Discussion of

The Lead-Lag Relationship between VIX Futures and SPX Futures
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Lead-lag relationship

One market often follows the movements of another with a time delay

- e.g., SPX vs. SPX futures/options; SPX vs. SPX-tracking ETFs

Why do we observe lead-lag patterns?

1. **Information channel**: some markets might reflect information faster

2. **Hedging channel**: market makers’ hedging activities create a cross-market relation

This paper focuses on the **latter channel** between **VIX futures** vs. **SPX futures**

- Very relevant topic with policy implications

- Pitfalls of dealers’ hedging/rebalancing
  - “Gamma traps” (DeSimone, 2020); “Vol-mageddon” (Augustin, Cheng & Van den Bergen, 2021)
Methodology

Ultra-high-frequency data (tick data in milliseconds)

- Main challenge: non-synchronously observed data points
- Solution: correlation curve (Hayashi & Yoshida, 2005; Hoffmann, Rosenbaum & Yoshida, 2013)

An illustrative example (Dao, McGroarty & Urquhart, 2018)

We can measure co-occurrences of market activities instead of price comovements

- CMAT and PCMA (Dobrev & Schaumburg, 2017)
Comment 1: Do we observe a clear lead-lag pattern?

On average, $LLT \approx 0$ and $CMAT \approx 0$

Moreover, $LLC$ (lead-lag strength) becomes high whenever $LLT \approx 0$

- The correlation between the two markets is strongest when $LLT \approx 0$
Comment 2: LLR vs. LLT/CMAT

Hence, the paper’s empirical analysis mainly relies on LLR

On average, LLR < 0, implying that VIX futures lead SPX futures.

- It is a bit self-serving to say that LLR is a “more robust measure of the lead-lag relation” because LLR produces more favorable results.

Can we say VIX futures lead SPX futures because LLR < 0 (when LLT \(\approx 0\))?  

- The authors argue that other market activities “could strengthen VIX futures’ lead while pushing the LLT towards zero”

- It would be nice to elaborate on this, making the mechanism clear.
Comment 3: Cross-market activities

The paper argues that cross-market activities mainly drive the lead-lag pattern

- LLR is regressed on $PCMA_t \cdot 1_{\{CMAT_t \leq 0\}}$ and $PCMA_t \cdot 1_{\{CMAT_t > 0\}}$

- **Main result**: LLR is loaded on $PCMA_t \cdot 1_{\{CMAT_t \leq 0\}}$ with a negative sign

But, CMAT is also a lead-lag measure

\[
\begin{align*}
VIX \text{ futures lag} & \Rightarrow LLR > 0 \quad \text{and} \quad PCMA_t \cdot 1_{\{CMAT_t \leq 0\}} = 0 \\
VIX \text{ futures lead} & \Rightarrow LLR \leq 0 \quad \text{and} \quad PCMA_t \cdot 1_{\{CMAT_t \leq 0\}} > 0
\end{align*}
\]

- Thus, the negative sign might be a **mechanical outcome**

Another issue: LLR is also loaded on $PCMA_t \cdot 1_{\{CMAT_t > 0\}}$ with a negative sign

- This is odd and inconsistent with the paper’s story

- Perhaps, this is why the authors do not regress LLR on PCMA itself?
Hidden premise: VIX futures’ lead comes from hedging activities

- I feel like there is a logical gap here
- What about the volatility feedback effect?

One simple thing to check

- The authors find that uninformed VIX futures trading (e.g., rebalancing of VIX ETPs) does affect SPX futures’ prices
- Do we observe a stronger lead-lag relationship on those rebalancing days?
Conclusion

• Interesting paper!

• Understanding the relation between the two markets is important

• The paper will benefit from a consistent story that can tie things together

• I am looking forward to reading the next version!