

Jumps, Leverage and Risk Premiums*

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Abstract

Jumps in asset prices are ubiquitous, yet the apparent high price of jump risk observed empirically is widely viewed as puzzling. Importantly however, in addition to direct price risks, jumps may also trigger simultaneous changes in other distributional features of asset returns. We develop model-free short-time expansions for two alternative risk-neutral variance measures, allowing us to more clearly delineate the role played by jumps in generating both price and variance risks and the pricing of “jump leverage” in particular. Relying on high-frequency S&P 500 index and options data, we find that the jump leverage effect commands a sizeable risk premium.

Keywords: jumps, options, tail risk, leverage effect, risk premiums, VIX index.

JEL classification: C51, C52, G12.

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1 Introduction

There is ample empirical evidence for the existence of jumps, or discontinuities, in the price of the market portfolio. There is also an extensive literature suggesting that the risk associated with market price jumps is priced differently from the risk associated with “smooth,” or continuous, price moves.¹ Jumps also feature prominently in many equilibrium-based models seeking to rationalize the joint behavior of stock and option markets.² These existing empirical analyses and theoretical models notwithstanding, the apparent “high” price of market jump risk is widely regarded as puzzling.

To more concretely demonstrate this apparent puzzle, consider the tail shape parameters of the actual and risk-neutral jump distributions that determine the rate at which the tails of the respective distributions decay to zero. Extreme Value Theory (EVT) dictates that this decay may be conveniently approximated by the tails of a generalized Pareto distribution. Relying on the S&P 500 high-frequency returns analyzed below, together with the nonparametric techniques of Bollerslev and Todorov (2011a), the resulting estimate for the observed left jump tail shape parameter equals 235. By comparison, the estimate for the risk-neutral left jump tail shape parameter based on the S&P 500 options data analyzed below, together with the methods of Bollerslev and Todorov (2014) and Bollerslev et al. (2015), only equals 20. This large gap in the rate at which the two jump tails decay is difficult to rationalize with most existing economic models. As a case in point, for the time-varying disaster risk equilibrium models of Wachter (2013) and Seo and Wachter (2019), which are explicitly designed to account for the rare occurrence of market price jumps, this difference in the tail shape parameters would necessitate a coefficient of risk-aversion for the representative agent in excess of two hundred.³

Set against this background we develop new model-free short-time expansions for alternative risk-neutral variance measures that allow us to more clearly delineate the nuanced

¹See e.g., Bates (2000, 2019), Pan (2002), Eraker et al. (2003), Eraker (2004), Broadie et al. (2007), Santa-Clara and Yan (2010), Bollerslev and Todorov (2011b), Bollerslev et al. (2015), Andersen et al. (2015), Aït-Sahalia et al. (2020), Aït-Sahalia et al. (2021) and Aleti (2022), among others.

²See e.g., Eraker and Shaliastovich (2008), Du (2011), Drechsler and Yaron (2011), Wachter (2013), Dew-Becker et al. (2017), Martin (2017), Seo and Wachter (2019), Schreindorfer (2020), Dew-Becker et al. (2021) and Eraker and Yang (2022), among others.

³This also echoes longstanding empirical evidence for the “expensiveness” of deep out-of-the-money put options; see, e.g., Aït-Sahalia et al. (2001) and Bondarenko (2014b), among others.

effects of jumps and the pricing thereof. Our resulting risk premium estimates for the S&P 500 market index indicate a much more important role for “jump leverage effects,” or simultaneous jumps in the price and the stochastic volatility and/or jump intensity of the market, than hitherto suggested in the literature. Correspondingly, the compensation for jump leverage risk also accounts for a sizeable portion of the much-studied market variance risk premium.⁴ In short, the puzzling “high” price of market jump risk may be explained by the fact that this risk is typically accompanied by other risks that are similarly disliked by investors.

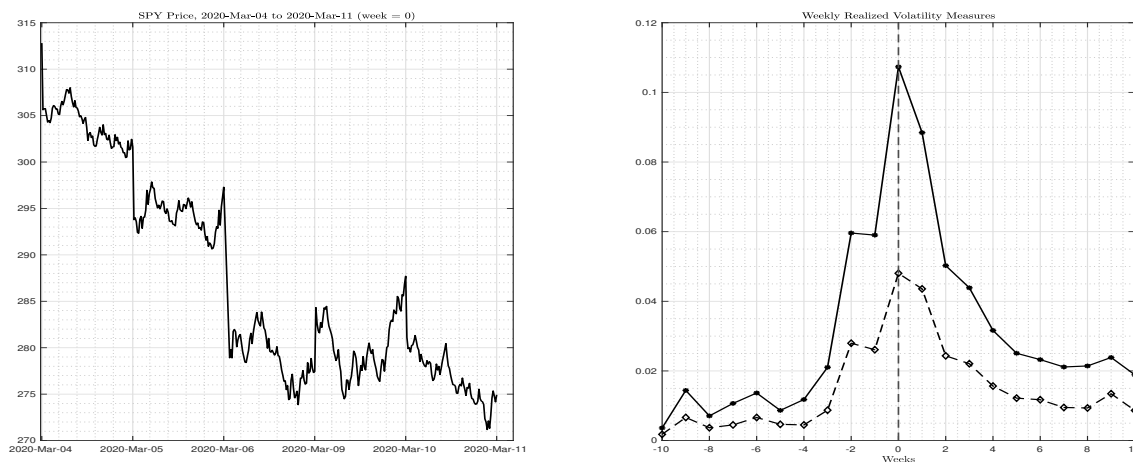


Figure 1: The left panel shows the SPY at a 5-minute intraday frequency for the week of March 4 - 11, 2020. The right plot shows the total weekly realized variation based on the summation of the squared 5-minute returns (solid line) and the realized jump variation (dashed line) constructed as the difference between the total variation and the bipower variation. The week of March 4 - 11, 2020 corresponds to week 0. Both of the variation measures are reported in annualized volatility units.

To help intuit the key empirical features underlying these findings, the left panel in Figure 1 shows the intraday prices for the S&P 500 market index at a 5-minute frequency for the week of March 4 - 11, 2020. Due to heightened fears about the global pandemic, the index dropped by more than 10% for that week as a whole. Meanwhile, even though

⁴Hu et al. (2022) have also recently documented a strong positive empirical relation between the variance risk premium and the conditional covariance between the market return and its variance, or the “leverage effect.” The discrete-time stochastic volatility model in Cheng et al. (2019) also relies on a related identification scheme.

the single large overnight decline on March 6 clearly stands out, there were also *many* other smaller intraday price jumps during that week. Moreover, most of these smaller price jumps appeared to be followed by heightened volatility. Corroborating that idea, the right panel in Figure 1 shows that while the realized jump risk for the week as a whole (indicated by the dashed line at week 0) did indeed increase compared to the week before (indicated by the dashed line at week -1), so did the total realized volatility inclusive of the diffusive price risk (indicated by the solid line).⁵ In other words, negative jumps in the market index tend to induce positive jumps in the aggregate market volatility and the intensity of future jump arrivals.⁶

A number of studies have previously highlighted the existence of such a jump leverage effect, or the tendency for negative market price jumps to trigger positive jumps in market volatility and/or market jump intensity.⁷ However, the pricing implications of this effect and its role in explaining the seemingly “high” price of market jump risk has not been thoroughly studied in the literature. We seek to fill this void and precise the impact of the jump leverage effect for explaining market jump risk premiums.

Our approach is decidedly non-parametric, relying on the rich information in short-dated options with different tenors for extracting information about the pricing of the jump leverage effect.⁸ Intuitively, while asymmetry in the jump distribution and the jump leverage effect will both contribute to asymmetry in the distributions of returns over very short horizons, it is possible to separate the two effects by also looking at slightly longer horizons and the term structure of skewness. In particular, while the asymmetry of jump risk should manifest

⁵The total weekly realized variation measures are based on the summation of the squared 5-minute returns over the week. The realized jump variation measures are constructed by subtracting the corresponding jump-robust bipower variation of Barndorff-Nielsen and Shephard (2004).

⁶These observations are also broadly consistent with the high-frequency-based parametric model estimates in Bates (2019), and the finding that large daily market moves typically represent the accumulation of a series of self-exciting intradaily volatility-price cojumps; see also the more recent estimates in Ewald and Zou (2021).

⁷See e.g., Jacod and Todorov (2010), Todorov and Tauchen (2011), Bandi and Renò (2012), Bandi and Renò (2016), Ait-Sahalia et al. (2017), Jacod et al. (2017), among others. There is also a large existing literature concerned with the nonparametric estimation of leverage effects more generally without separately considering the impact of jumps; see e.g., Bollerslev et al. (2006), Ait-Sahalia et al. (2013), Wang and Mykland (2014), Andersen et al. (2015) and Kalnina and Xiu (2017), among others.

⁸Jackwerth and Vilkov (2019) have also recently proposed an estimator for the risk-neutral leverage effect based on a parametric copula model and options on the S&P 500 and VIX index. By contrast, our method is fully nonparametric and seeks to estimate the instantaneous jump leverage effect.

the same across different horizons, the leverage effect should feature more prominently over longer return horizons.

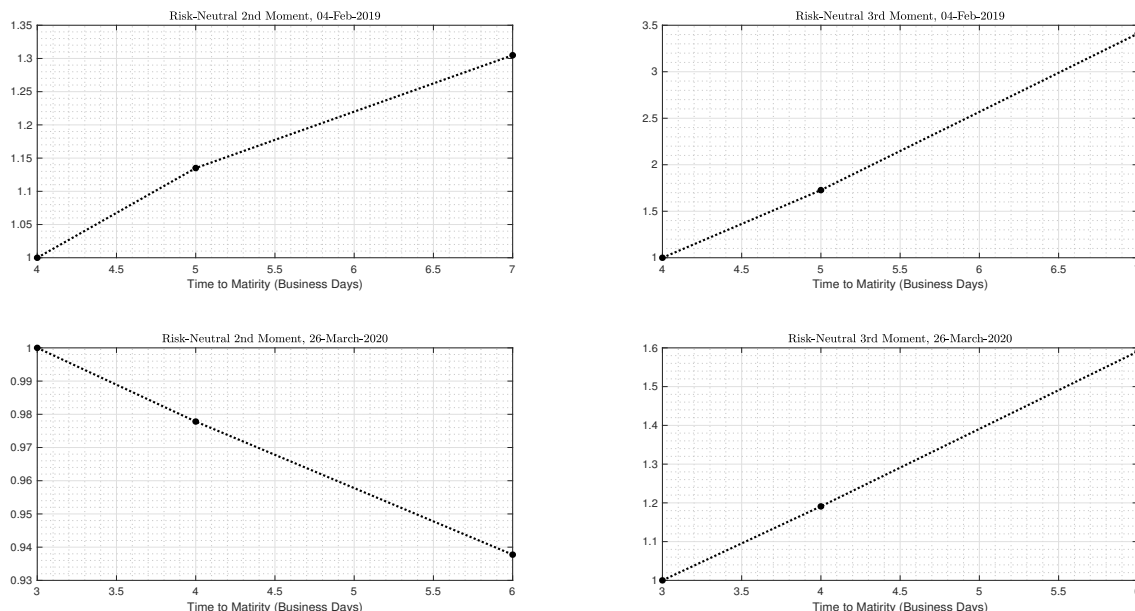


Figure 2: The left two panels show the second risk neutral moments calculated from short-dated S&P 500 options for different horizons on February 4, 2019 (top) and March 26, 2020 (bottom). The right two panels show the third risk neutral moments on the same two days. For easy of comparisons, the moments at the shortest horizons are normalized to unity.

To illustrate, Figure 2 plots the (normalized to unity at the shortest horizon) term structures of the risk-neutral variance and risk-neutral third moment calculated from short-dated S&P 500 options on two select days: February 4, 2019, a day with relatively low volatility, and March 26, 2020, a day with relatively high volatility.⁹ Since volatility is well-known to be mean reverting, the risk-neutral variance term structure is naturally upward sloping on the low volatility day and downward sloping on the high volatility day. By contrast, the term structure for the risk-neutral third moment is strongly upward sloping on *both* days. Looking across other high and low volatility days reveal the same general pattern: the risk-neutral third moment term structure is almost always upward sloping. This finding is difficult to reconcile with mean reversion in state variables, including the volatility and/or jump inten-

⁹The estimates for the variance and the third moment shown in the figure are based on the expressions for $\widehat{V}_{t,T}^Q$ and $3(\widehat{V}_{t,T}^Q - \widehat{V}_{t,T}^Q)$ formally defined in equations (21) and (22) below.

sity, as one would expect these effects to manifest in opposite directions on high and low volatility days. Instead, the systematic upward sloping term structure for the risk-neutral third moment points to the significant pricing of the jump leverage effect, with the steepness of the slopes further underscoring the seemingly large magnitude of the premium.¹⁰

In order to more formally assess this pricing, we begin our analysis by nonparametrically identifying the risk-neutral expectation of the jump leverage effect. We do so by developing new short-term expansions for two alternative risk-neutral variance measures: (i) the risk-neutral conditional expectation of the log-price, mirroring the measure used by the Chicago Board Options Exchange (CBOE) in their computation of the VIX index, and (ii) the risk-neutral conditional log-return variance. While both of our expansions are linear functions of the return time horizon and its square, with the leading term in both being equal to the risk-neutral spot variance, the slopes of the two expansions differ. Moreover, as formally demonstrated below, this difference is almost exclusively attributable to the jump leverage effect, allowing for its nonparametric estimation from actively traded short-dated S&P 500 index options. Comparing the resulting risk-neutral estimates with their realized counterparts estimated from high-frequency intraday S&P 500 index returns, in turn reveal highly significant compensation for short-term leverage risk, with more than half of the risk premium in third return moments at a horizon of two weeks attributable to jump leverage.

From a theoretical perspective, this significant pricing of the jump leverage effect is naturally associated with instantaneous changes in the investment opportunity set induced by jumps in the volatility and/or jump intensity. Accordingly, shocks to the stochastic volatility and/or jump intensity should be priced. Consistent with this thesis, we find that at a horizon of two weeks, close to one-fifth of the variance risk premium may be directly attributed to compensation for time variation in the stochastic volatility and/or jump intensity, as opposed to compensation for instantaneous variance risk. Further corroborating the non-trivial pricing of the jump leverage effect, we also find that the term structure of the variance risk premium substantially flattens for horizons beyond two weeks.¹¹

In addition to our main empirical findings based on the pricing of short-dated S&P 500 index options, we also provide additional supportive empirical evidence based on the pricing

¹⁰Relatedly, Kozhan et al. (2013) and Schneider et al. (2020) have previously emphasized the nontrivial pricing of skewness and coskewness risk embedded in options.

¹¹This flattening of the term structure over longer horizons is also in line with the empirical evidence of Dew-Becker et al. (2017).

of VIX index options. In particular, while the realized jump risk for the VIX index appears close to symmetric, we find that its risk-neutral counterpart implied from VIX index options and our new short-term expansions for the option-implied variance risk measures is highly right skewed, as would be the case if jump leverage risk is significantly priced.

The rest of the paper is organized as follows. We begin in Section 2 by formally developing the short-time expansions for the risk-neutral variance measures that underly our analyses. Sections 3 and 4, respectively, defines the different jump and leverage risk premium measures and discusses their practical implementation based on short-dated S&P 500 options and high-frequency intraday S&P 500 returns. Section 5 presents our main new empirical findings pertaining to the pricing of jump leverage risks. Section 6 provides additional supportive empirical evidence based on the pricing of VIX index options. We conclude with a brief informal discussion of various equilibrium-based models that can possibly help explain the nontrivial pricing of the jump leverage effect. All of the proofs, along with Monte Carlo simulation evidence underscoring the accuracy of the new variance expansions, are deferred to an Appendix.

2 Short-Time Expansions of Risk-Neutral Variance Measures

We will consider two alternative risk-neutral variance measures and corresponding expansions. The difference between the two expansions in turn allow us to identify and nonparametrically estimate the risk-neutral jump leverage effect from short-dated options.

Let X_t denote the underlying price process for the market, with the corresponding log-price process denoted x_t . We will assume that X_t is defined on the probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$, and that the price obeys the following general Itô semimartingale dynamics

$$\frac{dX_t}{X_{t-}} = \alpha_t dt + \sigma_t dW_t + \int_{\mathbb{R}} (e^z - 1)(\mu - \nu^{\mathbb{P}})(dt, dz), \quad (1)$$

where W_t is a Brownian motion, μ is an integer-valued jump measure counting the jumps in X with compensator $\nu^{\mathbb{P}}$, α_t denotes the drift capturing the instantaneous expected return, and σ_t refers to the instantaneous stochastic volatility. Under standard conditions, no-arbitrage implies the existence of a risk-neutral measure under which the cum-dividend discounted

price process is a local martingale. Since we will be focussing on short return horizons, for notational simplicity we will set the risk-free rate and the dividend yield identically equal to zero. Accordingly, the dynamics of X_t under the risk-neutral probability measure \mathbb{Q} may be expressed as

$$\frac{dX_t}{X_{t-}} = \sigma_t dW_t^{\mathbb{Q}} + \int_{\mathbb{R}} (e^z - 1)(\mu - \nu^{\mathbb{Q}})(dt, dz), \quad (2)$$

where $W_t^{\mathbb{Q}}$ is a \mathbb{Q} -Brownian motion and $\nu^{\mathbb{Q}}$ denotes the jump compensator under \mathbb{Q} .

Our first risk-neutral variance measure is simply given by the conditional variance of the log-return over some time horizon T under the above defined \mathbb{Q} measure. That is:

$$V_{t,t+T}^{\mathbb{Q}} \equiv \frac{1}{T} \text{Var}_t^{\mathbb{Q}}(x_{t+T} - x_t). \quad (3)$$

Our second variance measure is based on the conditional risk-neutral mean of the log-return over the horizon T ,

$$\mathcal{V}_{t,t+T}^{\mathbb{Q}} \equiv -\frac{2}{T} \mathbb{E}_t^{\mathbb{Q}}(x_{t+T} - x_t) = \frac{1}{T} \mathbb{E}_t^{\mathbb{Q}} \left(\int_t^{t+T} \sigma_s^2 ds + 2 \int_t^{t+T} \int_{\mathbb{R}} (e^z - 1 - z) \nu^{\mathbb{Q}}(ds, dz) \right), \quad (4)$$

where the second equality follows by an application of Itô's lemma. This second variance measure is also the measure used by the CBOE in the construction of the VIX volatility index.¹²

Letting the return horizon approach zero, the instantaneous counterparts of the above two variance measures are naturally defined by:

$$V_t^{\mathbb{Q}} \equiv \lim_{\Delta \rightarrow 0} V_{t,t+\Delta}^{\mathbb{Q}}, \quad \mathcal{V}_t^{\mathbb{Q}} \equiv \lim_{\Delta \rightarrow 0} \mathcal{V}_{t,t+\Delta}^{\mathbb{Q}}. \quad (5)$$

These instantaneous measures may also alternatively be expressed in terms of the risk-neutral asset characteristics as

$$V_t^{\mathbb{Q}} = \sigma_t^2 + \int_{\mathbb{R}} z^2 \nu_t^{\mathbb{Q}}(dz), \quad \mathcal{V}_t^{\mathbb{Q}} = \sigma_t^2 + 2 \int_{\mathbb{R}} (e^z - 1 - z) \nu_t^{\mathbb{Q}}(dz), \quad (6)$$

¹²The connection between the $V_{t,t+T}^{\mathbb{Q}}$ and $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$ variance measures has also previously been used by Du and Kapadia (2012) in the construction of an options-based jump index, while Bondarenko (2014a) has previously analyzed the connection between the two measures in regards to variance trading and replication strategies.

where the risk-neutral jump compensator is decomposed as $\nu^{\mathbb{Q}}(dt, dz) = dt\nu_t^{\mathbb{Q}}(dz)$. As these expressions make clear, both of the instantaneous variance measures only depend on the \mathbb{Q} probability measure through their second jump components. As such, the difference between the two measures may be entirely explained by jumps,

$$V_t^{\mathbb{Q}} - \mathcal{V}_t^{\mathbb{Q}} = \int_{\mathbb{R}} z^2 \nu_t^{\mathbb{Q}}(dz) - 2 \int_{\mathbb{R}} (e^z - 1 - z) \nu_t^{\mathbb{Q}}(dz) \approx -\frac{1}{3} \int_{\mathbb{R}} z^3 \nu_t^{\mathbb{Q}}(dz), \quad (7)$$

that is the (scaled) \mathbb{Q} third moment of market price jumps.

Over ultra short return horizons, or $T \approx 0$, the $V_{t,t+T}^{\mathbb{Q}}$ and $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$ variance measures may naturally be used as proxies for the instantaneous variance measures $V_t^{\mathbb{Q}}$ and $\mathcal{V}_t^{\mathbb{Q}}$, respectively. However, for longer return horizons, or $T > 0$, mean reversion in volatility and jump intensity will both contribute nontrivially to $V_{t,t+T}^{\mathbb{Q}}$ and $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$, thus rendering the difference between the two measures more difficult to interpret. In order to formally account for this, it is useful to define the instantaneous drift term of any arbitrary process z evaluated under \mathbb{Q} as:

$$m_t^{\mathbb{Q}}(z) \equiv \lim_{\Delta \rightarrow 0} \frac{\mathbb{E}_t^{\mathbb{Q}}(z_{t+\Delta} - z_t)}{\Delta}. \quad (8)$$

Of course, if the process z is stationary, then $\mathbb{E}^{\mathbb{Q}}(m_t^{\mathbb{Q}}(z)) = 0$. Meanwhile, any risk premium for variation in z will generally imply that $\mathbb{E}^{\mathbb{P}}(m_t^{\mathbb{Q}}(z)) \neq 0$, with the sign being positive (negative) if the specific variation in z is disliked (liked) by investors.

Now, utilizing the above notation, it is possible to show that

$$\mathcal{V}_{t,t+T}^{\mathbb{Q}} = \mathcal{V}_t^{\mathbb{Q}} + \frac{T}{2} \times m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) + T^2 \times C_t + O_p(T^3), \quad T \downarrow 0, \quad (9)$$

where C_t denotes some \mathcal{F}_t -adapted random variable.¹³ Notably, the leading term in the expansion, which does not depend upon T , is given by the instantaneous risk-neutral variance $\mathcal{V}_t^{\mathbb{Q}}$, while the slope of the expansion, when viewed as a function of T , is solely determined by the mean reversion of $\mathcal{V}_t^{\mathbb{Q}}$. The random variable C_t that dictates the T^2 term depends in a complicated way on various features of the volatility and jump intensity dynamics. However, this term is numerically small, and we will treat it as a nuisance parameter in what follows.

In order to establish an analogous expansion for the $V_{t,t+T}^{\mathbb{Q}}$ variance measure, it is instruc-

¹³Formal proofs of this expansion and the expansion presented below are provided in Appendix A.

tive to first formally define the notion of an instantaneous leverage effect, that is a measure of the co-dependence between innovations, or shocks, to the market price and return variation. To this end, for arbitrary processes y and z , denote their quadratic covariation by:

$$[y, z]_t = \text{plim}_{n \rightarrow \infty} \sum_{i=1}^{\lfloor nt \rfloor} [(y_{i/n} - y_{(i-1)/n})(z_{i/n} - z_{(i-1)/n})]. \quad (10)$$

Furthermore, denote the predictable counterpart of $[y, z]_t$ under the \mathbb{Q} measure by $\langle y, z \rangle_t^{\mathbb{Q}}$,

$$[y, z]_t = \langle y, z \rangle_t^{\mathbb{Q}} + \mathbb{Q}\text{-martingale}. \quad (11)$$

In parallel to the instantaneous variance and drift measures defined in (5) and (8), respectively, the instantaneous risk-neutral leverage effect is then naturally defined by:

$$L_t^{\mathbb{Q}} \equiv \lim_{\Delta \rightarrow 0} \frac{\langle x, \mathcal{V}^{\mathbb{Q}} \rangle_{t+\Delta}^{\mathbb{Q}} - \langle x, \mathcal{V}^{\mathbb{Q}} \rangle_t^{\mathbb{Q}}}{\Delta}. \quad (12)$$

Using this definition of $L_t^{\mathbb{Q}}$, the following short-term expansion obtains

$$V_{t,t+T}^{\mathbb{Q}} = V_t^{\mathbb{Q}} + \frac{T}{2} \times (m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}}) - L_t^{\mathbb{Q}}) + T^2 \times C_t + O_p(T^3), \quad T \downarrow 0, \quad (13)$$

where again C_t denotes some numerically small \mathcal{F}_t -adapted random variable.

The expressions for $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$ and $V_{t,t+T}^{\mathbb{Q}}$ in (9) and (13) are obviously very similar except for one important distinction, namely the presence of the $L_t^{\mathbb{Q}}$ leverage term in the slope of the latter expansion. Since the leverage effect and the dependence between price and variance innovations is generally found to be negative, this term will therefore tend to increase the term structure slope of $V_{t,t+T}^{\mathbb{Q}}$ compared to the slope of $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$. In fact, since $m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}}) - m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ is likely to be small in absolute value, as it depends on the third moment of jumps, the term structure slope of $V_{t,t+T}^{\mathbb{Q}} - \mathcal{V}_{t,t+T}^{\mathbb{Q}}$ will be predominantly determined by the $-L_t^{\mathbb{Q}}$ leverage term. As such, this helps explain the seemingly puzzling empirical evidence discussed in the introduction, and the systematically upward sloping term structure of the third risk neutral moment irrespective of the level of the volatility. We will return to this in our discussion of the estimated jump leverage risk premium below.

3 Jump and Leverage Risk Premiums

The risk-neutral measures defined in connection with the expansions discussed in the previous section naturally suggest the definition of corresponding risk premiums to help illuminate the pricing of jump risk, and jump leverage risk in particular.¹⁴

To begin, the instantaneous variance risk premiums for the two different variance measures may obviously be defined by

$$IVRP_t \equiv V_t^{\mathbb{Q}} - V_t^{\mathbb{P}}, \quad \mathcal{IVR}\mathcal{P}_t \equiv \mathcal{V}_t^{\mathbb{Q}} - \mathcal{V}_t^{\mathbb{P}}, \quad (14)$$

where $V_t^{\mathbb{P}}$ and $\mathcal{V}_t^{\mathbb{P}}$ denote the \mathbb{P} counterparts to the $V_t^{\mathbb{Q}}$ and $\mathcal{V}_t^{\mathbb{Q}}$ instantaneous variances. Since the diffusive spot volatility σ_t^2 must be identical under the \mathbb{P} and \mathbb{Q} measures to prevent arbitrage opportunities, the $IVRP_t$ and $\mathcal{IVR}\mathcal{P}_t$ risk premiums are solely determined by the pricing of jumps. However, that is not the case for the corresponding variance risk premiums defined over non-trivial time intervals $T > 0$,

$$VRP_{t,t+T} \equiv V_{t,t+T}^{\mathbb{Q}} - V_{t,t+T}^{\mathbb{P}}, \quad \mathcal{VR}\mathcal{P}_{t,t+T} \equiv \mathcal{V}_{t,t+T}^{\mathbb{Q}} - \mathcal{V}_{t,t+T}^{\mathbb{P}}. \quad (15)$$

The second of these two measures, in particular, has been extensively studied empirically in the recent literature, typically with T set to one-month mimicking the horizon of the popular VIX index used in place of $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$.¹⁵

Meanwhile, to help more formally determine what drives the $VRP_{t,t+T}$ and $\mathcal{VR}\mathcal{P}_{t,t+T}$ measures over for short horizons T , it is instructive to employ the expansions developed in the previous section. Doing so, we obtain the following two decompositions:

$$VRP_{t,t+T} = IVRP_t + \frac{T}{2} \times (m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(V_t^{\mathbb{Q}})) - \frac{T}{2} \times (L_t^{\mathbb{Q}} - L_t^{\mathbb{P}}) + O_p(T^2), \quad (16)$$

and

$$\mathcal{VR}\mathcal{P}_{t,t+T} = \mathcal{IVR}\mathcal{P}_t + \frac{T}{2} \times (m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})) + O_p(T^2). \quad (17)$$

As previously noted, the leading component in $VRP_{t,t+T}$ (resp. $\mathcal{VR}\mathcal{P}_{t,t+T}$), that is the

¹⁴To help fix ideas, Appendix B further discusses how the different risk premiums defined below manifest in the context of the parametric double-jump stochastic volatility model of Duffie et al. (2000).

¹⁵See, e.g., Bollerslev et al. (2009), Drechsler and Yaron (2011) and Bekaert and Hoerova (2014), along with many subsequent studies.

instantaneous variance risk premium $IVRP_t$ (resp. \mathcal{IVRP}_t), is solely determined by asset price jumps. On the other hand, the second $m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(V_t^{\mathbb{Q}})$ (resp. $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$) term, which depends linearly on the horizon T , reflects compensation demanded by investors for any changes in the investment opportunity set. In the context of the underlying general Itô semimartingale in (1), and its risk-neutral counterpart in (2), that is temporal variation in σ_t^2 and/or changes in the intensity of the jumps.

Importantly, the decomposition for $VRP_{t,t+T}$ contains an additional linear-in- T component specifically due to the compensation for leverage risk. We will refer to this “extra” term as the instantaneous leverage risk premium in the sequel, or

$$ILLRP_t \equiv L_t^{\mathbb{Q}} - L_t^{\mathbb{P}}. \quad (18)$$

In parallel to the instantaneous variance risk premiums, which are solely determined by jumps and the pricing thereof, $ILLRP_t$ is solely determined by the jump leverage effect and the pricing thereof, that is the risk premium for co-jumps between the log-price x and $\mathcal{V}^{\mathbb{Q}}$. Empirically, we would expect $ILLRP_t$ to be negative and thus contribute positively to $VRP_{t,t+T}$.

We turn next to a discussion of our model-free estimation of these different risk premiums.

4 Feasible Measures

Our feasible counterparts to the risk premium quantities defined in the previous section rely on portfolios of short-dated options to estimate the \mathbb{Q} risk-neutral quantities and high-frequency returns to proxy the corresponding \mathbb{P} risk measures.

4.1 Option-Based Risk Measures

Let $O_{t,T}(K)$ denote the time t price of a European-style out-of-the-money option expiring at time $t+T$ with strike K . With the risk-free rate and the dividend yield both set to zero, the general results in Bakshi and Madan (2000) and Carr and Madan (2001) then provide

the following two option spanning results:

$$\mathbb{E}_t^{\mathbb{Q}}(x_{t+T} - x_t)^2 = 2 \int_0^\infty \left(1 - \log\left(\frac{K}{X_t}\right)\right) \frac{O_{t,T}(K)}{K^2} dK, \quad (19)$$

and

$$\mathbb{E}_t^{\mathbb{Q}}(x_{t+T} - x_t) = - \int_0^\infty \frac{O_{t,T}(K)}{K^2} dK. \quad (20)$$

In practice, of course, we do not observe options on a continuum of strikes. Instead let the discrete grid of $N_{t,T}$ option prices observed at time t be denoted by $K_1 < \dots < K_{N_{t,T}}$, where for simplicity we suppress the dependence of the strike grid on the pair (t, T) . The actually observed option prices $\widehat{O}_{t,T}(K_j)$ are also subject to pricing errors, say $\widehat{O}_{t,T}(K_j) = O_{t,T}(K_j) + \epsilon_{t,T}(j)$ for $j = 1, \dots, N_{t,T}$. Following standard practice in the option pricing literature, we will assume that the $\epsilon_{t,T}(j)$ observation errors exhibit only weak spatial and temporal dependencies, and hence are “averaged out” in the estimation.

Using the observed option prices and the spanning results in (19) and (20), we construct the following estimates for $V_{t,t+T}^{\mathbb{Q}}$ and $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$:

$$\widehat{V}_{t,t+T}^{\mathbb{Q}} = \frac{2}{T} \sum_{j=2}^{N_{t,T}} \left(1 - \log\left(\frac{K_{j-1}}{X_t}\right)\right) \frac{\widehat{O}_{t,T}(K_{j-1})}{K_{j-1}^2} (K_j - K_{j-1}) - \frac{T}{4} \left(\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}\right)^2, \quad (21)$$

$$\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}} = \frac{2}{T} \sum_{j=2}^{N_{t,T}} \frac{\widehat{O}_{t,T}(K_{j-1})}{K_{j-1}^2} (K_j - K_{j-1}). \quad (22)$$

Guided by the expansions for the risk-neutral variance measures in (9) and (13), we then run the following linear regressions at each point time using all of the availability tenors:

$$\widehat{V}_{t,t+T_j}^{\mathbb{Q}} = b_{t,0} + b_{t,1}T_j + b_{t,2}T_j^2 + \epsilon_{t,T_j}, \quad \widehat{\mathcal{V}}_{t,t+T_j}^{\mathbb{Q}} = \beta_{t,0} + \beta_{t,1}T_j + \beta_{t,2}T_j^2 + \varepsilon_{t,T_j}. \quad (23)$$

Denoting the resulting OLS estimates by $\widehat{b}_{t,i}$ and $\widehat{\beta}_{t,i}$, respectively, our risk-neutral variance estimates are then simply defined by:

$$\widehat{V}_t^{\mathbb{Q}} = \widehat{b}_{t,0}, \quad \widehat{\mathcal{V}}_t^{\mathbb{Q}} = \widehat{\beta}_{t,0}. \quad (24)$$

In lieu of (9) and (13), $L_t^{\mathbb{Q}}$ may seemingly be estimated by twice the difference in the

estimated slopes from the two regressions in (23). However, this estimator will be biased due to the presence of the risk-neutral instantaneous drift $m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}} - \mathcal{V}_t^{\mathbb{Q}})$ and the mean reversion in $\int_{\mathbb{R}}(z^2 - 2(e^z - 1 - z))\nu_t^{\mathbb{Q}}(dz)$. Even though this bias will typically be much smaller than $\int_{\mathbb{R}} z^2 \nu_t^{\mathbb{Q}}(dz)$, if the mean reversion in the jump intensity and the diffusive volatility manifest in the same direction, we can easily correct for the bias. In particular, utilizing

$$\widehat{s}_t = \frac{\sum_{i=1}^{k_n} |\widehat{V}_{t-i/n}^{\mathbb{Q}} - \widehat{V}_{t-(i+1)/n}^{\mathbb{Q}}|}{\sum_{i=1}^{k_n} |\widehat{\mathcal{V}}_{t-i/n}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t-(i+1)/n}^{\mathbb{Q}}|}, \quad (25)$$

based on a “small” window of k_n variance increments, we obtain the following simple bias-corrected leverage estimator:

$$\widehat{L}_t^{\mathbb{Q}} = -2 \left(\widehat{b}_{t,1} - \widehat{s}_t \widehat{\beta}_{t,1} \right). \quad (26)$$

The Monte Carlo simulation results reported in Appendix C underscore the accuracy of each of the $\widehat{V}_t^{\mathbb{Q}}$, $\widehat{\mathcal{V}}_t^{\mathbb{Q}}$ and $\widehat{L}_t^{\mathbb{Q}}$ risk-neutral risk estimators.

We turn next to a discussion of our corresponding return-based risk estimates.

4.2 Return-Based Risk Measures

We assume that market prices and options data are sampled n times during the unit time interval. Relying on a window consisting of k_n return and variance increments prior to time t , we define our realized variance and leverage estimators as

$$\widehat{RV}_t = \frac{n}{k_n} \sum_{i=1}^{k_n} (x_{t-i/n} - x_{t-(i+1)/n})^2, \quad (27)$$

and

$$\widehat{RL}_t = \frac{n}{k_n} \sum_{i=1}^{k_n} [(x_{t-i/n} - x_{t-(i+1)/n})(\widehat{V}_{t-i/n}^{\mathbb{Q}} - \widehat{V}_{t-(i+1)/n}^{\mathbb{Q}})], \quad (28)$$

respectively. Under standard conditions, \widehat{RV}_t and \widehat{RL}_t consistently, for $n \rightarrow \infty$, estimate $QV_{t-k_n/n,t}$ and $QL_{t-k_n/n,t}$, formally defined as the quadratic variation of x and the quadratic covariation of x and $\mathcal{V}^{\mathbb{Q}}$,

$$QV_{t-k_n/n,t} \equiv \frac{n}{k_n} ([x, x]_t - [x, x]_{t-k_n/n}), \quad (29)$$

and

$$QL_{t-k_n/n,t} \equiv \frac{n}{k_n}([x, \mathcal{V}^{\mathbb{Q}}]_t - [x, \mathcal{V}^{\mathbb{Q}}]_{t-k_n/n}). \quad (30)$$

Moreover, if $k_n \rightarrow \infty$ and $k_n/n \rightarrow 0$, the $QV_{t-k_n/n,t}$ and $QL_{t-k_n/n,t}$ estimands naturally converge to their spot counterparts.

4.3 Feasible Realized Risk Premium Estimates

Armed with the above risk-neutral and realized risk estimates, our feasible versions of $IVRP_t$ and $ILRP_t$, defined in (14) and (18), are simply obtained as

$$\widehat{RIVRP}_t \equiv \widehat{V}_t^{\mathbb{Q}} - \widehat{RV}_t, \quad (31)$$

and

$$\widehat{RILRP}_t \equiv \widehat{L}_t^{\mathbb{Q}} - \widehat{RL}_t, \quad (32)$$

respectively. These estimators of the instantaneous variance and leverage risk premiums purposely rely on the easy-to-calculate model-free realized \widehat{RV}_t and \widehat{RL}_t measures, rather than their conditional expectations, $\mathcal{V}_t^{\mathbb{P}}$ and $L_t^{\mathbb{P}}$. Practical estimation of the conditional expectations would necessitate additional modeling assumptions pertaining to the dynamics of \widehat{RV}_t and \widehat{RL}_t . Importantly, however, relying on the realized values instead of their conditional expectations, does not alter any of our main conclusions and average risk premium estimates. In particular, ignoring the (negligible) contribution stemming from estimation error, it readily follows that

$$\widehat{RIVRP}_t = IVRP_t + \mathbb{P}\text{-martingale}, \quad \widehat{RILRP}_t = ILRP_t + \mathbb{P}\text{-martingale},$$

where the \mathbb{P} -martingale terms reflect the differences between the respective realizations of the discrete-time stochastic processes and their conditional expectations. Although these \mathbb{P} -martingale terms are not necessarily “small” numerically, they are by definition mean-zero and unpredictable, and hence do not affect the expected values, nor the dynamic dependencies, of the estimated risk premiums.

5 Market Jump and Leverage Risk Premiums

We begin the discussion of our main empirical findings with an account of the data underlying our results, followed by a summary of the daily risk estimates, before finally considering the implications of our model-free estimates for the pricing of jumps and jump leverage risk.

5.1 Data

Our empirical analyses is based on high-frequency returns and option data for the S&P 500 index spanning the period 2007-2020. The options are European style and are traded on the CBOE. We obtained the option data from the CBOE Data Shop. We rely on high-frequency price records of the SPY exchange traded fund (ETF) designed to track the S&P 500 index. The SPY data is obtained from the TAQ database. We record the SPY and options prices at a 5-minute frequency during the trading day, starting at 9.35 EST and ending at 15.55 EST, resulting in a total of 77 price records per day.

We apply standard filters and cleaning procedures to the data. In so doing, we remove any days for which the number of SPY zero returns exceeds twenty percent of the total daily number of high-frequency returns. This mostly eliminates trading days around holidays. We also remove the four 15-minute periods on March 9, 12, 16 and 18, 2020, where trading was halted due to market-wide circuit breakers. We take the option mid-quotes as our option price observations. We remove any options with zero bids and options for which the ratio of the ask relative to the bid exceeds ten. We also remove date and maturity pairs for which the minimum of the ratio of the out-of-the-money option price relative to its strike exceeds five percent of the maximum of this ratio. We further remove date and maturity pairs for which the maximum strike gap around the money exceeds twenty. Finally, to avoid anomalous results associated with large event risks, we also exclude any options for which the horizons span the dates of the 2016 Brexit and 2020 U.S. Presidential elections.

For the actual estimation, at each point in time we use the first two available shortest maturity options with at least three business days to expiration. We then keep adding options with tenors up until twelve business days. If on a given day the number of available tenors is just two, or if the gap between the shortest and longest available tenor is less than six business days, we drop the squared tenor term in the regressions in (23) in order to avoid any issues with multicollinearity (this mostly happens in the early part of the sample). Also,

if at a given point in time, any of the estimated intercept terms in (23) are negative, we simply remove those observations from the analysis (again this rarely happens, and mostly so around holidays).

5.2 Risk Estimates

We begin by computing $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ and $\widehat{V}_{t,t+T}^{\mathbb{P}}$ at each observation time, for all available tenors, based on the expressions in (21) and (22), respectively. To account for the well-known intraday pattern in volatility, we compute time-to-maturity on a “business time scale,” with the length of the within day windows set to equalize the contribution to the intraday volatility. To allow for possible dynamic changes in the intraday pattern, we further calculate this on a rolling basis relying on the intraday and overnight realized volatilities over the past year. Using the regressions in (23), we then calculate the instantaneous variance estimates $\widehat{V}_t^{\mathbb{Q}}$ and $\widehat{V}_t^{\mathbb{P}}$ as defined in (24). We rely on (26) for our computation of $\widehat{L}_t^{\mathbb{Q}}$, with the \widehat{s}_t adjustment term in (25) calculated from the high-frequency return and option observations on the specific trading day. For each trading day, we then average all the high-frequency estimates and henceforth rely on these as our daily estimates.

Our realized risk measures \widehat{RV}_t and \widehat{RL}_t are similarly computed over windows of one trading day. To allow for comparison with the risk-neutral measures, we further normalize the high-frequency return measures using an overnight adjustment factor, based on the ratio of the intraday to the overnight realized volatilities computed on a one-year rolling basis. Table 1 reports the full-sample means for the resulting daily \mathbb{P} and \mathbb{Q} estimates, together with robust standard errors in parentheses.¹⁶ Since all the distributions are heavily right skewed, we also report the corresponding quantiles.

Looking first at the volatility estimates in the top-portion of the table, the $\widehat{V}_t^{\mathbb{Q}}$ spot estimates are naturally lower than the short and long maturity $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ estimates. Meanwhile, it is noteworthy that the relative difference between $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ and $\widehat{V}_t^{\mathbb{Q}}$ is the smallest for the highest quantile. This may be explained by the fact that mean reversion in volatility and the leverage effect impact the term structure of $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ in opposite directions, thus diminishing the relative difference between $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ and $\widehat{V}_t^{\mathbb{Q}}$ when volatility is high. Also, even though there is obviously a large gap between $\widehat{V}_t^{\mathbb{Q}}$ and \widehat{RV}_t , consistent with a positive $IVRP_t$ and the

¹⁶We purposely report the 1% truncated means here and throughout to avoid the full-sample values being unduly influenced by a few very large “crisis” observations.

Table 1: **Risk Measures**

Statistic	Mean	Quantiles		
		Q_{25}	Q_{50}	Q_{75}
$\widehat{V}_{t,t+T}^{\mathbb{Q}}$, short T	0.0395 (0.0074)	0.0130	0.0224	0.0429
$\widehat{V}_{t,t+T}^{\mathbb{Q}}$, long T	0.0418 (0.0078)	0.0151	0.0248	0.0478
$\widehat{V}_t^{\mathbb{Q}}$	0.0378 (0.0068)	0.0111	0.0201	0.0396
\widehat{RV}_t	0.0282 (0.0044)	0.0064	0.0127	0.0278
$3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$, short T	0.0028 (0.0007)	0.0005	0.0010	0.0021
$3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$, long T	0.0060 (0.0016)	0.0011	0.0019	0.0051
$3(\widehat{V}_t^{\mathbb{Q}} - \widehat{\mathcal{V}}_t^{\mathbb{Q}})$	0.0011 (0.0003)	-0.0001	0.0002	0.0008
$-\widehat{L}_t^{\mathbb{Q}}$	0.0579 (0.0106)	0.0089	0.0203	0.0464
$-\widehat{RL}_t$	0.0331 (0.0074)	0.0019	0.0056	0.0179

Note: The table reports truncated at 1% sample means, with Newey-West robust standard errors in parentheses, and quantiles for the different daily risk estimates defined in Sections 4.1 and 4.2. The estimates are constructed from short-dated S&P 500 index options and high-frequency SPY returns spanning 2007-2020.

widely documented significant variance risk premium over longer horizons, the gap between $\widehat{V}_t^{\mathbb{Q}}$ and $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ for short T is also fairly large, pointing to a sizable risk premium for shocks to the instantaneous risk-neutral variance.

Turning next to the $3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$ estimates, and measures of asymmetry in the risk-neutral return distribution, reported in the mid-portion of the table, the values seemingly depend strongly on the horizon T . Recall from the discussion in Section 2, that the leading term in an asymptotic expansion of $3(V_{t,t+T}^{\mathbb{Q}} - \mathcal{V}_{t,t+T}^{\mathbb{Q}})$ for small T equals $3 \int_{\mathbb{R}} (z^2 - 2(e^z - 1 - z)) \nu_t^{\mathbb{Q}}(dz)$. Meanwhile, the second-order term in said expansion that depends on the horizon T may be traced to mean-reversion in volatility and jump intensity, as well as the leverage effect. The pairwise differences in the sample means and the various quantiles of $3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$ for short and long tenors thus directly underscore the importance of these latter effects. Fur-

ther supporting this conjecture, the mean and the median of the instantaneous $3(\widehat{V}_t^{\mathbb{Q}} - \widehat{\mathcal{V}}_t^{\mathbb{Q}})$ estimates are also substantially lower than those of $3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$ for short T . Looking at the $3(\widehat{V}_t^{\mathbb{Q}} - \widehat{\mathcal{V}}_t^{\mathbb{Q}})$ estimates, it is also worth noting that with $3(V_t^{\mathbb{Q}} - \mathcal{V}_t^{\mathbb{Q}})$ being approximately equal to the risk-neutral third moment of the jumps (recall equation (7)), the numbers in the table point to very little asymmetry in the implied \mathbb{Q} jump size distribution.

5.3 Leverage Risk and Pricing

The final two rows in Table 1 summarize the results for our leverage risk estimates. Consistent with the idea of a nontrivially sized instantaneous leverage risk premium $ILLRP_t$, the sample mean of the risk-neutral $-\widehat{L}_t^{\mathbb{Q}}$ exceeds that of the realized $-\widehat{RL}_t$ by more than twice of what the sample mean of the risk neutral spot variance $\widehat{V}_t^{\mathbb{Q}}$ exceeds that of the realized variance \widehat{RV}_t . The different quantiles for $-\widehat{L}_t^{\mathbb{Q}}$ are also all substantially higher than the corresponding quantiles for $-\widehat{RL}_t$.

To help visualize and further appreciate the pricing of the jump leverage effect, and how it differs from the pricing of variance risk, Figure 3 plots the time series of $\widehat{V}_t^{\mathbb{Q}}$ and \widehat{RV}_t , while Figure 4 plots $-\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$. All of the measures obviously varied quite significantly over the 2007-2020 sample period, generally increasing during periods of crisis. As is to be expected, the realized measures, \widehat{RV}_t and $-\widehat{RL}_t$, also typically exceed their risk-neutral counterparts, $\widehat{V}_t^{\mathbb{Q}}$ and $-\widehat{L}_t^{\mathbb{Q}}$, at the onset of a crisis, with that ordering reversed in the aftermath of a crisis. As a case in point, during the financial crisis in the Fall of 2008, \widehat{RV}_t exceeded $\widehat{V}_t^{\mathbb{Q}}$ for the month of September and beginning of October, while $\widehat{V}_t^{\mathbb{Q}}$ exceeded \widehat{RV}_t by quite a wide margin for more than a year thereafter. This same general pattern is also evident for the $-\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$ leverage risk measures.

Meanwhile, the by far largest increase in all of the risk measures occurred in the Spring of 2020 coincident with the start of the COVID-19 pandemic. At that time $\widehat{V}_t^{\mathbb{Q}}$ reached a peak of roughly doubled that of its peak observed during the financial crisis of 2008. The heightened values of $\widehat{V}_t^{\mathbb{Q}}$ in the Spring of 2020 was, however, noticeable shorter-lived than the highs attained during the 2008 financial crisis. While these same general features manifest in $-\widehat{L}_t^{\mathbb{Q}}$ as well, there are also some noticeable differences.¹⁷ Most obviously, $-\widehat{L}_t^{\mathbb{Q}}$ increased

¹⁷Incidentally, these differences in the time series behavior of the variance and leverage risk measures also indicate that the jump risks have a much more complicated structure than portrayed by most parametric models hitherto employed in the literature, the popular double-jump stochastic volatility model discussed in

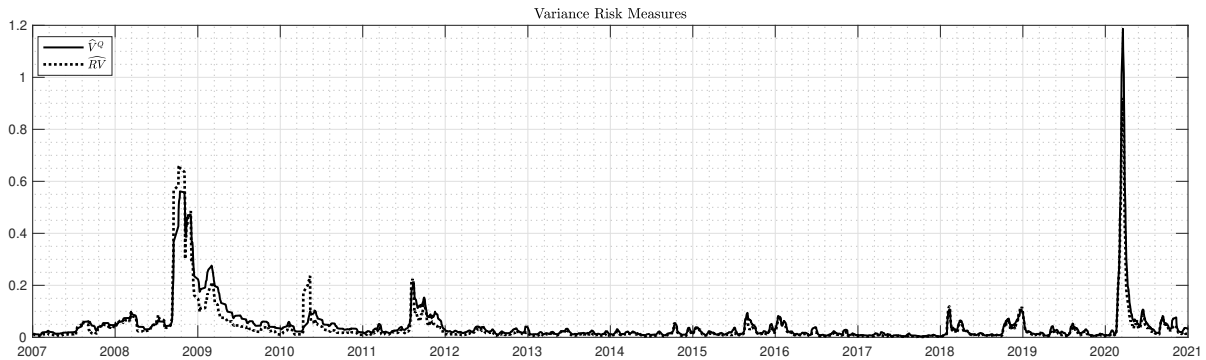


Figure 3: The figure plots the ten-day moving averages of the daily \widehat{V}_t^Q and \widehat{RV}_t variance risk estimates.

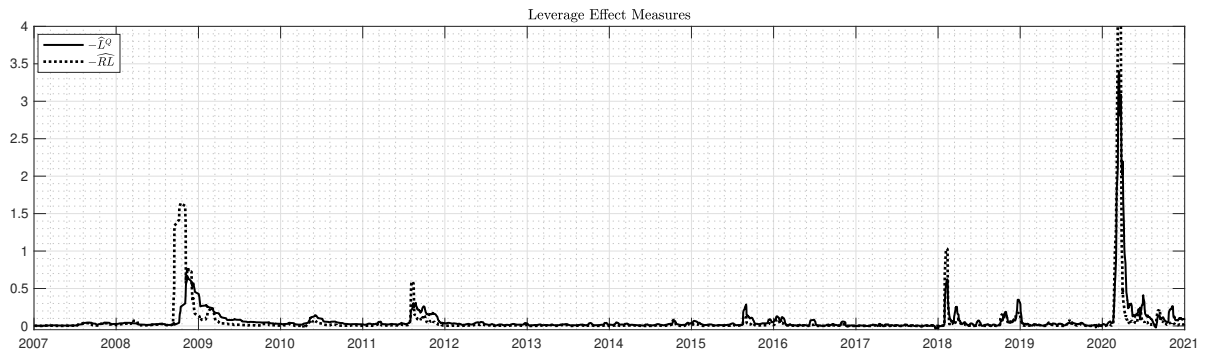


Figure 4: The figure plots the ten-day moving averages of the daily $-\widehat{L}_t^Q$ and $-\widehat{RL}_t$ leverage risk estimates.

even more dramatically during the recent pandemic, reaching a high of almost five-fold its high observed during the 2008 financial crises. In other words, if anything it appears as if leverage risk has become an even bigger concern to investors more recently than it has been historically.

To more specifically illustrate the behavior of the volatility and leverage risks during periods of market turmoil, Figure 5 plots the SPY price and the instantaneous risk-neutral volatility measure \widehat{V}_t^Q at a five-minute frequency for the three weeks in the sample when the market fell by more than 10%, namely the week of November 5-12, 2008 (displayed in the Appendix B included).

first panel), the week of August 3-10, 2011 (displayed in the second panel), and the week of March 4-11, 2020 (displayed in the third panel).¹⁸ Figure 6 plots the SPY and the leverage risk measure $-\widehat{L}_t^Q$ for the same three weeks. Looking first at Figure 5, there is obviously a strong negative correlation between the price changes and the changes in the \widehat{V}_t^Q spot volatility estimates during each of these three turbulent weeks. These large realized leverage effects also clearly manifest in the $-\widehat{L}_t^Q$ leverage risk estimates displayed in Figure 6.

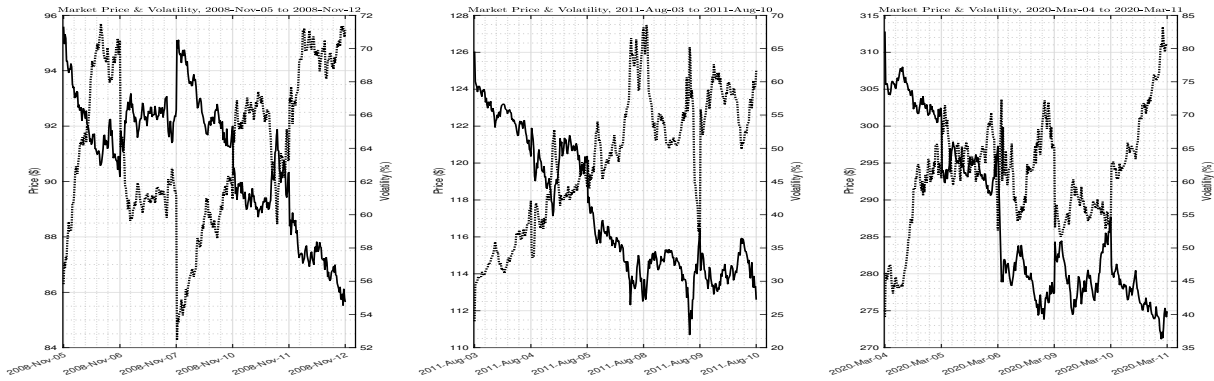


Figure 5: The figure shows the price of the SPY at a five-minute frequency (solid line) together with \widehat{V}_t^Q in annualized percentage form (dashed line) for November 5-12, 2008 (first panel), August 3-10, 2011 (second panel), and March 4-11, 2020 (third panel).

Although both of the two risk measures generally appear to be strongly negatively correlated with the price, there are also some important differences between the three episodes. In particular, while the average values of \widehat{V}_t^Q for the first week of November 2008 and the second week of March 2020 were very similar, at 64.0% and 61.2%, respectively, the volatility stayed within a relatively narrow range of around 55% – 70% for the first of the two weeks, while it fluctuated much more widely from a low of around 40% to a high of around 85% for the second of the two weeks. The volatility mean reversion and the leverage effect also both manifested very differently for these two different episodes. For instance, while the average value of $(252 \times \widehat{V}_{t,t+30}^Q)^{1/2}$, as proxied by the VIX, equalled 59.2% for the first week of November 2008, it was “only” equal to 43.1% for the second of March 2020. Recalling again the short-time expansions for the risk-neutral volatility measures in Section 2, this

¹⁸To allow for easier interpretation, the volatility measures in Figure 5 are displayed in annualized volatility units, or $(252 \times \widehat{V}_t^Q)^{1/2}$. The market also fell by more than 10% the first week of October 2008. However, due to issues with missing observations and quality of the options data, we omit that week from the display.

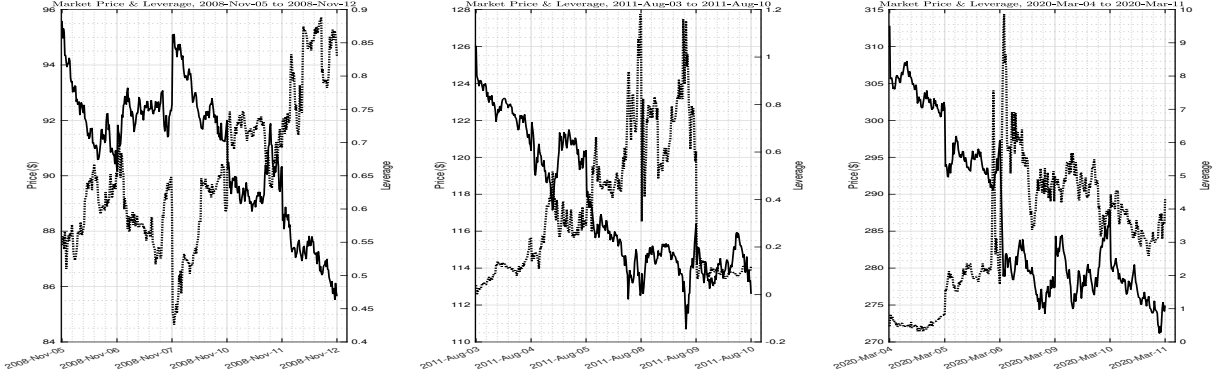


Figure 6: The figure shows the price of the SPY at a five-minute frequency (solid line) together with $-\widehat{L}_t^{\mathbb{Q}}$ (dashed line) for November 5-12, 2008 (first panel), August 3-10, 2011 (second panel), and March 4-11, 2020 (third panel).

noticeably lower average value of the VIX for the second of the two weeks may naturally be attributed to stronger mean reversion during that week. As previously noted, this strong volatility mean reversion was also accompanied by more pronounced leverage effects during the pandemic. Indeed, as Figure 6 shows, the estimates for $-\widehat{L}_t^{\mathbb{Q}}$ are an order of magnitude larger for the second week of March 2020 compared to the first week of November 2008.

The above discussed evidence for the presence and pricing of jump leverage risk holds that shocks in the form of jumps to the stochastic volatility and/or the jump intensity should be priced. If so, we should also expect the mean of the instantaneous drift $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ to be positive. Consistent with that idea and the expansion in (9), the full-sample mean of the estimates for $\beta_{t,1}$ from the regression in (23) equals 0.0636, corresponding to an estimate of 0.1272 for $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$. We can similarly estimate the forward quantity, say $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ for some $T > 0$, by employing options recorded at time t expiring shortly after time $t + T$. Doing so with options expiring soon after two weeks, the full-sample mean estimate of $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ only equals 0.0050, compared to 0.1272 for the mean instantaneous drift. A formal test for the null hypothesis that the mean of $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ equals that of $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ is also strongly rejected with a t -statistic of 4.34. This significantly lower estimate for $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ compared to the estimate for $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ thus again suggests that the risk premium associated with the variation in $\mathcal{V}_t^{\mathbb{Q}}$ is mostly due to the compensation for short-lived spikes, or jumps.

To further buttress this thesis, as an additional independent robustness check we next

look at the pricing of VIX index options.

6 Jump Leverage Risk Premiums Embedded in VIX Options

If the jump leverage effect constitutes an important source of risk that requires compensation by investors, then some of the jumps to volatility, namely the ones directly related to price jumps, should also be priced. In fact, based on existing empirical evidence (see, e.g., Jacod and Todorov (2010), Todorov and Tauchen (2011) and Caporin et al. (2017), among others), one would naturally expect most volatility jumps to be accompanied by simultaneous price jumps in the opposite direction.¹⁹ As such, this allows for independent verification of the importance of the jump leverage risk premium through the use of options written on the VIX index and the pricing thereof.

The VIX index computed by the CBOE is formally based on the $\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}$ risk-neutral variance measure defined in (4) together with the two available tenors T closest to 30 calendar days.²⁰ We obtain end-of-day prices for VIX options spanning the 2007-2020 period from OptionMetrics. We further collect 5-minute price records for the VIX index itself from the CBOE Data Shop. We apply standard cleaning procedures and similar filters to the ones discussed in Section 5.1 used for processing the S&P 500 price and options data.²¹ However, since the number of available short-tenor VIX index options is significantly less than the number of short-dated S&P 500 options, especially for the early part of the sample, we retain a somewhat wider set of VIX index options with maturities ranging from three up to thirty business days.

¹⁹This strong negative dependence between jumps in the market and the volatility of the market also naturally arises in many equilibrium models, the recent model by Eraker and Yang (2022) included. Counter to the empirical evidence, however, this particular theoretical model, as well as other exponentially-affine models, cannot account for negative volatility jumps; see also the discussion in Amengual and Xiu (2018).

²⁰The CBOE uses a calendar day convention in their computation of the VIX, while as previously noted we rely on a business day convention in our high-frequency calculations of $\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}$. However, since we are not actually modeling the connection between the VIX and our $\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}$ measure, this difference is immaterial.

²¹We also further apply bounce-back filters similar to the ones proposed by Barndorff-Nielsen et al. (2009) to remove rare large intraday price swings caused in part by the rules applied by the exchange for the inclusion of deep out-of-the-money options in the computation of the VIX; see Andersen et al. (2015) for a more detailed discussion of this issue.

Using the VIX index option data and the expressions in (21) and (22), we begin by computing the two separate estimates for the risk-neutral variances of the VIX. We then run the regressions in (23), omitting the T^2 terms due to the more limited availability of different tenors. We refer to the resulting estimated intercepts, and instantaneous risk-neutral variance estimates of the log(VIX) index, by $\widehat{VV}_t^{\mathbb{Q}}$ and $\widehat{\mathcal{V}\mathcal{V}}_t^{\mathbb{Q}}$, respectively. In parallel to our analysis of the S&P 500 discussed above, we then compare these option-implied volatilities for the VIX with their daily realized counterparts,

$$\widehat{RVV}_t = \frac{n}{k_n} \sum_{i=1}^{k_n} (\Delta_{t,i}^n VIX)^2, \quad \widehat{\mathcal{R}\mathcal{V}\mathcal{V}}_t = \frac{2n}{k_n} \sum_{i=1}^{k_n} (e^{\Delta_{t,i}^n VIX} - 1 - \Delta_{t,i}^n VIX), \quad (33)$$

where $\Delta_{t,i}^n VIX \equiv \log(VIX_{t-i/n}) - \log(VIX_{t-(i-1)/n})$ denote the intraday high-frequency logarithmic returns on the VIX.

Given the aforementioned strong empirical evidence for negative dependence between market price jumps and jumps in the VIX, together with the strong empirical evidence for the significant pricing of market price jumps, one would naturally expect volatility jump risk to be priced with the opposite sign to that of market price jumps. In particular, $3(\widehat{VV}_t^{\mathbb{Q}} - \widehat{\mathcal{V}\mathcal{V}}_t^{\mathbb{Q}})$, which by the discussion in Section 2 provides a proxy for the third risk-neutral moment of the jumps in the log-VIX index, would naturally be expected to be less than the corresponding realized measure $3(\widehat{RVV}_t - \widehat{\mathcal{R}\mathcal{V}\mathcal{V}}_t)$, with the gap stemming from the pricing of volatility jump risk. Consistent with that conjecture, the full-sample mean of $3(\widehat{VV}_t^{\mathbb{Q}} - \widehat{\mathcal{V}\mathcal{V}}_t^{\mathbb{Q}})$ equals -0.1102 , while that of $3(\widehat{RVV}_t - \widehat{\mathcal{R}\mathcal{V}\mathcal{V}}_t)$ only equals -0.0007 . The t -statistic of 19.16 for testing whether the risk-neutral and realized third moments of the jumps to the market volatility are the same is also highly significant. In other words, jumps to the state variables that drive the stochastic volatility and risk-neutral jump intensity of the market are obviously priced by investors, further corroborating the practical importance of the jump leverage effect and the sizeable magnitude of the corresponding jump leverage risk premium.

7 Concluding Remarks

Jumps in asset prices trigger two types of risks: (i) risk stemming from changes in the instantaneous volatility, and (ii) risk stemming from changes in the future investment op-

portunity set due to jumps in the diffusive volatility and/or jump intensity. We provide new model-free evidence in support of the nontrivial pricing of the latter type of jump risk, and so-called jump leverage risk in particular. Our analyses rely on novel short-time risk-neutral variance expansions, together with short-dated S&P 500 index options and high-frequency S&P 500 returns for estimating different risk measures and quantifying the corresponding risk premiums.

Our new model-free risk premium estimates also provide useful guidance for the future development of empirically more realistic equilibrium-based asset pricing models able to explain the significant pricing of jump risks. Most modern general equilibrium asset pricing models hold that the market and its instantaneous risk-neutral volatility are functions of some underlying state vector S_t :

$$X_t = F(S_t), \quad \mathcal{V}_t^{\mathbb{Q}} = G(S_t).$$

The state vector S_t is typically assumed to follow a Markov jump-diffusion. The specific variables entering S_t , and the functional forms of F and G , will depend on the particular equilibrium model. The market price and its volatility, X_t and $\mathcal{V}_t^{\mathbb{Q}}$, may also depend on different components of S_t . The exact form of these dependencies in turn determine the risks of the state variables that command risk premiums and the signs and connection between the instantaneous variance and leverage risk premiums, \mathcal{IVRP}_t and \mathcal{ILRP}_t , emphasized here. To further clarify, it is instructive consider three different classes of equilibrium models that have recently been proposed in the literature to help explain option data and the pricing of jump risks in particular.

- A rare disaster risk model with time-varying probabilities of disasters and a representative agent with Epstein-Zin utility (e.g., Wachter (2013) and Seo and Wachter (2019)). In this class of models, the aggregate consumption exhibits jumps (or rare disasters), which carries over to the aggregate dividend of the market. The state vector S_t includes the aggregate dividend and the time-varying probability of jumps (or rare disasters). Importantly, X_t depends on both state variables, while the risk-neutral spot variance $\mathcal{V}_t^{\mathbb{Q}}$ only depends on the second state variable. Consequently, even though X_t is subject to occasional jumps, X_t and $\mathcal{V}_t^{\mathbb{Q}}$ do not co-jump. Consequently, while $\mathcal{IVRP}_t > 0$, in this class of models $\mathcal{ILRP}_t \equiv 0$.

- A rare disaster risk model and a representative agent with external habit formation risk preferences (e.g., Du (2011)). In this model the aggregate consumption is again subject to jumps. The state variable S_t is comprised of the aggregate level of consumption and the level of habit. Since the time-varying risk aversion of the representative agent is determined by habit formation, which depends on the value of the aggregate consumption, jumps in risk aversion occur at the same time as jumps in the aggregate consumption. A negative jump in consumption therefore triggers a positive jump in risk-aversion, which in turn implies a positive jump in $\mathcal{V}_t^{\mathbb{Q}}$. As a result, $\mathcal{IVRP}_t > 0$ and $\mathcal{ILRP}_t > 0$, both due to jumps in consumption.
- A model with jumps in the volatility of consumption growth and a representative agent with Epstein-Zin utility (e.g., Eraker and Shaliastovich (2008), Drechsler and Yaron (2011) and Eraker and Yang (2022)). In this class of models, the state variable S_t includes consumption growth, which does not jump, and the volatility of consumption growth, which varies over time and may jump. Since investors have Epstein-Zin preferences, shocks to the time-varying volatility, including volatility jumps, will be priced. Positive jumps in consumption volatility will therefore also trigger negative jumps in the market, as well as positive jumps in the risk-neutral variance $\mathcal{V}_t^{\mathbb{Q}}$. Consequently, $\mathcal{IVRP}_t > 0$ and $\mathcal{ILRP}_t > 0$, both due to jumps in consumption growth volatility.

Although formally distinct, the economic channels through which the jump leverage risk premium arise in the latter two equilibrium classes of models discussed above both depend on price jumps being accompanied by sudden changes in the investment opportunity set and/or the investor’s risk aversion. It would be interesting to further calibrate the models and quantify the magnitudes of these different channels to help precise the economic mechanisms that drive the empirically “large” jump leverage risk premium document here. We leave further work along these lines for future research.

A Appendix: Derivations of Short-Term Variance Expansions

A.1 Assumptions

We set the risk-free rate and the dividend yield to zero and make the normalization $x_0 = 0$. We further denote the martingale component of x with

$$M_T = \int_0^T \sigma_s dW_s^{\mathbb{Q}} + \int_0^T \int_{\mathbb{R}} z(\mu - \nu^{\mathbb{Q}})(ds, dz).$$

To derive the expansions, we assume that the \mathbb{Q} -dynamics of z , for z being one of the processes $\mathcal{V}^{\mathbb{Q}}$, $V^{\mathbb{Q}}$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, $m^{\mathbb{Q}}(V^{\mathbb{Q}})$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$, $L^{\mathbb{Q}}$, $m^{\mathbb{Q}}(L^{\mathbb{Q}})$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M)$, $[\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}]$ and $m^{\mathbb{Q}}([\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}])$, is of the form

$$z_t = z_0 + \int_0^t m_s^{\mathbb{Q}}(z) ds + \mathbb{Q} - \text{martingale},$$

with the above martingale being \mathcal{F}_0 -conditionally square-integrable, $m_t^{\mathbb{Q}}$ being a process with càdlàg paths, and $\mathbb{E}_0^{\mathbb{Q}}(m_t^{\mathbb{Q}})^2 < \infty$. We further denote the martingale part of $\mathcal{V}^{\mathbb{Q}}$ by $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$. We assume that the above decomposition also holds for the processes $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$.

The above assumptions are rather weak and generally satisfied for the continuous-time asset pricing models used in finance, including standard reduced-form models, as well as equilibrium-based models. In particular, the assumptions hold when the asset price dynamics is embedded in a general stochastic differential equation (SDE), and the coefficients of the SDE have finite conditional moments of certain order.

A.2 Proofs

Given the dynamics of x under \mathbb{Q} and using the notation of M above, it follows by an application of Itô's formula that

$$x_T = -\frac{1}{2} \int_0^T \mathcal{V}_s^{\mathbb{Q}} ds + M_T.$$

Another application of Itô's formula for the product of two processes, leads to

$$\begin{aligned}
T \times V_{0,T}^{\mathbb{Q}} &= \mathbb{E}_0^{\mathbb{Q}} \left(-\frac{1}{2} \int_0^T (\mathcal{V}_s^{\mathbb{Q}} - \mathbb{E}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}})) ds + M_T \right)^2 \\
&= \frac{1}{4} \text{var}_0^{\mathbb{Q}} \left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds \right) + \mathbb{E}_0^{\mathbb{Q}}(M_T^2) - \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds M_T \right) \\
&= \frac{1}{4} \text{var}_0^{\mathbb{Q}} \left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds \right) + \int_0^T \mathbb{E}_0^{\mathbb{Q}}(V_s^{\mathbb{Q}}) ds - \int_0^T \text{cov}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}, M_s) ds,
\end{aligned}$$

and

$$T \times \mathcal{V}_{0,T}^{\mathbb{Q}} = \int_0^T \mathbb{E}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}) ds.$$

Using our assumption for $\mathcal{V}_s^{\mathbb{Q}}$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$, we further have

$$\begin{aligned}
\mathbb{E}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}) &= \int_0^s \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})) du = \int_0^s \mathbb{E}_0^{\mathbb{Q}} \left(m_0^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) + \int_0^u m_v^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})) dv \right) du \\
&= \int_0^s \mathbb{E}_0^{\mathbb{Q}} \left(m_0^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) + u \times m_0^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})) + \int_0^u \int_0^v m_z^{\mathbb{Q}}(m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))) dz dv \right) du \\
&= s \times m_0^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) + s^2 \times C_0 + \tilde{C}_0(s),
\end{aligned}$$

for some \mathcal{F}_0 -adapted random variable C_0 and \mathcal{F}_0 -adapted random function $\tilde{C}_0(s)$ satisfying $|\tilde{C}_0(s)| \leq s^3 \times \tilde{C}'_0$, where \tilde{C}'_0 is another \mathcal{F}_0 -adapted random variable. A similar expansion for $\mathbb{E}_0^{\mathbb{Q}}(V_s^{\mathbb{Q}})$ also readily obtains. Next, applying Itô's formula and using the definition of the processes M and $L^{\mathbb{Q}}$ (note in particular that $M_0 = 0$), as well as our assumptions for the dynamics of $L^{\mathbb{Q}}$, $m^{\mathbb{Q}}(L^{\mathbb{Q}})$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M)$, we have

$$\begin{aligned}
\text{cov}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}, M_s) &= \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^s m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) du M_s \right) + \mathbb{E}_0^{\mathbb{Q}}(Z_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) M_s) \\
&= \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^s m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) M_u du \right) + \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^s L_u^{\mathbb{Q}} du \right) \\
&= \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^s \int_0^u m_v^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M) dv du \right) + \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^s \left(L_0^{\mathbb{Q}} + \int_0^u m_v^{\mathbb{Q}}(L^{\mathbb{Q}}) dv \right) du \right) \\
&= s \times L_0^{\mathbb{Q}} + s^2 \times C_0 + \tilde{C}_0(s),
\end{aligned}$$

where C_0 and $\tilde{C}_0(s)$ satisfy the same properties as above. Finally, direct expansion and the

fact that $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ is a martingale leads to:

$$\begin{aligned} \text{var}_0^{\mathbb{Q}} \left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds \right) &= \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T \int_0^s (m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))) duds \right)^2 \\ &+ \mathbb{E}_0 \left(\int_0^T \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) Z_v^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) dudv \right) \\ &+ 2\mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T \int_0^s (m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))) duds \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) du \right). \end{aligned}$$

Using the definition of quadratic variation and the fact that $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ is a martingale, we may write

$$\mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) Z_v^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) dudv \right) = 2 \int_0^T (T-s) \mathbb{E}_0^{\mathbb{Q}}([\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}]_s) ds.$$

Next, using again the fact that $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ is a martingale,

$$\begin{aligned} &\mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T \int_0^s (m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))) duds \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) du \right) \\ &= \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T (T-s) \int_0^s (m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))) du Z_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) ds \right) \\ &\quad + \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T \int_0^s (m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))) du \int_0^s Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) duds \right) \\ &= 2\mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T (T-s) m_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) Z_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) ds \right) \\ &\quad + \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T \int_0^s (m_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})) Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) duds \right). \end{aligned}$$

From here, by making use of our assumptions for $[\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}]$, $m^{\mathbb{Q}}([\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}])$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, it follows that

$$\text{var}_0^{\mathbb{Q}} \left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds \right) = T^3 \times C_0 + \tilde{C}_0(T),$$

where C_0 is some \mathcal{F}_0 -adapted random variable, and the \mathcal{F}_0 -adapted random function $\tilde{C}_0(T)$ satisfies $|\tilde{C}_0(T)| \leq T^4 \times \tilde{C}'_0$ for some some \mathcal{F}_0 -adapted random variable \tilde{C}'_0 .

B Appendix: Jumps and Leverage Risk in a Parametric Model

This Appendix demonstrates how the risk premium measures defined in Section 3 manifest in the popular parametric double-jump stochastic volatility model of Duffie et al. (2000). This particular model has been extensively used in the empirical asset pricing literature.

The model postulates the following asset dynamics under \mathbb{P}

$$\frac{dX_t}{X_{t-}} = \alpha_t dt + \sigma_t dW_t^{\mathbb{P}} + \int_{\mathbb{R}^2} (e^z - 1)(\mu - \nu^{\mathbb{P}})(ds, dz, dy),$$

$$d\sigma_t^2 = \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - \sigma_t^2)dt + \eta\sigma_t dB_t^{\mathbb{P}} + \int_{\mathbb{R}^2} y(\mu - \nu^{\mathbb{P}})(ds, dz, dy),$$

where α_t is a linear function of σ_t^2 , $(W_t^{\mathbb{P}}, B_t^{\mathbb{P}})$ is a bivariate Brownian motion with correlation ρ , and the jump compensator takes the form

$$\nu^{\mathbb{P}}(ds, dz, dy) = (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}}\sigma_t^2) \frac{e^{-\frac{(z-\mu_z^{\mathbb{P}})^2}{2v_z^{\mathbb{P}}}}}{\sqrt{2\pi v_z^{\mathbb{P}}}} \frac{e^{-y/\mu_y^{\mathbb{P}}}}{\mu_y^{\mathbb{P}}} 1_{\{y>0\}}.$$

This specification in turn accounts for a number of key empirically relevant features: volatility is time-varying, price and volatility can both jump, and the jumps typically arrive together, the intensity of the jumps is an affine function of σ_t^2 and therefore time-varying. Also, even though the price and volatility jumps arrive together, the size of the jumps are independent with normal and exponential distributions, respectively.

Per the discussion in the main text, assume for simplicity that the risk-free interest rate and the dividend yield are both zero. The dynamics of X under the \mathbb{P} and \mathbb{Q} measures then formally coincide, although the pricing of various risks may cause the values of the risk-neutral parameters, superscripted with \mathbb{Q} in the following, to differ from the corresponding \mathbb{P} superscripted parameters. Importantly, however, the ρ and η parameters do not change under this equivalent change of measure.

To begin, consider the instantaneous variance risk premium. Using the shorthand notation $\tilde{v}_z^{\mathbb{Q}} = 2 \left(e^{\mu_z^{\mathbb{Q}} + v_z^{\mathbb{Q}}/2} - 1 - \mu_z^{\mathbb{Q}} \right)$, the two differently defined risk-neutral instantaneous

variances may be expressed as

$$V_t^{\mathbb{Q}} = \sigma_t^2 + (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2)((\mu_z^{\mathbb{Q}})^2 + v_z^{\mathbb{Q}}), \quad \mathcal{V}_t^{\mathbb{Q}} = \sigma_t^2 + (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2) \tilde{v}_z^{\mathbb{Q}}.$$

The same expressions also obtain for $V_t^{\mathbb{P}}$ and $\mathcal{V}_t^{\mathbb{P}}$, except for the \mathbb{Q} -parameters being replaced by their \mathbb{P} counterparts. Correspondingly, the $IVRP_t$ instantaneous variance risk premium takes the specific form

$$IVRP_t = (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2)((\mu_z^{\mathbb{Q}})^2 + v_z^{\mathbb{Q}}) - (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}} \sigma_t^2)((\mu_z^{\mathbb{P}})^2 + v_z^{\mathbb{P}}).$$

As previously noted, $IVRP_t$ generally only reflects the pricing of price jump risk, which in the context of the double-jump stochastic volatility model can arise from either different jump intensities, or different jump distributions under the \mathbb{P} and \mathbb{Q} measures.

Turning next to the leverage risk, we have

$$\frac{1}{T} ([x, \mathcal{V}^{\mathbb{Q}}]_{t+T} - [x, \mathcal{V}^{\mathbb{Q}}]_t) = \frac{1}{T} (1 + \lambda_1^{\mathbb{Q}} \tilde{v}_z^{\mathbb{Q}}) \left(\rho \eta \int_t^{t+T} \sigma_s^2 ds + \sum_{s \in [t, t+T]} \Delta x_s \Delta \sigma_s^2 \right).$$

The dependence of the leverage risk on the \mathbb{Q} -parameters stems from the fact that this risk is defined as a co-movement between x and $\mathcal{V}^{\mathbb{Q}}$, which depends on the \mathbb{Q} -parameters because of jumps in the price and the pricing thereof. Meanwhile, the first term in the parentheses above stems from the dependence between the Brownian motions driving the price and volatility, while the second term is due to the price-volatility co-jumps. Taking conditional expectations and letting $T \downarrow 0$, it further follows that

$$L_t^{\mathbb{Q}} = (1 + \lambda_1^{\mathbb{Q}} \tilde{v}_z^{\mathbb{Q}}) [\rho \eta \sigma_t^2 + \mu_z^{\mathbb{Q}} \mu_y^{\mathbb{Q}} (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2)],$$

and

$$L_t^{\mathbb{P}} = (1 + \lambda_1^{\mathbb{Q}} \tilde{v}_z^{\mathbb{Q}}) [\rho \eta \sigma_t^2 + \mu_z^{\mathbb{P}} \mu_y^{\mathbb{P}} (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}} \sigma_t^2)].$$

These expressions for the instantaneous leverage effects in turn implies the following instantaneous leverage risk premium

$$ILLRP_t = (1 + \lambda_1^{\mathbb{Q}} \tilde{v}_z^{\mathbb{Q}}) [\mu_z^{\mathbb{P}} \mu_y^{\mathbb{P}} (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}} \sigma_t^2) - \mu_z^{\mathbb{Q}} \mu_y^{\mathbb{Q}} (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2)].$$

Since the terms in $L_t^{\mathbb{Q}}$ and $L_t^{\mathbb{P}}$ due to the continuous leverage effect coincide, these terms cancel in the $ILLRP_t$ risk premium, which therefore only depends on price-volatility co-jumps. As such, $ILLRP_t$ is effectively a difference of, re-scaled by constants, \mathbb{P} and \mathbb{Q} jump intensities. This also directly parallels the expression for $IVRP_t$. Correspondingly, $IVRP_t$ and $ILLRP_t$ are both affine functions of σ_t^2 , and thus exhibit the same dynamics. This result is a particular feature of the double-jump volatility model. If the intensity of the price jumps that arrive together with the volatility jumps differed from those that do not, this would naturally cause the dynamics of $IVRP_t$ and $ILLRP_t$ to differ.

C Appendix: Monte Carlo Simulations

This appendix evaluates the accuracy of the new $\widehat{V}_t^{\mathbb{Q}}$, $\widehat{\nu}_t^{\mathbb{Q}}$ and $\widehat{L}_t^{\mathbb{Q}}$ feasible risk-neutral measures discussed in Section 4.1. We rely on the double-jump stochastic volatility model discussed in Appendix B to generate the true option prices. The specific values of the model parameters used in the simulations are reported in Table C.1.

Table C.1: **Parameter Values**

Parameter	Value	Parameter	Value	Parameter	Value
$\kappa^{\mathbb{Q}}$	30	$\theta^{\mathbb{Q}}$	0.018	η	0.2
ρ	-0.9	$\lambda_0^{\mathbb{Q}}$	0	$\lambda_1^{\mathbb{Q}}$	385
$\mu_z^{\mathbb{Q}}$	-0.05	$\mu_y^{\mathbb{Q}}$	0.0234	$\sqrt{v_z^{\mathbb{Q}}}$	0.01

Note: The table reports risk-neutral parameter values for the double-jump stochastic volatility model in (B)-(B) in Appendix B used in the simulations.

The parameter values imply that $\mathbb{E}^{\mathbb{Q}}(\sigma_t^2) = 0.0257$, together with a relatively fast mean reversion of volatility with a half-life of around 8 days only. The jump parameters are set in a way so that $\int_{\mathbb{R}^2} z^2 \nu_t(dz, dy) = \sigma_t^2$, implying that jumps contribute half of the risk-neutral jump variation at any point in time. We also allow for nontrivial volatility jumps, with a mean value equal to 0.0234, very close to that of the risk-neutral mean of σ_t^2 . The observed

option prices used in the estimation are contaminated by the following errors

$$\widehat{O}_{t,T}(K_j) = O_{t,T}(K_j)(1 + 0.025 \times z_{t,T}(j)), \quad j = 1, \dots, N_{t,T},$$

where $\{z_{t,T}(j)\}_{j=1}^{N_{t,T}}$ are sequences of *i.i.d.* standard normal independent random variables. The size of the observation error is calibrated to roughly match the bid-ask spreads of the S&P 500 index options used in our empirical analyses.

We initialize the simulations by setting the time- t value of the spot variance to a low, average and a high value of 0.0170, 0.0204 and 0.0267, corresponding to the 25-th, 50-th and 75-th quantiles, respectively, of the unconditional distribution of σ_t^2 . The initial level of the underlying stock price is set to 4,500. For each (t, T) pair the strike grid is equidistant with gaps between strikes of 5. Starting from the at-the-money strike of 4,500, the strikes are then extended on both sides until the true out-of-the-money option price falls below 0.075. We employ three different tenors in the estimation, namely $T_1 = 3/252$, $T_2 = 5/252$ and $T_3 = 10/252$, corresponding to 3, 5 and 10 business days to expiration, respectively. Again, this specification of the strike grids and the choice of tenors mimics those of the S&P 500 index options used in the actual estimation. Finally, the \widehat{s}_t bias-adjustment term for \widehat{L}_t^Q is computed from 80 high-frequency return and option observations, approximately corresponding to a day worth of observations when sampling at the 5-minute frequency.

Table C.2 reports the quantiles of the resulting \widehat{V}_t^Q , $\widehat{\mathcal{V}}_t^Q$ and \widehat{L}_t^Q risk estimates based on 1,000 Monte Carlo replications. As the table shows, the new estimates generally perform admirably, with the medians almost exactly equal to the true values, together with fairly narrow interquartile ranges.

Table C.2: Monte Carlo Simulation Results

Estimand	True Value	Estimates		
		Q_{25}	Q_{50}	Q_{75}
Initial Volatility $\sigma_t^2 = 0.0170$				
$V_t^{\mathbb{Q}}$	0.0340	0.0336	0.0339	0.0343
$\mathcal{V}_t^{\mathbb{Q}}$	0.0337	0.0333	0.0337	0.0340
$-L_t^{\mathbb{Q}}$	0.0214	0.0212	0.0223	0.0233
Initial Volatility $\sigma_t^2 = 0.0204$				
$V_t^{\mathbb{Q}}$	0.0408	0.0403	0.0407	0.0412
$\mathcal{V}_t^{\mathbb{Q}}$	0.0404	0.0400	0.0404	0.0408
$-L_t^{\mathbb{Q}}$	0.0257	0.0245	0.0256	0.0267
Initial Volatility $\sigma_t^2 = 0.0267$				
$V_t^{\mathbb{Q}}$	0.0534	0.0528	0.0533	0.0538
$\mathcal{V}_t^{\mathbb{Q}}$	0.0529	0.0524	0.0528	0.0534
$-L_t^{\mathbb{Q}}$	0.0336	0.0305	0.0319	0.0333

Note: The table reports the simulated quantiles of the $V_t^{\mathbb{Q}}$, $\mathcal{V}_t^{\mathbb{Q}}$, and $-L_t^{\mathbb{Q}}$ risk estimates for different initial volatility levels based on 1,000 Monte Carlo replications.

References

- Aït-Sahalia, Y., J. Fan, R. J. Laeven, C. D. Wang, and X. Yang (2017). Estimation of the continuous and discontinuous leverage effects. *Journal of the American Statistical Association* 112(520), 1744–1758.
- Ait-Sahalia, Y., J. Fan, and Y. Li (2013). The leverage effect puzzle: Disentangling sources of bias at high frequency. *Journal of Financial Economics* 109(1), 224–249.
- Aït-Sahalia, Y., J. Jacod, and D. Xiu (2021). Inference on risk premia in continuous-time asset pricing models. *National Bureau of Economic Research, Working Paper*.
- Aït-Sahalia, Y., M. Karaman, and L. Mancini (2020). The term structure of equity and variance risk premia. *Journal of Econometrics* 219(2), 204–230.
- Aït-Sahalia, Y., Y. Wang, and F. Yared (2001). Do option markets correctly price the probabilities of movement of the underlying asset? *Journal of Econometrics* 102(1), 67–110.
- Aleti, S. (2022). The high-frequency factor zoo. *Duke University, Working Paper*.
- Amengual, D. and D. Xiu (2018). Resolution of policy uncertainty and sudden declines in volatility. *Journal of Econometrics* 203(2), 297–315.
- Andersen, T. G., O. Bondarenko, and M. T. Gonzalez-Perez (2015). Exploring return dynamics via corridor implied volatility. *Review of Financial Studies* 28(10), 2902–2945.
- Andersen, T. G., N. Fusari, and V. Todorov (2015). The risk premia embedded in index options. *Journal of Financial Economics* 117(3), 558–584.
- Bakshi, G. and D. Madan (2000). Spanning and derivative-security valuation. *Journal of Financial Economics* 55(2), 205–238.
- Bandi, F. M. and R. Renò (2012). Time-varying leverage effects. *Journal of Econometrics* 169(1), 94–113.
- Bandi, F. M. and R. Renò (2016). Price and volatility co-jumps. *Journal of Financial Economics* 119(1), 107–146.

- Barndorff-Nielsen, O. E., P. Hansen, A. Lunde, and N. Shephard (2009). Realized kernels in practice: trades and quotes. *The Econometrics Journal* 12, C1–C32.
- Barndorff-Nielsen, O. E. and N. Shephard (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of Financial Econometrics* 2(1), 1–37.
- Bates, D. S. (2000). Post-'87 crash fears in the S&P 500 futures option market. *Journal of Econometrics* 94(1-2), 181–238.
- Bates, D. S. (2019). How crashes develop: Intradaily volatility and crash evolution. *Journal of Finance* 74(1), 193–238.
- Bekaert, G. and M. Hoerova (2014). The VIX, the variance premium and stock market volatility. *Journal of Econometrics* 183(2), 181–192.
- Bollerslev, T., J. Litvinova, and G. Tauchen (2006). Leverage and volatility feedback effects in high-frequency data. *Journal of Financial Econometrics* 4(3), 353–384.
- Bollerslev, T., G. Tauchen, and H. Zhou (2009). Expected stock returns and variance risk premia. *Review of Financial Studies* 22(11), 4463–4492.
- Bollerslev, T. and V. Todorov (2011a). Estimation of jump tails. *Econometrica* 79(6), 1727–1783.
- Bollerslev, T. and V. Todorov (2011b). Tails, fears, and risk premia. *Journal of Finance* 66(6), 2165–2211.
- Bollerslev, T. and V. Todorov (2014). Time-varying jump tails. *Journal of Econometrics* 183(2), 168–180.
- Bollerslev, T., V. Todorov, and L. Xu (2015). Tail risk premia and return predictability. *Journal of Financial Economics* 118(1), 113–134.
- Bondarenko, O. (2014a). Variance trading and market price of variance risk. *Journal of Econometrics* 180(1), 81–97.
- Bondarenko, O. (2014b). Why are put options so expensive? *Quarterly Journal of Finance* 4(3), 1450015.

- Broadie, M., M. Chernov, and M. Johannes (2007). Model specification and risk premia: Evidence from futures options. *Journal of Finance* 62(3), 1453–1490.
- Caporin, M., A. Kolokolov, and R. Renò (2017). Systemic co-jumps. *Journal of Financial Economics* 126(3), 563–591.
- Carr, P. and D. Madan (2001). Optimal positioning in derivative securities. *Quantitative Finance* 1, 19–37.
- Cheng, X., E. Renault, and P. Sangrey (2019). Identification robust inference for risk prices in structural stochastic volatility models. *University of Pennsylvania, Working Paper*.
- Dew-Becker, I., S. Giglio, and B. Kelly (2021). Hedging macroeconomic and financial uncertainty. *Journal of Financial Economics* 142(2), 23–45.
- Dew-Becker, I., S. Giglio, A. Le, and M. Rodriguez (2017). The price of variance risk. *Journal of Financial Economics* 123(2), 225–250.
- Drechsler, I. and A. Yaron (2011). What’s vol got to do with it. *Review of Financial Studies* 24(1), 1–45.
- Du, D. (2011). General equilibrium pricing of options with habit formation and event risks. *Journal of Financial Economics* 99(2), 400–426.
- Du, J. and N. Kapadia (2012). The tail in the volatility index. *University of Massachusetts Amherst, Working Paper*.
- Duffie, D., J. Pan, and K. Singleton (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica* 68, 1343–1376.
- Eraker, B. (2004). Do stock prices and volatility jump? Reconciling evidence from spot and option prices. *Journal of Finance* 59(3), 1367–1403.
- Eraker, B., M. Johannes, and N. Polson (2003). The impact of jumps in volatility and returns. *Journal of Finance* 58(3), 1269–1300.
- Eraker, B. and I. Shaliastovich (2008). An equilibrium guide to designing affine pricing models. *Mathematical Finance* 18(4), 519–543.

- Eraker, B. and A. Yang (2022). The price of higher order catastrophe insurance: The case of vix options. *Journal of Finance*, *forthcoming*.
- Ewald, C. and Y. Zou (2021). Stochastic volatility: A tale of co-jumps, non-normality, gmm and high frequency data. *Journal of Empirical Finance* 64, 37–52.
- Hu, G., K. Jacobs, and S. B. Seo (2022). Characterizing the variance risk premium: The role of the leverage effect. *Review of Asset Pricing Studies*, *forthcoming*.
- Jackwerth, J. and G. Vilkov (2019). Asymmetric volatility risk: Evidence from option markets. *Review of Finance* 23(4), 777–799.
- Jacod, J., C. Klüppelberg, and G. Müller (2017). Testing for non-correlation between price and volatility jumps. *Journal of Econometrics* 197(2), 284–297.
- Jacod, J. and V. Todorov (2010). Do price and volatility jump together? *Annals of Applied Probability* 20(4), 1425–1469.
- Kalnina, I. and D. Xiu (2017). Nonparametric estimation of the leverage effect: A trade-off between robustness and efficiency. *Journal of the American Statistical Association* 112(517), 384–396.
- Kozhan, R., A. Neuberger, and P. Schneider (2013). The skew risk premium in the equity index market. *Review of Financial Studies* 26(9), 2174–2203.
- Martin, I. (2017). What is the expected return on the market? *Quarterly Journal of Economics* 132(1), 367–433.
- Pan, J. (2002). The jump-risk premia implicit in options: Evidence from an integrated time-series study. *Journal of Financial Economics* 63(1), 3–50.
- Santa-Clara, P. and S. Yan (2010). Crashes, volatility and the equity premium: Lessons from S&P 500 options. *Review of Economics and Statistics* 92(2), 435–451.
- Schneider, P., C. Wagner, and J. Zechner (2020). Low-risk anomalies? *Journal of Finance* 75(5), 2673–2718.

- Schreindorfer, D. (2020). Macroeconomic tail risks and asset prices. *Review of Financial Studies* 33(8), 3541–3582.
- Seo, S. B. and J. A. Wachter (2019). Option prices in a model with stochastic disaster risk. *Management Science* 65(8), 3449–3469.
- Todorov, V. and G. Tauchen (2011). Volatility jumps. *Journal of Business and Economic Statistics* 29(3), 356–371.
- Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *Journal of Finance* 68(3), 987–1035.
- Wang, C. D. and P. A. Mykland (2014). The estimation of leverage effect with high-frequency data. *Journal of the American Statistical Association* 109(505), 197–215.