma_{1,\mathrm{rv}} = \gamma_{2,\mathrm{vdr}} = 1$ and $\gamma_{2,\mathrm{rv}} = \gamma_{1,\mathrm{vdr}} = 0$.

Table IA2: This table presents the results of the predictive regressions of equations (31) and (32) in which the expected realized variance and expected risk premia are the independent variables. t-statistics are represented in parentheses and are computed using Newey-West standard errors with number of lags equal to T.

T	$\gamma_{1,\text{rv}}$ (t-stat.)	$\gamma_{2,\text{rv}}$ (t-stat.)	$\gamma_{1,\text{vdr}}$ (t-stat.)	$\gamma_{2,\text{vdr}}$ (t-stat.)	$\gamma_{1,\text{rv}}$ (t-stat.)	$\gamma_{2,\text{rv}}$ (t-stat.)	$\gamma_{1,\text{vdr}}$ (t-stat.)	$\gamma_{2,\text{vdr}}$ (t-stat.)
	Specification 1				Specification 2			
18	2.184	-0.341	-1.163	1.299	1.026	-0.284	-0.004	1.223
	(5.94)	(-1.86)	(-3.07)	(7.11)	(3.92)	(-1.39)	(-0.01)	(5.81)
6	1.072	0.014	-0.057	0.939	1.024	-0.025	-0.011	0.973
	(8.23)	(0.06)	(-0.43)	(3.80)	(8.27)	(-0.10)	(-0.08)	(4.12)
1	1.075	0.765	-0.075	0.234	1.075	0.760	-0.075	0.239
	(16.23)	(1.91)	(-1.14)	(0.58)	(16.17)	(1.90)	(-1.12)	(0.60)
	Specification 3				Specification 4			
18	1.723	-0.161	-0.780	1.143	0.985	-0.058	-0.015	1.032
	(6.77)	(-0.81)	(-3.18)	(6.06)	(5.55)	(-0.32)	(-0.08)	(5.64)
6	1.076	-0.016	-0.067	0.988	1.013	0.075	-0.009	0.915
	(8.72)	(-0.07)	(-0.53)	(4.32)	(9.98)	(0.42)	(-0.13)	(4.87)
1	1.072	0.754	-0.075	0.245	1.037	0.257	-0.038	0.743
	(16.36)	(1.91)	(-1.13)	(0.62)	(18.51)	(1.17)	(-0.67)	(3.39)

Table IA2 shows that many VAR specifications are not able to obtain the one-month variance discount rate, and long-term variance expectations. In particular, specifications 1 through 3 fail in obtaining the one-month variance discount rate effectively. It follows that the default spread is an important variable in order to obtain this discount rate. Furthermore, specification 1 and 3 predict long-term variance expectations which exceed the realized variance. Finally, specification 4 does a good job in obtaining short-term risk premia as well as long-term variance expectations, and the results are very similar to the results of the benchmark specification of the VAR.

Internet Appendix C

VAR using quarterly data

In this section, I show that the results continue to hold if the analyses are done at the quarterly rather than monthly frequency. This is done to alleviate concerns that the estimates based on monthly frequency overstate the persistence of the variables and, therefore, create a bias in the variance expectations. The VAR is used to calculate quarterly stock market variance expectations and for this reason the variance swap rate with three months to maturity is the shortest maturity considered in this exercise. Table IA3 presents the estimation results.

Table IA3: This table shows the estimated coefficients of the VAR of equation (15) with t-values in parentheses. All variables are normalized to have mean equal to zero, and $pc_t^{(1)}$ and $pc_t^{(1)}$ are additionally standardized to have standard deviation equal to one. The sample period for the dependent variables is March 1996 to June 2019, with 94 quarterly data points.

Coefficients VAR model					
	rv_t	$pc_t^{(1)}$	$pc_t^{(2)}$	DEF_t	R^2
rv_{t+1}	-0.031	0.468	-0.267	0.448	0.459
(t-stat.)	(-0.17)	(3.07)	(-3.28)	(1.66)	
$pc_{t+1}^{(1)}$	-0.013	0.750	0.003	0.104	0.585
(t-stat.)	(-0.07)	(5.02)	(0.04)	(0.40)	
$pc_{t+1}^{(2)}$	-0.108	0.294	0.696	0.140	0.619
(t-stat.)	(-0.60)	(1.99)	(8.82)	(0.54)	
DEF_{t+1}	0.024	-0.025	-0.007	0.847	0.709
(t-stat.)	(0.46)	(-0.59)	(-0.33)	(11.50)	

Overall, the estimation results based on quarterly frequency are very similar to the results based on monthly frequency. Variance expectations and risk premia are calculated using these estimates and by adjusting equations (17) and (18) accordingly. Table IA4 presents the results.

Table IA4: This table shows the results of the variance decomposition of variance swap rates using equation (19), based on the VAR estimated on quarterly data. Note that the (co)variances of the third, fourth, and fifth columns are scaled with the variance of the second column such that the sum of the three (co)variances equals one.

T	$\mathrm{var}(vs)$	$\frac{\mathrm{var}(E_{\mathrm{rv}})}{\mathrm{var}(vs)}$	$\frac{\mathrm{var}(E_{\mathrm{vdr}})}{\mathrm{var}(vs)}$	$\frac{-2\cdot\mathrm{cov}(E_{\mathrm{rv}},E_{\mathrm{vdr}})}{\mathrm{var}(vs)}$
18	0.220	0.218	0.665	0.117
12	0.227	0.434	0.505	0.060
6	0.274	0.824	0.206	-0.030
3	0.348	1.021	0.085	-0.107

The results of the decomposition of variance swap rates in Table IA3 are remarkably close to the results of Table 3. Therefore, my results are robust whether the frequency of the VAR is monthly or quarterly. Finally, I also decompose the variance swap rate using the predictive regressions of equations (12) and (13). Table IA5 presents the results.

Table IA5: This table shows the results of the predictive regressions of equations (12) and (13) in which the variance swap price is the independent variable. The frequency of the data is quarterly. t-statistics are represented in parentheses and are computed using Newey-West standard errors with number of lags equal to $\frac{1}{3} \cdot T$.

Dependent variable:	$y_{\text{rv},t+T}$		$y_{\text{vdr},t+T}$	
Maturity	b_{rv} (t-stat.)	R^2	b_{vdr} (t-stat.)	R^2
18	0.240	0.029	-0.741	0.218
	(1.21)		(-3.76)	
12	0.540	0.129	-0.452	0.092
	(3.32)		(-2.71)	
6	0.836	0.303	-0.162	0.016
	(5.48)		(-1.04)	
3	0.964	0.417	-0.036	0.001
	(8.05)		(-0.30)	

The similarity between the results of Table 1 and Table IA5 indicate that the results of predictive regressions are robust to decreasing the frequency to the quarterly level.

Internet Appendix D

Appendix asset pricing models

In the following subsections, I discuss more the models considered in this paper in detail. Section Internet Appendix D.1 discusses the model by Gabaix (2012), Section Internet Appendix D.2 discusses Wachter (2013), and Section Internet Appendix D.3 discusses Drechsler and Yaron (2011).

Internet Appendix D.1

Variable disaster risk and CRRA preferences

In this subsection, I discuss the variable rare disaster model of Gabaix (2012). I use the following specification from Dew-Becker et al. (2017):

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + \sigma_c \epsilon_{c,t+1} + J_{c,t+1}, \\ L_{t+1} &= \big(1-\rho_L\big)\bar{L} + \rho_L L_t + \sigma_L \epsilon_{L,t+1} \quad \text{and} \\ \Delta d_{t+1} &= \eta \sigma_c \epsilon_{c,t+1} - L_t \cdot \mathbb{1}_{J_{c,t} \neq 0},\end{aligned}$$

where $\epsilon_{c,t+1}, \epsilon_{L,t+1} \sim N(0,1)$ and $J_{c,t+1}$ is the jump process (rare disaster). The state variable L_t captures the exposure of the dividend process toward the rare disaster, and this exposure varies over time. During times when L_t is large, the stock market is affected more by consumption disasters than when L_t is low. The rare disaster process is modeled as a compound Poisson process and is defined as follows:

$$J_t = \sum_{i=1}^{N_t} \xi_{i,t}, \text{ where } N_t \sim \text{Poisson}(\lambda_t) \text{ and } \xi_{i,t} \sim N\big(\mu_d, \sigma_d\big). \tag{33}$$

Note that in the model by Gabaix (2012) $\lambda_t = \lambda$; that is the jump intensity does not vary over time. The representative agent in the model has power utility preferences with risk aversion parameter γ, which yields the following stochastic discount factor:

$$M_{t+1} = \delta \Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma},$$

where δ is the utility discount rate. I use the calibration from Dew-Becker et al. (2017), which is calibrated to match the risk premium on one-month variance swaps, and is given in Table IA9 of Appendix Internet Appendix D.1.

In order to obtain an equation for the realized stock market variance, I use the following log-linear stock market return approximation:

$$r_{m,t+1} \approx \kappa_0 + \kappa_1 pd_{t+1} - pd_t + \Delta d_{t+1}, \tag{34}$$

where κ_0, κ_1 are log-linearization constants and I use the follow approximation: $pd_t \approx z_0 + z_1 L_t$. The stock market return is then driven by two Gaussian shocks $\big(\epsilon_{c,t+1}$ and $\epsilon_{L,t+1}\big)$ and a jump shock $\big(L_t \cdot \mathbb{1}_{J_{c,t}\neq 0}\big)$. I follow Dew-Becker et al. (2017), who assume that, in the absence of a disaster, the shocks to consumption and the variable disaster have a deterministic variance. In the case of a disaster occurring, Dew-Becker et al. (2017) assume that the largest daily decline in the value of the stock market is $F\%$. Under these assumptions, realized variance over the next period equals the following:

$$RV_{t+1} = \kappa_1^2 z_1^2 \sigma_L^2 + \eta^2 \sigma_c^2 + F \cdot L_t \mathbb{1}_{J_{c,t}\neq 0}, \tag{35}$$

where the first two summands correspond to the variance from the consumption process and variable rare disaster process, and the last summand is the realized variance from the jump process.

Equation (35) offers a first insight into the drivers of variance risk in the model by Gabaix (2012). Realized variance depends on whether the consumption disaster hits the economy. Given that the consumption disaster is a (very) undersirable outcome of the agent, she is willing to pay a large price to hedge this risk. Furthermore, it follows from equation (35) that the disaster size L_t drives variation in expected stock market variance.

The variance swap rate at time t with T months to maturity is computed as the sum of risk-neutral realized variances of equation (35), as follows:

$$VS_t^{(T)} = \sum_{t=1}^{T} \mathbb{E}_t^{\mathbb{Q}}\big(RV_{t+i}\big) = T \cdot v_0 + v_1 \sum_{i=i}^{T} \mathbb{E}_t\big(L_{t+i-1}\big), \tag{36}$$

where $v_0 = \kappa_1^2 z_1^2 \sigma_L^2 + \eta^2 \sigma_c^2$ is the diffusive variance and $v_1 = F \cdot \mathbb{E}^{\mathbb{Q}}\big(\mathbb{1}_{J_c \neq 0}\big)$. These equations show that the size of the disaster L_t also drives the risk premium embedded in the variance swap rate.

The calibration of the model by Gabaix (2012) is from Dew-Becker et al. (2017) and given in the following table.

Table IA6: Calibration of the model by Gabaix (2012).

Parameter	Value	Parameter	Value
μ_c	$0.01/12$	σ_c	$0.02/\sqrt{12}$
μ_d	-0.3	σ_d	0.15
$\bar{L}$	$-\log(0.5)$	σ_L	0.04
ρ_L	$0.87^{1/12}$	η	5
β	$0.96^{1/12}$	γ	7
λ	$\frac{0.01}{12}$		

Note that in the calibration of Dew-Becker et al. (2017) the risk-aversion is raised to 7 in order to match the Sharpe ratio on one-month variance swaps.

In the following, I present more details of the results from the simulation study for the model by Gabaix (2012). First, I present sample statistics of realized variance and risk premia in the model. The results of this simulation study are represented in the following table.

Table IA7: This table presents sample statistics of the realized variance and risk premia in the model by Gabaix (2012). The mean, standard deviation and Sharpe Ratio of the 18-, 12-, 6-, 3-, and 1-month simple risk premia are presented. The second column consists of the empirical result, and the third, fourth, and fifth columns represent the 5%, 50%, and 95% quantile of the simulation study, respectively.

Statistic	Data	Model		
	Est.	5%	50%	95%
Realized variance				
$\mathbb{E}(RV)$	0.162	0.115	0.116	0.117
$\sigma(RV)$	0.095	0.000	0.021	0.043
risk premia				
$\mathbb{E}(r^{(18)})$	-0.006	-0.024	-0.021	-0.017
$\sigma(r^{(18)})$	0.187	0.021	0.056	0.109
$SR(r^{(18)})$	-0.106	-4.091	-1.278	-0.572
$\mathbb{E}(r^{(12)})$	-0.013	-0.037	-0.032	-0.025
$\sigma(r^{(12)})$	0.227	0.021	0.080	0.161
$SR(r^{(12)})$	-0.202	-5.976	-1.326	-0.578
$\mathbb{E}(r^{(6)})$	-0.050	-0.075	-0.063	-0.050
$\sigma(r^{(6)})$	0.316	0.023	0.155	0.319
$SR(r^{(6)})$	-0.544	-11.028	-1.362	-0.584
$\mathbb{E}(r^{(3)})$	-0.098	-0.150	-0.127	-0.099
$\sigma(r^{(3)})$	0.447	0.028	0.308	0.636
$SR(r^{(3)})$	-0.756	-17.644	-1.370	-0.585
$\mathbb{E}(r^{(1)})$	-0.285	-0.451	-0.380	-0.296
$\sigma(r^{(1)})$	0.676	0.068	0.926	1.905
$SR(r^{(1)})$	-1.458	-22.305	-1.366	-0.585

Table IA7 confirms the finding of Figure 3 that the model by Gabaix (2012) is able to capture the strongly increasing term structure of expected variance swap returns documented in the data. Moreover, Table IA7 shows that the volatility of variance swap returns varies a lot across simulation sets, and this results from the fact that the probability of a disaster is small (1% p.a.). If no disasters occur in a simulation set, the volatility of variance swap returns is very low. Finally, I conclude from Table IA7 that the model by Gabaix (2012) is not able to capture the dynamics of empirical stock market volatility.

In the following, I decompose variance swap rates in the model by Gabaix (2012) for each simulation set seperately. Table IA8 presents the results.

Table IA8: This table presents the results of the simple variance decomposition of variance swap rates in the data and in the model by Gabaix (2012). The results of the data are from Table 1, with standard errors in parentheses. The regression coefficients of the model are estimated for each simulation set, and the mean and standard deviation of the regression coefficients are represented in the table.

Maturity	Data		Model	
	b_{rv}	b_{vdr}	b_{rv}	b_{vdr}
18	0.245	-0.728	0.002	-0.985
	(0.185)	(0.188)	(0.028)	(0.125)
12	0.558	-0.419	0.002	-0.989
	(0.151)	(0.158)	(0.028)	(0.101)
6	0.833	-0.168	0.002	-0.994
	(0.119)	(0.122)	(0.028)	(0.069)
3	0.957	-0.040	0.002	-0.997
	(0.083)	(0.086)	(0.027)	(0.047)
1	1.101	0.101	0.002	-0.998
	(0.056)	(0.056)	(0.027)	(0.027)

Table IA8 shows that the result of Figure 1 is stable across the simulation sets. In particular, short-term variance swap rates are solely driven by risk premia, and this number is very similar across simulations, and, therefore, it is strong evidence that the model is not in line with the data.

Internet Appendix D.2

Time-varying disaster risk and Epstein-Zin preferences

In this subsection, I discuss a discrete version of the model by Wachter (2013). Similar to Gabaix (2012), the consumption disaster risk varies over time. However, in the model by Wachter (2013), the disaster intensity, rather than the disaster size, varies over time. Furthermore, the agent in the model has preferences as in Epstein and Zin (1989), rather than CRRA preferences.

Consumption and dividend growth in the model are given by

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + \sigma_c \epsilon_{c,t+1} + J_{t+1} \quad \text{and} \\ \Delta d_{t+1} &= \eta \Delta c_{t+1}, \end{aligned}$$

where $\epsilon_c \sim N(0,1)$ and J_t is a compound-Poisson as in equation (33) of the model by Gabaix (2012). However, in this model the intensity of the consumption disaster is time-varying and follows the following square-root process:

$$\lambda_{t+1} = \phi \lambda_t + (1-\phi)\mu_\lambda + \sigma_\lambda \sqrt{\lambda_t} \epsilon_{\lambda,t+1},$$

where $\epsilon_{\lambda,t} \sim N(0,1)$. The investor has Epstein-Zin utility with elasticity of intertemporal substitution (EIS) equal to one and, therefore, is the log-utility given by

$$v_t = (1-\beta)c_t + \frac{\beta}{1-\alpha} \log \mathbb{E}_t \exp\big(v_{t+1}(1-\alpha)\big),$$

where β is the utility discount rate and $\gamma = 1-\alpha$ is the risk aversion parameter. The calibration of the model is from Dew-Becker et al. (2017) and is given in Table IA9 of Appendix Internet Appendix D.2.

An equation for the realized variance in the model by Wachter (2013) follows from the log-linear market return which is given by

$$r_{m,t+1} \approx \kappa_0 + \kappa_1 pd_{t+1} - pd_t + \Delta d_{t+1},$$

where κ_0, κ_1 are log-linearization constants for the log-market return and pd_t is the log price-dividend ratio and is approximately linear in the state variable: $pd_t \approx z_0 + z_1\lambda_t$. Under these assumptions, realized variance in this model given by

$$RV_{t+1} = \eta^2\sigma_c^2 + \kappa_1^2 z_1^2 \sigma_\lambda^2 \lambda_t - F\eta J_{t+1}, \tag{37}$$

where the first two summands correspond the variances of the diffusive shocks $\epsilon_{c,t+1}$ and $\epsilon_{\lambda,t+1}$ and the last summand corresponds to the realized variance from the consumption disaster.

Equation 37 offers a first insight into the pricing of variance risk in the model by Wachter (2013). The first summand of equation (35) is constant over time; however, the second summand scales with the level of the intensity of the consumption disaster. This results from the fact that the disaster intensity follows a square-root process, which indicates that future variance of the disaster intensity scales with the current level of the disaster intensity. Therefore, even in the absence of consumption disasters, stock market variance is time-varying in this model. This result is different from the model by Gabaix (2012) in which the variance of the stock market only varies if a disaster hits the economy. The third summand of equation (37) corresponds to the stock market variance that follows from the disaster process.

Variance swap rates are computed as the risk-neutral expectation of the sum of realized variances of equation (37) of period $t+1$ until $t+T$, as follows:

$$VS_t^{(T)} = \sum_{t=1}^{T} \mathbb{E}_t^{\mathbb{Q}}\big(RV_{t+i}\big) = T \cdot v_0 + v_1 \sum_{i=1}^{T} \mathbb{E}^{\mathbb{Q}}(\lambda_{t+i-1}), \tag{38}$$

where $v_0 = \eta^2\sigma_c^2$ and $v_1 = \kappa_1^2 z_1^2 \sigma_\lambda^2 - F\eta \exp\big(-\alpha\mu_d + \frac{1}{2}\alpha\sigma_d^2\big)(\mu_d - \alpha\sigma_d^2)$. Due to the Epstein-Zin preferences of the agent, the risk-neutral dynamics of the disaster intensity are different from the real-world dynamics in the sense that states with low lifetime utility, which correspond to states with high disaster intensity, receive a larger risk-neutral probability. This yields the agent a premium for instruments that offer protection against states in which disaster intensity is high, and this feature is not present in a model with CRRA preferences.

The calibration of the model is given in Table IA9.

Table IA9: This table shows the calibration of the model by Wachter (2013).

Parameter	Value	Parameter	Value
μ_c	$0.0252/12$	σ_c	$0.02/\sqrt{12}$
μ_d	-0.15	σ_d	0.10
μ_λ	$0.0355/12$	σ_λ	$0.067/12$
ϕ	$\exp(-0.08/12)$	β	$\exp(-0.012/12)$
η	2.6	γ	$4.9 = 1-\alpha$

Note that in the calibration of Dew-Becker et al. (2017) the risk-aversion is raised to 4.9 in order to match the Sharpe ratio on one-month variance swaps as closely as possible.

In the following, I present more details of the results from the simulation study for the model by Wachter (2013). First, I present sample statistics of realized variance and risk premia in the model. The results of this simulation study are represented in the following table.

Table IA10: This table presents sample statistics of the realized variance and risk premia in the model by Wachter (2013). The mean, standard deviation, and Sharpe ratio of the 18-, 12-, 6-, 3-, and 1-month simple risk premia are presented. The second column consists of the empirical result, and the third, fourth, and fifth columns represent the 5%, 50%, and 95% quantile of the simulation study, respectively.

Statistic	Data	Model		
	Est.	5%	50%	95%
		Realized variance		
$\mathbb{E}(RV)$	0.162	0.094	0.123	0.168
$\sigma(RV)$	0.095	0.028	0.048	0.073
		risk premia		
$\mathbb{E}(r^{(18)})$	-0.006	-0.015	-0.010	-0.005
$\sigma(r^{(18)})$	0.187	0.080	0.101	0.179
$SR(r^{(18)})$	-0.106	-0.556	-0.357	-0.097
$\mathbb{E}(r^{(12)})$	-0.013	-0.019	-0.014	-0.005
$\sigma(r^{(12)})$	0.227	0.084	0.118	0.256
$SR(r^{(12)})$	-0.202	-0.721	-0.412	-0.075
$\mathbb{E}(r^{(6)})$	-0.050	-0.032	-0.024	-0.007
$\sigma(r^{(6)})$	0.316	0.086	0.183	0.502
$SR(r^{(6)})$	-0.544	-1.284	-0.464	-0.048
$\mathbb{E}(r^{(3)})$	-0.098	-0.060	-0.045	-0.009
$\sigma(r^{(3)})$	0.447	0.074	0.338	1.008
$SR(r^{(3)})$	-0.756	-2.674	-0.459	-0.031
$\mathbb{E}(r^{(1)})$	-0.285	-0.173	-0.127	-0.018
$\sigma(r^{(1)})$	0.676	0.045	0.993	3.058
$SR(r^{(1)})$	-1.458	-12.294	-0.442	-0.019

Table IA10 confirms the finding of Figure 3 that the model by Wachter (2013) is not able to capture the strongly increasing term structure of expected variance swap returns documented in the data. Moreover, Table IA10 shows that also in the model by Wachter (2013) the volatility of variance swap returns varies a lot across simulation sets, and this results from the fact that the probability of a disaster is, on average, small (3.55% p.a.). If no disasters occur in a simulation set, the volatility of variance swap returns is very low. Finally, I conclude from Table IA7 that the model by Wachter (2013) does a better job than the model by Gabaix (2012) of capturing the empirical dynamics of stock market volatility.

In the following, I decompose variance swap rates in the model by Wachter (2013) for each simulation set separately. Table IA11 presents the results.

Table IA11: This table presents the results of the simple variance decomposition of variance swap rates in the data and in the model by Wachter (2013). The results of the data are from Table 1, with standard errors in parentheses. The regression coefficients of the model are estimated for each simulation set, and the mean of the regression coefficients is represented in the table with the standard deviation in parentheses.

Maturity	Data		Model	
	b_{rv}	b_{vdr}	b_{rv}	b_{vdr}
18	0.245	-0.728	0.973	-0.037
	(0.185)	(0.188)	(0.040)	(0.040)
12	0.558	-0.419	0.963	-0.033
	(0.151)	(0.158)	(0.029)	(0.032)
6	0.833	-0.168	0.954	-0.033
	(0.119)	(0.122)	(0.021)	(0.024)
3	0.957	-0.040	0.950	-0.039
	(0.083)	(0.086)	(0.019)	(0.020)
1	1.101	0.101	0.948	-0.052
	(0.056)	(0.056)	(0.018)	(0.018)

Table IA11 confirms the finding of Figure 1 that variance swap rates are driven by variance expectations in the model by Wachter (2013). Moreover, this result is very stable across the simulation sets, as indicated by the low standard deviation of b_{rv}. Therefore, this is strong evidence that the model is not in line with the data because my analysis shows that long-term variance swaps are mostly driven by risk premia.

In the following subsection, I discuss long-run risk model by Drechsler and Yaron (2011).

Internet Appendix D.3

Long-run risk

In this subsection, I discuss the long-run risk model by Drechsler and Yaron (2011). This model is a generalization of the long-run risk model by Bansal and Yaron (2004) in order to incorporate stylized facts regarding the variance risk premium. The model is generalized in the sense that the long-run mean consumption growth and the stochastic volatility incorporate jump shocks. Moreover, the long-run mean of the stochastic volatility process varies over time. The agent in the model has Epstein-Zin preferences, as is standard in long-run risk models. An important difference between the long-run risk model and the previously discussed consumption disaster models is that there are no consumption disasters in the long-run risk model. However, the state variables, which govern the future consumption growth rate and future consumption volatility, are exposed to jump risk.

Drechsler and Yaron (2011) specify the state vector of the economy as a VAR with Gaussian and jump shocks, as follows:

$$Y_{t+1} = \begin{pmatrix} \Delta c_{t+1} \\ x_{t+1} \\ \bar{\sigma}^2_{t+1} \\ \sigma^2_{t+1} \\ \Delta d_{t+1} \end{pmatrix} = \mu + FY_t + G_t z_{t+1} + J_{t+1}, \tag{39}$$

where, μ is a vector with the means of each state variable, F is specified as follows:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \rho_x & 0 & 0 & 0 \\ 0 & 0 & \rho_{\bar{\sigma}} & 0 & 0 \\ 0 & 0 & (1-\tilde{\rho}_\sigma) & \rho_\sigma & 0 \\ 0 & \phi & 0 & 0 & 0 \end{pmatrix}, \tag{40}$$

$G_t G_t'$ is the variance-covariance matrix, $z_{t+1} \sim N(0, I)$ is a vector of Gaussian shocks, and J_{t+1} is a vector of jump shocks. Jumps are compound-Poisson as in equation (33) with intensity λ_t, which can vary over time, similar to the model by Wachter (2013). Drechsler and Yaron (2011) consider a specification with jumps in x_t and σ_t^2, where $J_{x,t}$ is compound normal distributed and $J_{\sigma,t}$ is compound gamma distributed.

The first and last element of Y_t are the consumption and dividend growth, respectively. These processes have a time-varying mean, which is driven by the persistent process x_t, the second element of Y_t. The third element of Y_t is the long-run mean $\bar{\sigma}_t^2$ of the stochastic volatility process σ_t^2, the fourth element of Y_t.

The variance-covariance matrix, $G_t G_t'$, which governs the stochastic volatility of the model, and the jump intensity, λ_t, are affine in the state variable σ_t^2:

$$G_t G_t' = h + H_\sigma \sigma_t^2 \quad \text{and}$$
$$\lambda_t = l_0 + l_1 \sigma_t^2,$$

and, therefore, all variation in either the jump intensity or stochastic volatility is driven by σ_t^2.

The representative agent in the model has Epstein-Zin utility for which the stochastic discount factor is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, δ is the utility discount rate, γ is the risk aversion, ψ is the EIS, and $r_{c,t+1}$, the return on wealth. Drechsler and Yaron (2011) solve a log-linear version of the model and use $pd_{t+1} \approx A_{0,m} + A'_m Y_{t+1}$, which says that the log price-dividend ratio is linear in the state variables. Under these conditions is the log-linearized market return, written as follows:

$$r_{m,t+1} = r_0 + \left(B'_r F - A'_m \right) + B'_r G_t z_{t+1} + B'_r J_{t+1}, \tag{41}$$

where $A_{0,m}$, A'_m, r_0 and B'_r are given in equations (8) and (9) of Drechsler and Yaron (2011). Realized variance during period $t+1$ is equal to

$$RV_{t+1} = B'_r h B_r + B'_r H_\sigma \sigma_t^2 B_r + B'_r J_{t+1} J'_{t+1} B_r. \tag{42}$$

The assumption underlying this realized variance equation is that the Gaussian shocks z_{t+1} occur diffusively during period $t+1$, while jumps happen on a single day.

Equation (42) offers a first insight into the pricing of variance risk in the model by Drechsler and Yaron (2011). The first summand corresponds to the constant variance coming from the Gaussian shocks in the model. The second summand corresponds to the stochastic variance coming from the Gaussian shocks for which the variance is governed by the state variable σ_t^2. Finally, the third summand corresponds to the realized variance coming from the jump realizations in the state variables x_t and σ_t^2. Similar to the model by Wachter (2013) is time-variation in the realized variance coming from stochastic variance of Gaussian shocks and from the jump shocks.

The variance swap rate at time t with maturity T is computed as the risk-neutral expectation of the realized variance of equation (42), as follows:

$$VS_t^{(T)} = \sum_{t=1}^{T} \mathbb{E}_t^{\mathbb{Q}}\big(RV_{t+i}\big) = T \cdot v_0 + v_1 \sum_{t=1}^{T} \mathbb{E}_t^{\mathbb{Q}}\big(\sigma_{t+i-1}^2\big),$$

where,

$$v_0 = B_r' h B_r \text{ and } v_1 = B_r' H_\sigma B_r + l_1 \cdot B_r' \Psi^{\mathbb{Q}} B_r.$$

In the last equation, $\Psi^{\mathbb{Q}}$ is a matrix that has the risk-neutral variance of the disaster realization on the diagonal and corresponds to equation (21) of Drechsler and Yaron (2011). In order to derive these equations, I used $l_{0,x} = l_{0,\sigma} = 0$ and $l_{1,x} = l_{1,\sigma}$ from the calibration of Drechsler and Yaron (2011). The full calibration of the model is from Drechsler and Yaron (2011) and presented in Table 5 of their paper, and I use the calibration in which jump shocks in the x_t process follow a compound-Poisson in combination with a normal distribution.

In the following, I present more details of the results from the simulation study for the model by Drechsler and Yaron (2011).[IA1] First, I present sample statistics of realized variance and risk premia in the model. The results of this simulation study are represented in the following table.

[IA1] I thank Friedrich Lorenz for sharing the codes to solve the model.

Table IA12: This table presents sample statistics of the realized variance and risk premia in the model by Drechsler and Yaron (2011). The mean, standard deviation, and Sharpe ratio of the 18-, 12-, 6-, 3-, and 1-month simple variance swap returns are presented. The second column consists of the empirical result, and the third, fourth, and fifth columns represent the 5%, 50%, and 95% quantile of the simulation study, respectively.

Statistic	Data	Model		
	Est.	5%	50%	95%
	Realized variance			
$\mathbb{E}(RV)$	0.162	0.157	0.169	0.187
$\sigma(RV)$	0.095	0.051	0.087	0.134
	risk premia			
$\mathbb{E}(r^{(18)})$	-0.006	-0.036	-0.026	-0.014
$\sigma(r^{(18)})$	0.187	0.191	0.276	0.387
$SR(r^{(18)})$	-0.106	-0.654	-0.333	-0.128
$\mathbb{E}(r^{(12)})$	-0.013	-0.045	-0.032	-0.015
$\sigma(r^{(12)})$	0.227	0.232	0.343	0.488
$SR(r^{(12)})$	-0.202	-0.659	-0.326	-0.109
$\mathbb{E}(r^{(6)})$	-0.050	-0.063	-0.043	-0.017
$\sigma(r^{(6)})$	0.316	0.304	0.477	0.734
$SR(r^{(6)})$	-0.544	-0.686	-0.314	-0.084
$\mathbb{E}(r^{(3)})$	-0.098	-0.089	-0.060	-0.020
$\sigma(r^{(3)})$	0.447	0.394	0.671	1.144
$SR(r^{(3)})$	-0.756	-0.736	-0.309	-0.064
$\mathbb{E}(r^{(1)})$	-0.285	-0.176	-0.116	-0.027
$\sigma(r^{(1)})$	0.676	0.708	1.352	2.697
$SR(r^{(1)})$	-1.458	-0.820	-0.292	-0.036

Table IA12 confirms the finding of Figure 3 that the model by Drechsler and Yaron (2011) is not able to capture the strongly increasing term structure of expected variance swap returns documented in the data. Moreover, it shows that the model predicts, for each maturity, a volatility of variance swap returns, which is larger than observed empirically. Finally, I conclude from Table IA7 that the model by Drechsler and Yaron (2011) does a good job of capturing the empirical dynamics of stock market volatility.

In the following, I decompose variance swap rates in the model by Drechsler and Yaron (2011) for each simulation set separately. Table IA13 presents the results.

Table IA13: This table presents the results of the simple variance decomposition of variance swap rates in the data and in the model by Drechsler and Yaron (2011). The results of the data are from Table 1, with standard errors in parentheses. The regression coefficients of the model are estimated for each simulation set and the mean of the regression coefficients are represented in the table with the standard deviation in parentheses.

Maturity	Data		Model	
	b_{rv}	b_{vdr}	b_{rv}	b_{vdr}
18	0.245	-0.728	0.349	-0.560
	(0.185)	(0.188)	(0.105)	(0.108)
12	0.558	-0.419	0.412	-0.502
	(0.151)	(0.158)	(0.105)	(0.107)
6	0.833	-0.168	0.506	-0.418
	(0.119)	(0.122)	(0.097)	(0.098)
3	0.957	-0.040	0.567	-0.379
	(0.083)	(0.086)	(0.087)	(0.086)
1	1.101	0.101	0.615	-0.385
	(0.056)	(0.056)	(0.075)	(0.075)

Table IA13 confirms the finding of Figure 1 that short-term variance swap rates are driven by variance expectations and long-term variance swap rates by risk premia. Moreover, this result is stable across the simulation sets, as indicated by the low standard deviations of b_{rv} and b_{vdr}. Therefore, this is strong evidence that the model predicts a variation in short-term risk premia, which is substantially larger than observed empirically.