The Lead-Lag Relationship between VIX Futures and SPX Futures

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Preliminary draft

Abstract

We study the lead-lag relationship between VIX futures and SPX futures on a sample of high-frequency data using the cross-correlation function. The analysis reveals large time-variation in the lead-lag relation. We find that cross-market activity explains a major part of the lead-lag relation and that days of high activity are associated with a strengthened VIX futures lead over SPX futures. As VIX futures hedging by dealers generate cross-market activity, this indicates that the strengthening of the VIX futures leadership could be partly explained by dealers' hedging activities. We also find evidence that the hedging channel can move the SPX futures market for reasons unrelated to price discovery.

Keywords: Lead-lag relation; volatility hedging; price impact; high-frequency data; cross-correlation; cross-market activity; informed trading.

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1 Introduction

The market for VIX futures has witnessed an impressive growth since the introduction of the first VIX futures contract in 2004 on the Chicago Board of Options Exchange (CBOE) and the VIX index itself has become a widely recognized yardstick of stock market risk. Since their launch, VIX futures gained popularity as tools to hedge volatility exposure or diversify portfolios (Whaley, 2009). In 2009, the first VIX exchange-traded product (VIX ETP) hit the market and, since then, investors have increasingly used products tied to VIX futures to speculate in future volatility outlook (Bollen et al., 2017; Bhansali and Harris, 2018). Typically, major dealers in financial markets take the other side of the VIX futures trade. Market makers and dealers are subject to strict risk requirements and profit from their flow of transactions and not from risk taking. In order to hedge their positions in volatility, dealers typically employ various options based hedging strategies (Chang, 2017). The dynamic nature of these strategies entail that in order to maintain the hedge after either a change in volatility or a change in their net position in volatility, the dealers have to trade the underlying index. Hence, the mechanisms of the hedging activities have the potential to impact the lead-lag relation between VIX futures and SPX futures. The observation that hedging activities can impact the lead-lag relation between two markets if market makers in one market trade in the other market to hedge is not new. The delta hedging by option dealers is an important example of this (Easley et al., 1998; Chan et al., 2002; Schlag and Stoll, 2005).

In this paper we study the lead-lag relation between VIX futures and SPX futures on a sample of transactions time-stamped down to the millisecond and collected over the period from January 2013 to September 2020. We find that the lead-lag relation is dynamic with the VIX futures leading SPX futures on average terms. We study the determinants of the lead-lag relation and find that the level of cross-market trading has a positive and significant impact on the strength of the VIX futures lead over SPX futures. Since cross-market trading can arise from hedging by VIX futures dealers, this can be an indicator that VIX futures hedging influences the lead-lag relationship. In addition, we estimate the aggregate net gamma position of market makers and find that a negative gamma position strengthens the VIX futures lead while a positive gamma position weakens the lead. However, after controlling for the cross-market trading the effect of the gamma position almost disappear. The literature on lead-lag relations is mostly concerned with which markets first reflect new information (where informed traders prefer to trade), and hedging can be a channel through which information is transmitted to the market for the asset that is used as hedging instrument (see e.g. Kaul et al. (2004); Hu (2014)). However, a lead-lag relation generated by hedging activities does not necessarily reflect the diffusion of information. Rather it might materialize if VIX futures market makers' hedging activities have market impact on the instrument used for hedging. To discriminate between the two hedging-based explanations of the lead-lag relation, we exploit the presence of uninformed VIX futures trading by VIX ETPs to analyze the nature of the VIX futures hedging channel. We find that the trades move the SPX futures market in the direction expected under the VIX futures hedging mechanisms. Since the trades in VIX futures performed by the VIX ETPs are uninformed, this indicates a market impact which is driven by factors other than the transmission of information across markets.

Besides the hedging-based explanations, we also connect the strength of the lead-lag relation to other more classical variables such as stock market returns, the volatility of the stock market, and the level of liquidity in the two futures markets. The SPX index returns and the level of the VIX index have no clear impact on the lead-lag relation when controlling for a set of other variables. Moreover, and in line with a price discovery channel, we find that an increase (decrease) in the liquidity of VIX futures relative to the liquidity of the SPX futures results in a strengthening (weakening) of the VIX futures lead but again the explanatory power appears to vanish once the cross-market trading is accounted for.

The findings have relevance for policy makers who are concerned about market fragility. If hedging activities of VIX futures dealers have the potential to destabilize or magnify market drops it is vital for regulators to understand the driving mechanisms behind the price movements. In fact, regulators are starting to worry that, due to the rebalancing of the hedge ratios of dealers, a sudden increase in volatility or a drop in the underlying can trigger a sell-off of the underlying that magnifies the market decline (Bank for International Settlements, 2018). The market movements on February 5 2018 serve as anecdotal evidence in support of this belief. On this day the VIX spiked and incurred the largest relative move of 116% since its inception in 1993. The sharp increase in the VIX was accompanied by a drop to the SPX index of 4.1%. These movements materialized despite the fact that no clear macro-economic event occurred (Augustin et al., 2021).

Evidence of a transitory price impact from hedging activities has also been found for other markets. For instance, Kao et al. (2018) find that VIX option trading has a temporary impact on changes in the VIX index and attribute this to the use of VIX options for hedging by SPX option market makers. The research is also related to the literature on stock-pinning, namely the phenomenon that option dealers' hedging demands can change the dynamics of the price of the underlying security. This is shown by Avellaneda and Lipkin (2003); Ni et al. (2005); Golez and Jackwerth (2012), who find that if the open interest of an option for a given strike is sufficiently large, delta hedging of dealers can push the stock price to the strike price of the option at the option expiration.

The evidence on the leadership of the VIX futures may be somewhat surprising in light of the large and important SPX futures market. Nevertheless, the findings are consistent with other research showing that VIX futures plays a dominant role in relation to other SPX-related markets. In particular, several studies show that the VIX futures lead the VIX index (Shu and Zhang, 2012; Frijns et al., 2016; Bollen et al., 2017; Chen and Tsai, 2017; Kao et al., 2018), which by construction of the VIX index translates into VIX futures leading SPX options.[1]

The paper is also related to the literature on the relation between lagged stock market returns and volatility: Carr and Wu (2006) study the cross-correlation function for SPX index returns and VIX index changes at a daily frequency and find marginal evidence of SPX returns having some predictive power for VIX index changes. Similarly, Bollerslev et al. (2006) find significant negative correlation between the absolute value of SPX returns (volatility proxy)

[1]The mentioned papers interpret this in terms of greater information content in VIX futures as opposed to the VIX index. Although not being the scope of this paper, we note here that the VIX futures lead can also be the consequence of dealers hedging their VIX futures exposure in the SPX options markets.

and lagged SPX returns both sampled at a five-minute frequency. At the same time, correlations between returns and lagged absolute returns are close to zero. The same pattern is found by Bollerslev et al. (2012) using the squared VIX index as the volatility measure. Their results shed light on the relevance of the leverage and volatility feedback effect as two competing explanations for the relation between returns and volatility.

A large number of studies on lead-lag relations among SPX and its derivatives already exist. The findings of Frijns et al. (2016) reveal that VIX futures lead the SPX index. While it has also been shown that SPX futures lead the SPX index (Chu et al., 1999; Hasbrouck, 2003), these findings leave it unclear to which extend VIX futures lead or lag SPX futures. As mentioned above, there is also evidence that VIX futures lead the VIX index which is equivalent to leading SPX options. Moreover, it has been shown by Chen et al. (2016) that SPX futures provide greater contribution to price discovery than SPX options. Thus, both VIX futures and SPX futures lead SPX options, but again the VIX futures and SPX futures lead-lag relation cannot be inferred from these studies. While Lee et al. (2017) show that the VIX futures basis (the difference between VIX futures and the VIX index) has some predictable power for the SPX futures returns, the information contained in the VIX futures price and VIX futures basis may be distinct. We therefore contribute to the literature on lead-lag relations among the SPX-related markets by providing the first evidence on the lead-lag relation between the VIX futures and SPX futures market. Contrary to Lee et al. (2017) using daily data, we analyze the lead-lag relation from high-frequency data which allows for examining its time-variation.

The remainder of the paper is structured as follows: First, Section 2 details the channels through which VIX futures hedging can impact SPX futures. Next, we introduce the methodology to quantify the lead-lag relation in Section 3. Section 4 presents the data and the results of the empirical analysis. Finally, Section 5 concludes.

2 Hedging induced market spillovers

Market makers in VIX futures typically risk manage their volatility exposure by trading other volatility sensitive products such as European options. For instance, a new position in a VIX futures (VX) can be hedged with a delta-hedged position in a European option on the SPX index. Hence, trading in VIX futures leads to subsequent trading in the underlying index, which in the context of VIX futures, is typically carried out via SPX futures (ES). Consider a dealer with a short position in a VIX futures with price P_t^{VX}. The dealer hedges by buying options on the SPX index while delta hedging using SPX futures. Denote by P_t^{ES} the price of the SPX futures used for hedging and by V_t its instantaneous volatility. In order to obtain zero sensitivity to changes in the SPX futures (i.e. a delta neutral hedge) to the short VIX futures position, the dealer invests X_t^Q in options (either calls or puts) with price P_t^Q and X_t^{ES} in the underlying SPX futures simultaneously. Hence, at any given point in time t the dealer faces the two equations:

$$-\frac{\partial P_t^{VX}}{\partial V_t} + \frac{\partial P_t^Q}{\partial V_t} X_t^Q = 0 \tag{1}$$

$$X_t^{ES} + \frac{\partial P_t^Q}{\partial P_t^{ES}} X_t^Q = 0. \tag{2}$$

Solving equation (1) reveals that to obtain zero sensitivity to changes in volatility, the amount of options the dealer has to enter equals $X_t^Q = \frac{\partial P_t^{VX}/\partial V_t}{\partial P_t^Q/\partial V_t} > 0$. Solving (2) reveals that $X_t^{ES} < 0$ if call options are used to hedge the volatility exposure and $X_t^{ES} > 0$ if put options are used.

Moreover, as providers of liquidity to SPX options demand, dealers tend to be short in put options and long in call options (Garleanu et al., 2008; Goyenko and Zhang, 2019). High volatility scenarios are typically associated with a distressed stock market where end user demand in long put options is high. Hence, the aggregate dealer position in gamma tend to follow the business cycle in the sense that it is negative in downturns and positive in upturns (Baltussen et al., 2021). For this reason, market makers run the risk of being part of the feedback cycle illustrated in Figure 1. An increase in volatility due to increased demand in long VIX futures

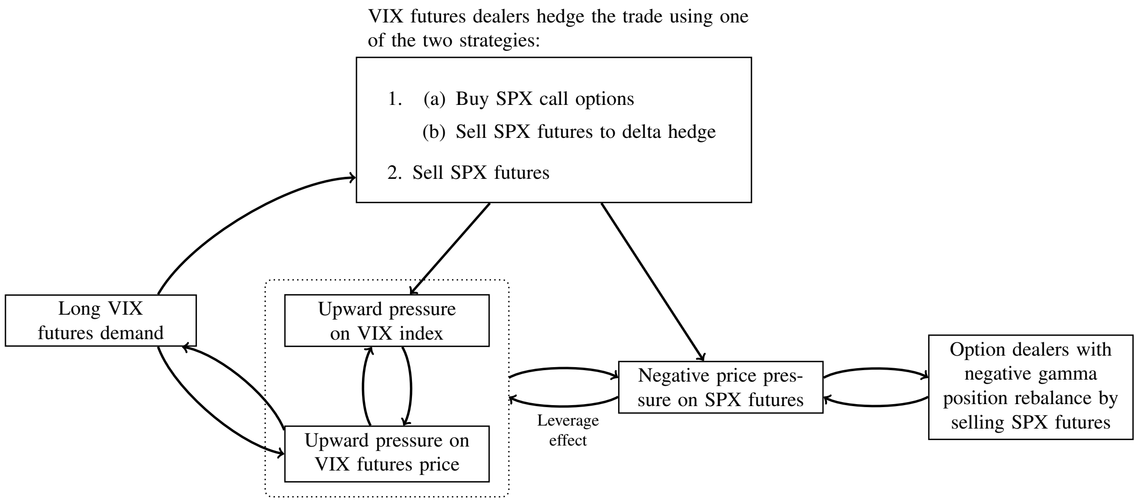

Figure 1: Feedback effect from initial long demand in VIX futures.

leads to a decrease in SPX futures prices via two channels: First, the long VIX futures demand translates into a short VIX futures position of the dealer which has to be hedged in the options market. Being a short volatility position the dealer has to hedge with a long option position. In principle, the dealer can enter into put or call options, however, in practice the dealer has to buy call options since this matches the short position of the end user demand. In addition, the positive delta of the call position has to be hedged by a short SPX futures position. Alternatively, dealers could turn to a more approximate hedge utilizing the negative correlation between VIX futures and SPX futures prices. Under this approach, long demand in VIX futures (short position of the dealer) is simply hedged by a short position in SPX futures. In either case, this result in negative price pressure on SPX futures. Second, the increase in volatility also impacts the SPX futures price negatively via the leverage effect. Being in a negative gamma position, the dealers sell additional SPX futures in order to rebalance hedges. This puts additional price pressure on SPX futures, which again translates into increasing VIX futures prices. The full feedback cycle materializes as increasing volatility and VIX futures prices generate additional VIX futures demand which creates more VIX futures hedging activity.

In terms of the lead-lag relation between VIX futures and SPX futures, Figure 1 illustrates how hedging activities may influence this relation. The long VIX futures demand triggers SPX futures selling by VIX futures dealers. In turn, this forces option dealers to rebalance. Hence, the initial impact from VIX futures hedging may generate further SPX futures selling when option dealers are negative gamma. For this reason the negative gamma position of option dealers is a potential amplifier of the VIX futures hedging mechanism's impact on the lead-lag relation. However, even for the case when the net gamma position is positive, it can still be the case that the VIX futures dealer's hedging activities drive the lead-lag relation in the direction of a stronger VIX futures lead via the channels depicted in the upper part of Figure 1. On the other hand, it can also be the case that SPX futures lead VIX futures via the leverage effect. Hence, it is an empirical question which of the channels dominate.

3 Lead-lag methodology

Many studies on lead-lag relations are concerned with assets that are closely linked together such that a cointegrating relation of the prices can be assumed. In those settings, the information share (Hasbrouck, 1995) or the common factor component weight approach (Gonzalo and Granger, 1995) are often applied. However, it is inappropriate to impose the assumption of cointegration when describing the relation between VIX futures and SPX futures. Instead we analyze the lead-lag relationship through the cross-correlation function and the cross-market activity measure. In section 3.1 we describe how to obtain the cross-correlation function using the techniques of Hayashi and Yoshida (2005); Hoffmann et al. (2013), and section 3.2 presents three different quantifications of the lead-lag relation based on the cross-correlation function. Finally, Section 3.3 presents the methodology behind the cross-market activity measure of Dobrev and Schaumburg (2017).

3.1 Estimation of the cross-correlation function

Based on Hayashi and Yoshida (2005) and Hoffmann et al. (2013), the cross-correlation for two assets, A and B, is estimated as

$$\hat{\rho}_{HY}(\vartheta) = \frac{\sum_{i=1}^{n_A} \sum_{j=1}^{n_B} \Delta_{t_i^A} X^A \Delta_{t_j^B} X^B 1_{\left\{(t_{i-1}^A, t_i^A] \cap (t_{j-1}^B - \vartheta, t_j^B - \vartheta] \neq \emptyset\right\}}}{\sqrt{\sum_{i=1}^{n_A} \left(\Delta_{t_i^A} X^A\right)^2} \sqrt{\sum_{j=1}^{n_B} \left(\Delta_{t_j^B} X^B\right)^2}} \tag{3}$$

which we shall refer to as the HY estimator. Here $\Delta_{t_i^k} X^k = X_{t_i^k}^k - X_{t_{i-1}^k}^k$ is the log-return of asset k, and $i = 1, \ldots, n_k$ meaning that the returns entering in equation (3) are computed between each single tick, t_i^k. The product of two returns is included in the sum whenever the time intervals over which the returns are realized is overlapping. Inspired by Dao et al. (2018), we use Figure 2 to illustrate this. By focusing on the first two line segments, we ignore the possibility of shifting the time-stamps so $\vartheta = 0$. As an example, considering the three intervals of asset B, J_1, J_2 and J_3. J_1 intersects with I_2, J_2 intersects with I_2, while J_3 intersects with I_2, I_3 and I_4. Thus, the contribution to the sum based on each of the three intervals is $\Delta_{t_1^B} X^B \Delta_{t_2^A} X^A$, $\Delta_{t_2^B} X^B \Delta_{t_2^A} X^A$ and $\Delta_{t_3^B} X^B (\Delta_{t_2^A} X^A + \Delta_{t_3^A} X^A + \Delta_{t_4^A} X^A)$, respectively. Hence, it is possible that the same return can contribute to the sum more than once as it will enter every time the interval intersects with one of the intervals of the other asset. Note that when implementing equation (3), the return over I_1 will not influence the correlation as the indicator function equals zero for intervals that do not intersect with any of the intervals of the other asset.

Repeatedly adjusting the time-stamps of the second asset by different values of ϑ, allows us to compute the cross-correlation function. The shift of the time-stamps is illustrated in the lower part of Figure 2. Note that only the time-stamps of one of the two assets are shifted while the time-stamps of the other remain fixed. The returns $\Delta_{t_i^k} X^k$ for $k = A, B$ are invariant to the shift of the time-stamps meaning that only the indicator function changes as ϑ changes. Thus, it is the same returns that enter equation (3) for each ϑ but they are multiplied and summed in different ways.

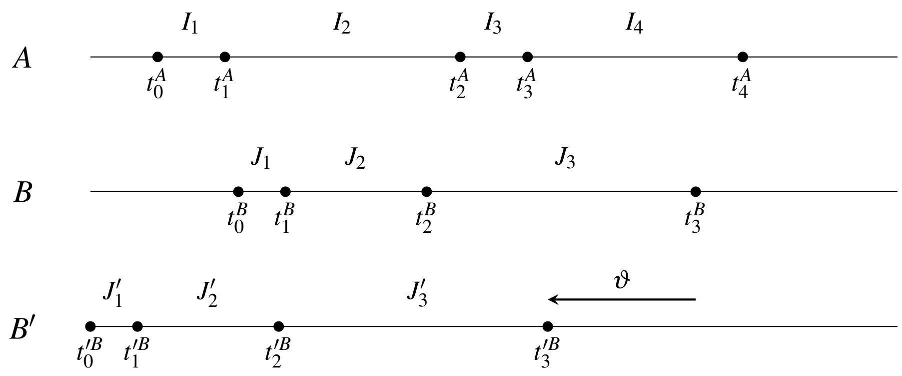

Figure 2: Illustration of the time-stamp adjustment for the cross-correlation functions.

3.2 Lead-lag time, lead-lag correlation and lead-lag ratio

To measure the lead-lag relation between two assets, Hoffmann et al. (2013) define the lead-lag time (LLT) as the value of ϑ that maximizes the absolute value of the cross-correlation function, $|\hat{\rho}_{HY}(\vartheta)|$, across all ϑ on some grid. If the absolute correlation is maximized at a point $\vartheta \neq 0$ then one asset is leading the other. Under certain assumptions, the point is a consistent estimator of the true LLT (Hoffmann et al., 2013).

While LLT measures the amount of time by which an asset leads the other, knowledge of the value of the cross-correlation function at the point corresponding to LLT is also informative about the nature of the lead-lag relation. The value of the cross-correlation at this point is referred to as the lead-lag correlation (LLC) (Dao et al., 2018).

Both LLT and LLC focus on a single point of the cross-correlation function. However, the rest of the cross-correlation function also contains relevant information about the strength of the lead-lag relation. The lead-lag ratio (LLR) of Huth and Abergel (2014) accounts exactly for this by compressing the entire cross-correlation function into a single measure of the lead-lag relation. Considering all the positive time-stamp adjustments $(\vartheta_1, \ldots, \vartheta_p)$, LLR is defined as

$$LLR = \frac{\sum_{i=1}^{p} \hat{\rho}_{HY}^2(\vartheta_i)}{\sum_{j=1}^{p} \hat{\rho}_{HY}^2(-\vartheta_j)}. \tag{4}$$

The ratio captures the relative forecasting ability of one asset over the other. When LLR > 1 it means that the correlations at positive lags are overall larger than the correlations at negative lags. Thus, the asset for which the time-stamps are kept fixed will lead the asset for which the time-stamps are adjusted (asset A will lead asset B in Figure 2). The conclusion of the leadership is the opposite if LLR < 1 where the asset with fixed time-stamps lags the other (asset B leads asset A). Compared with LLT, LLR takes into account the overall predictive power of the returns of one asset on the returns of the other asset by summing the squared correlations. The conclusion on the lead-lag relation drawn from LLT is generally more sensitive to the shape of the cross-correlation function as small variations in its shape could shift LLT from a positive to negative value and vice versa. LLT may also have limited ability to capture differences in the strength of the leadership that may exist even when two LLTs are close to each other. For instance, consider the two cases illustrated in Figure 3 where LLT and LLC are the same but the behavior of the cross-correlation function to the right of LLT is very different. In the plot to the left, the cross-correlation function goes to zero very quickly while in the plot to the right, the cross-correlation slowly decays to zero for values of ϑ higher than LLT. Hence, the lead-lag relation depicted in the plot to the left is much stronger than the one to the right. Still LLT and LLC will be identical and only LLR will capture this difference in the strength of the lead-lag relation. LLR could in fact lead to a different conclusion on the lead-lag relation than LLT. In order to strengthen the robustness of our results, LLR is therefore considered an important measure of the lead-lag relation in the following analysis.

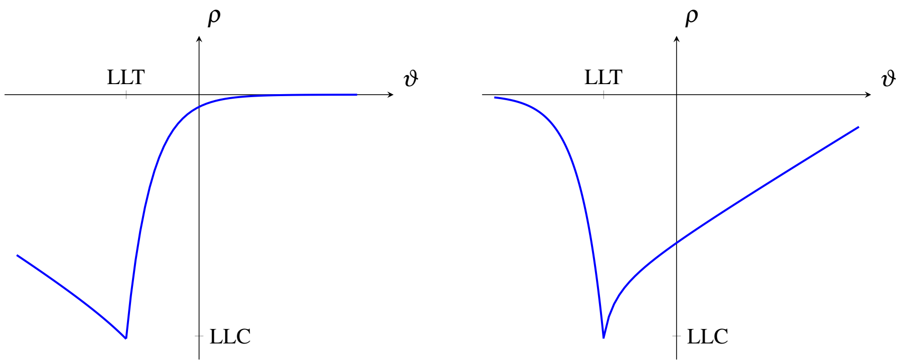

Figure 3: Illustration of cross-correlation functions.

3.3 Measuring cross-market activity

While the lead-lag measures of Section 3.2 are based on prices, we here present another measure of the lead-lag relation based on Dobrev and Schaumburg (2017) which is model-free and does not utilize prices. Instead the time-stamps of a well-defined activity, such as trading are used. The idea is to identify all so-called active time-stamps. A time-stamp is active if the specified activity takes place at that time-stamp. The total number of time-stamps with simultaneous activity is then summed over the trading day and shows how often both markets are active at the same time. Assuming that $\vartheta = 0$, this number can be obtained as

$$X_{\vartheta}^{raw} = \sum_{i=|\vartheta|+1}^{N-|\vartheta|} 1_{\{\text{market } A \text{ active in period } i\} \cap \{\text{market } B \text{ active in period } i+\vartheta\}} \tag{5}$$

where N is the total number of time-stamps. With data at millisecond frequency, this is the total number of milliseconds over the trading day. The number of cross-active time-stamps can be scaled by the total number of active time-stamps in the least active market to measure cross-market activity as a proportion of the total activity

$$X_{\vartheta}^{rel} = \frac{X_{\vartheta}^{raw}}{\min\left\{\sum_{i=|\vartheta|+1}^{N-|\vartheta|} 1_{\{\text{market } A \text{ active in period } i\}}, \sum_{i=|\vartheta|+1}^{N-|\vartheta|} 1_{\{\text{market } B \text{ active in period } i+\vartheta\}}\right\}}. \tag{6}$$

In order to capture only the activity which has a cross-market dimension, a further adjustment is implemented to account for the simultaneous activity which would occur simply by randomness. This gives cross-market activity in excess of what would be expected by coincidence given that activity in the two markets is independent of each other. It is defined as

$$X_{\vartheta} = X_{\vartheta}^{rel} - X_{\infty}^{rel} \tag{7}$$

where the adjustment term is defined as $X_{\infty}^{rel} = 1/\left(2\left(T_2 - T_1\right)\right)\sum_{|\vartheta|=T_1+1}^{T_2} X_{\vartheta}^{rel}$ for sufficiently large $T_2 > T_1$. In addition to simultaneous activity where $\vartheta = 0$, time-stamps are shifted forward or backward in time when considering non-zero values of ϑ. For a set of different values of

ϑ, a full curve for the proportion of cross-market activity can be obtained. We illustrate this in Figure 4, and denote the value of the time-stamp shift corresponding to the maximum of the curve by the cross-market activity time (CMAT), while the peak cross-market activity (PCMA) is the value of the cross-market activity, X_{ϑ}, at the point.

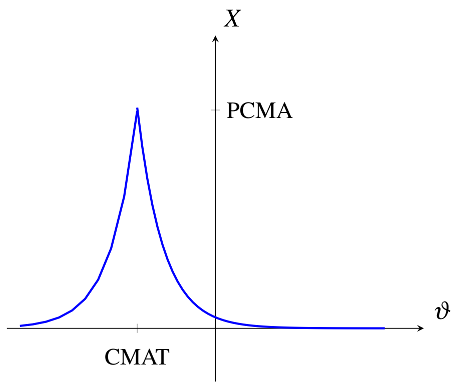

Figure 4: Illustration of cross-market activity function.

4 Empirical analysis

In this section, we detail the empirical analysis of the paper. First, Section 4.1 describes the data used for the analysis. Next, Section 4.2 presents the results on the overall lead-lag relationship between VIX futures and SPX futures. Section 4.3 shows the results of the cross-market activity analysis. In Section 4.4, we introduce the regression model for the dynamics of the lead-lag relation and present the regression results. Finally, Section 4.5 examines if VIX futures hedging impacts the SPX futures market through an uninformed channel.

4.1 Data

For the analysis we collect data over the period from January 2013 to September 2020. Tick-by-tick trade data on E-mini S&P 500 futures and VIX futures are obtained from the Tick Data database. For each sample date, the VIX futures contract used for the analysis is the one closest to an expiry of 30 days and the SPX futures contract used is the one closest to

Table 1: Descriptive statistic of VIX futures and SPX futures markets

		VIX futures	SPX futures
Trading volume	Mean	78,224	1,292,320
	Median	68,311	1,199,596
	Min	18,432	286,808
	Max	386,637	3,983,301
	Std. dev.	43,221	465,899
Dollar volume (in mm)	Mean	1,355	149,368
	Median	1,113	136,824
	Min	280	29,911
	Max	10,365	567,136
	Std. dev.	941	60,032

Statistics are computed from daily observations over the sample period. For each sample date, the VIX futures contract used is the one with an expiry closest to 30 days. The SPX futures contract is the one closest to expiration except when this is less than six days. Trading and dollar volume are obtained over the time interval 9:30-16:15 EST.

Table 2: VIX exchanged-traded products

Ticker	Name	Leverage	First day of trading	Last day of trading
VXX	iPath Series S&P 500 VIX Short Term Futures ETN	1	20090130	
VIXY	ProShares VIX Short-Term Futures ETF	1	20110104	
VIIX	VelocityShares Daily Long VIX Short-Term ETN	1	20101130	20200702
VMAX	REX VolMAXX Long VIX Futures Strategy ETF	1	20160503	20180724
UVXY	ProShares Ultra VIX Short-Term Futures ETF	2[a]	20111004	
TVIX	VelocityShares Daily 2x VIX Short-Term ETN	2	20101130	20200702
IVO	iPath Inverse S&P 500 VIX Short-Term Futures ETN	-1	20110114	20110916
IVOP	iPath Inverse S&P 500 VIX Short Term Futures ETN	-1	20110919	20180323
SVXY	ProShares Short VIX Short-Term Futures ETF	-1[b]	20111004	
XIV	VelocityShares Daily Inverse VIX Short-Term ETN	-1	20101130	20180215
VMIN	REX VolMAXX Short VIX Futures Strategy ETF	-1	20160503	20181126

[a] The target leverage was changed from 2 to 1.5 as of February 28, 2018.
[b] The target leverage was changed from -1 to -0.5 as of February 28, 2018.

expiry except when time to expiry is less than six days where we shift to the next contract. We focus only on trades during the regular trading hours of VIX futures, 9:30-16:15 EST. Any date where the exchanges closed earlier is removed from the sample. Trades with a negative price are removed. For the purpose of computing the lead-lag measures of Section 3, trades sharing the same time-stamp are replaced by a single trade with a price equal to the median price of the trades. Table 1 shows trading and dollar volume for VIX futures and SPX futures. Clearly, SPX futures are more heavily traded than the VIX futures both when measured in terms of trading and dollar volume.

Time series of the SPX index, the VIX index, and VIX futures closing prices and open interest are collected from the CBOE homepage. Dates of scheduled release of information on the U.S. Consumer Price Index, Producer Price Index, Employment Situation or Gross Domestic Product are obtained from Archival Federal Reserve Economic Data (ALFRED). The net gamma position of dealers is estimated utilizing daily options data provided by OptionMetrics. Daily data on the assets under management (AUM) and prices of VIX ETPs are obtained from Bloomberg. Table 2 contains a list of the VIX ETPs included in the sample.

4.2 The lead-lag relationship based on the cross-correlation function

In this section, we present the overall results on the lead-lag relation based on the measures of the lead-lag relation detailed in Section 3.1 and 3.2. For the computation of cross-correlations, we keep the time-stamp of the SPX futures trades fixed and shift the time-stamps of the VIX futures trades. To estimate the cross-correlation function, the grid of ϑ is chosen such that it is finer around zero and less dense as we move away from zero. This is since we expect that the lead-lag time to be small so we want to be able to capture variations in the correlation at a higher detail around zero. Hence, the grid is chosen as

$$-60,-59.9,\ldots,-1.1,-1,-0.99,\ldots,-0.11,-0.1,-0.099,\ldots,-0.001,0,0.001,\ldots,0.099,0.1,0.11,\ldots,0.99,1,1.1,\ldots,59.9,60$$

where the numbers are in seconds and where the largest value ($\pm$60 seconds) reflect the maximum allowed lead-lag. When the grid is most narrow, the length between two grid points is one millisecond which corresponds to the precision at which the trades are measured.

Figure 5 depicts the median of the cross-correlation. The peak of the function is around a lag of zero and the cross-correlation function is close to zero for lags greater than approximately 20 seconds in absolute value. Zooming in on lags within 2 seconds, we observe a skewed shape of the cross-correlation function with more weight on the left part of the curve. On average, this translates into a LLR measure less than 1. Hence, on average VIX futures lead the SPX futures.

Figure 6 shows the time series of the three lead-lag measures of Section 3.2 together with time series of the VIX index and the SPX index. The shaded areas of the chart represent the dates with a VIX level belonging to the 60% upper quantile corresponding to values above

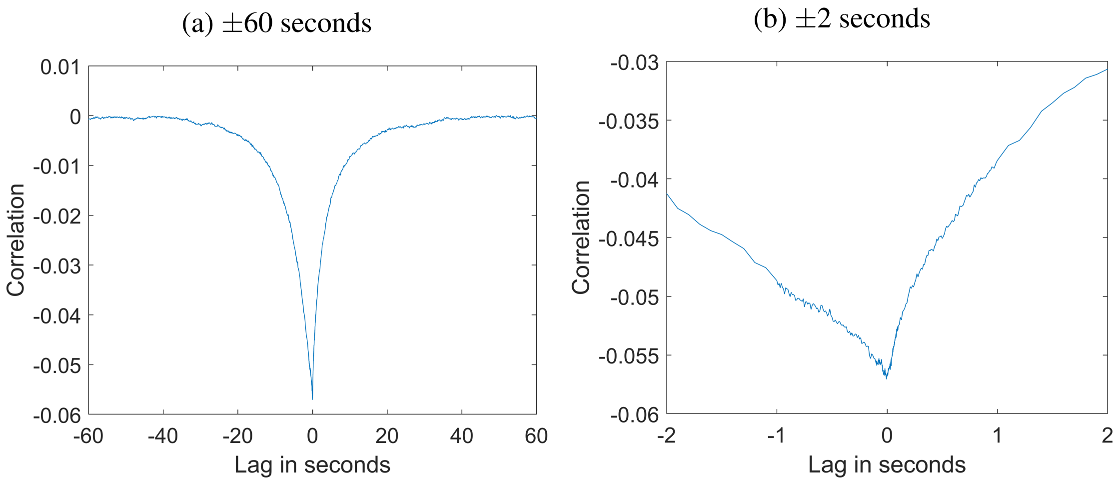

Figure 5: Median cross-correlation functions.

15.14%. Inspection of the upper panel of Figure 6 confirms what Figure 5 indicates, namely that on most dates the VIX futures lead the SPX futures (LLR less than one). We also notice some interesting features of the lead-lag measures on dates of high volatility: First, LLR seems to be more stable and around a level of approximately 0.8. Second, LLT is erratic when volatility is low while close to zero in high volatility regimes. Third, LLC gets more pronounced when volatility is high.

Table 3 reports descriptive statistics on the three lead-lag measures. Based on the observations connected to Figure 6, we also compute the statistics conditioned on the volatility being in the upper quartile. Additionally, we split the sample into the period before and after the beginning of the covid-19 crisis using February 20, 2020 as the cut-off date. In terms of the mean and median values, the LLR measure is relative stable across all four samples. However, the variability in the LLR measure is much lower conditioned on the volatility being high. Considering LLT, the picture is more extreme. On the full sample, LLT varies in the interval $[-60 \text{ seconds}, 60 \text{ seconds}]$ but with an average/median value of -0.63/-0.01 seconds. In comparison and on the high volatility sample, LLT is much more concentrated around 0 with a minimum value of -2.90 seconds and a maximum value equal to 0.66 seconds. Focusing on the two sub-periods defined by the onset of the covid-19 crisis, it seems that the covid-19 period is associated with a much tighter lead-lag time and with much lower variability in all the lead-lag measures. However, we note that with respect to the LLT measure, the covid-19 period differs

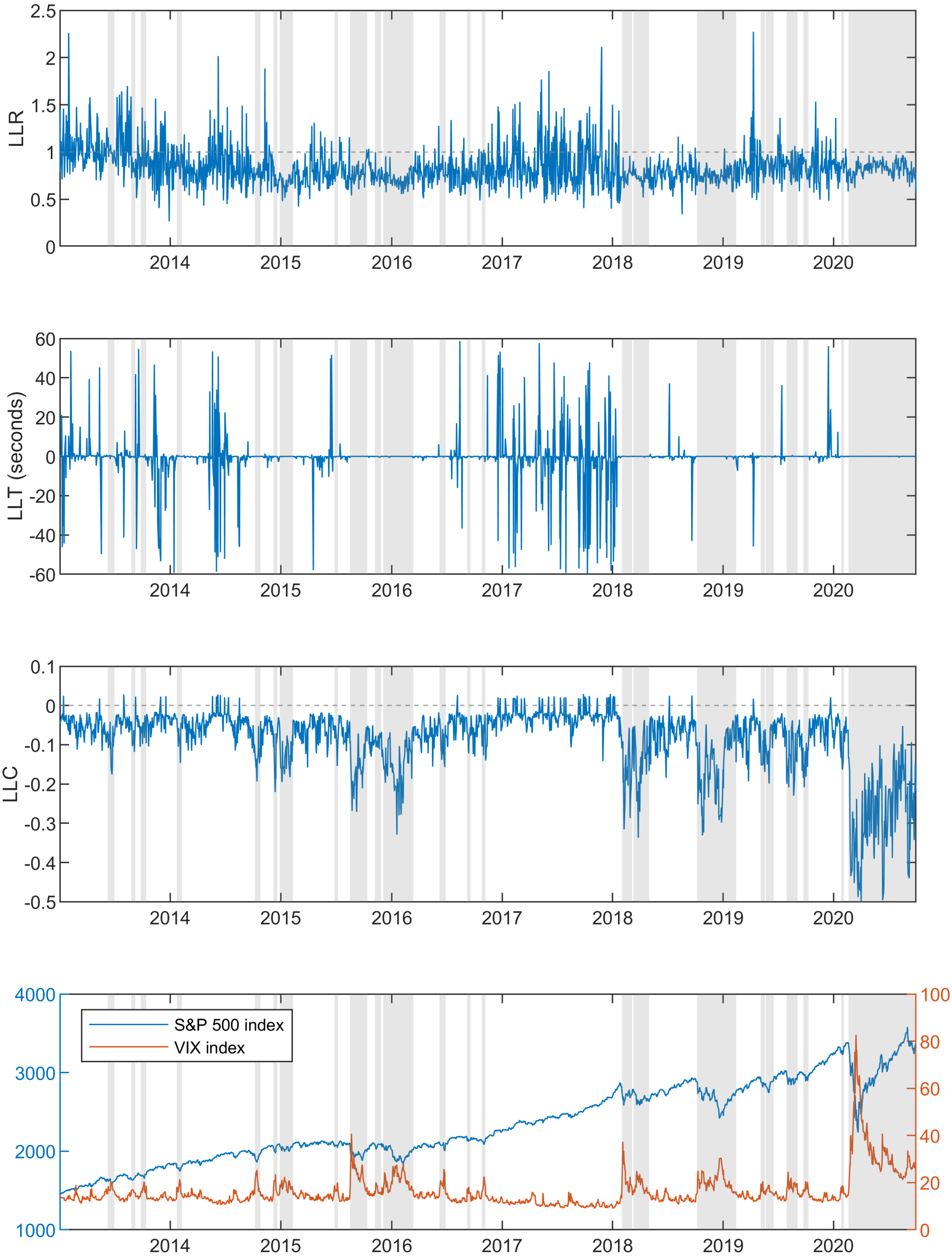

Figure 6: LLR, LLT, LLC and VIX and SPX index over the sample period. The shaded areas represent the dates with a level of the VIX index belonging to the 60% upper quantile.

Table 3: Descriptive statistic of lead-lag measures.

	Full sample			VIX upper quartile			Before covid-19			Covid-19 period		
	LLR	LLT	LLC	LLR	LLT	LLC	LLR	LLT	LLC	LLR	LLT	LLC
Mean	0.84	-0.63	-0.09	0.78	-0.01	-0.19	0.84	-0.68	-0.07	0.82	0.01	-0.28
Median	0.80	-0.01	-0.06	0.77	0.00	-0.17	0.80	-0.02	-0.06	0.83	0.02	-0.27
Min	0.27	-59.70	-0.50	0.51	-2.90	-0.50	0.27	-59.70	-0.34	0.57	-0.59	-0.50
Max	2.27	58.70	0.03	1.16	0.66	-0.02	2.27	58.70	0.03	0.97	0.04	-0.05
Std. dev.	0.20	10.55	0.08	0.10	0.16	0.09	0.21	11.00	0.06	0.08	0.05	0.10

The covid-19 period covers all sample dates after February 20, 2020.

from the sample conditioned on a high volatility level, where the measure based on the former sample is on average slightly positive while slightly negative in the latter.

We illustrate in Figure 7 kernel densities of the three measures of lead-lag strength computed for the sample conditioned on the VIX index being above and below its upper quartile. The figure confirms the findings reported in Table 3. When the level of volatility is high, the densities associated with LLT and LLR are more peaked with the mass more concentrated around the mean of the distribution. Focusing on the third chart, we see that LLC gets more pronounced in high volatility regimes in comparison to regimes with low VIX index values. Hence, the lead-lag relation is strengthened (measured by LLC and LLR) but is short-lived during high volatility (measured by LLT). A similar result is found in Buccheri et al. (2021), where the lead-lag correlation is found to strengthen among stocks when volatilities are high, while being more erratic in low volatility regimes. Other studies find that correlations at a daily frequency tend to increase between the VIX index and the SPX index when market movements are big (Cont and Kokholm, 2013; Todorov and Tauchen, 2011). The connection between the lead-lag relationship and the level of volatility can possibly be prescribed to different types of trading. First, in the context of high-frequency observations, Buccheri et al. (2021); Zhang (2010); Dobrev and Schaumburg (2017) argue that the relation between high-volatility and lead-lag relationships can be ascribed to high-frequency traders exploiting statistical dependencies across markets appearing when the volatility is high. In relation to VIX futures and SPX futures, this means that stronger negative correlation in periods of high volatility is possibly exploited by high-frequency traders reducing LLT to almost zero. Alternatively, in high

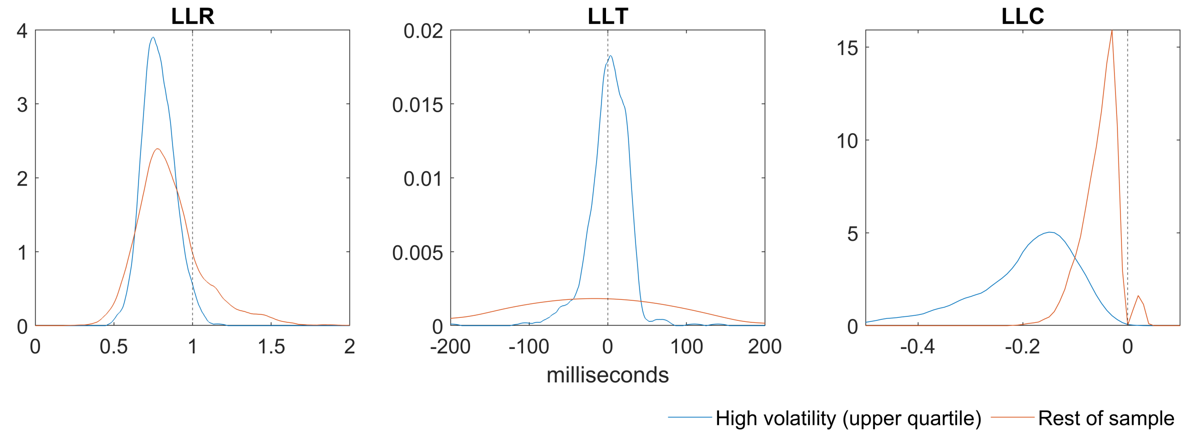

Figure 7: Kernel densities of lead-lag measures.

volatility regimes the demand for long positions in volatility tend to be high, with resulting high level of VIX futures hedging activities performed by market makers. These hedging activities could strengthen the VIX futures lead while pushing LLT towards zero.

4.3 Cross-market trading analysis

In this section, we quantify the cross-market activity described in Section 3.3 by using trading as the activity of interest. The measure is computed with a time-shift, ϑ, within $[-1000, 1000]$ where increments are of one millisecond and with $T_1 = 500$ and $T_2 = 1000$ as in Dobrev and Schaumburg (2017). For each sample date, Figure 8 plots the millisecond at which the cross-market activity peaks (CMAT) and the value of cross-market activity at the peak (PCMA) together with the VIX and SPX index. From the third panel showing PCMA, we see a clear connection with the level of the VIX index as the peak of cross-market activity increases during periods of high VIX indicated by the shaded areas. PCMA reaches its highest level at the beginning of the covid-19 pandemic. To the extend that cross-market activity is driven by high-frequency trading, the relation between cross-market activity and VIX is consistent with a feedback effect between volatility and high-frequency trading described by Dobrev and Schaumburg (2017). Heightened levels of volatility attracts more high-frequency trading and the increased presence of high-frequency traders generates even higher levels of volatility. Comparing this with LLC in Figure 6, we see that they appear be to negatively re-

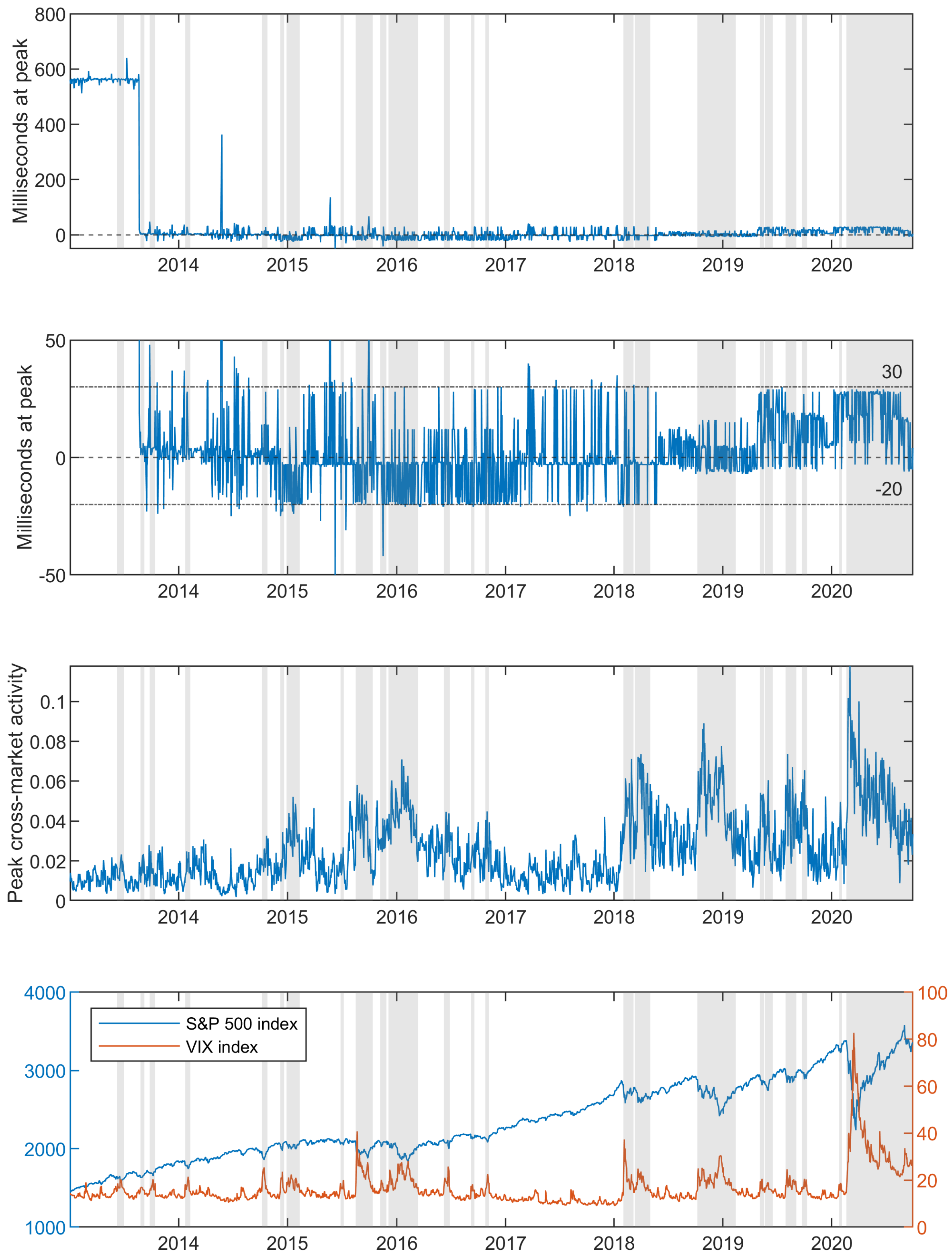

Figure 8: CMAT (milliseconds at peak), PCMA (peak cross-market activity) and VIX and SPX index over the sample period. The shaded areas represent the dates with a level of the VIX index belonging to the 60% upper quantile.

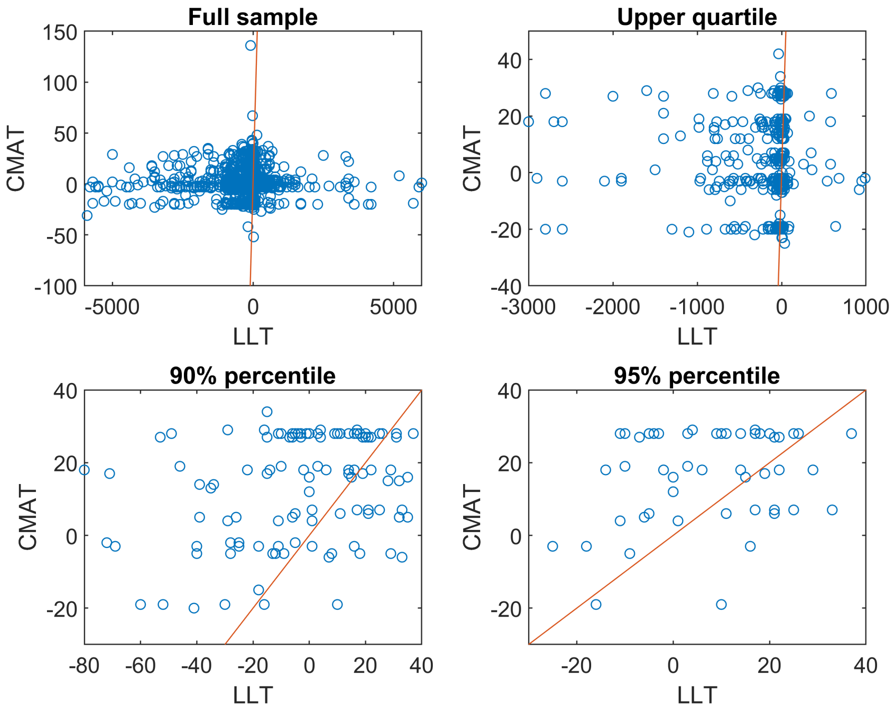

Figure 9: Comparison of LLT and CMAT. The subsamples are based on the level of the VIX index. The red line is a 45-degree line meaning that when points are above (below) the line, LLT indicates a greater (smaller) lead of the VIX futures compared to CMAT. The values of both LLT and CMAT are shown in milliseconds.

lated, and a computation of the correlation between the two time series reveals a value of -0.84. If high-frequency trading is a source of cross-market trading, this means that high levels of high-frequency trading, strong negative correlation and high volatility occurs simultaneously. On the other hand, if VIX futures hedging is a source of cross-market trading, the observations are also consistent with PCMA rising due to increasing VIX futures hedging activity spurred by the demand for VIX futures under high volatility.

As shown in the first panel of Figure 8, CMAT fluctuates at a level of approximately 560 milliseconds during the first part of the sample and hereafter exhibit a clear shift to values around zero. Possibly the break can be attributed to some technological change affecting latencies or is a result of how trades are registered.[2] The second panel zooms in on CMAT to examine its fluctuations after the break. Except for a few dates, the series appear to be bounded within the range of [-20,+30] milliseconds as indicated by the dotted lines. During sub-periods,

[2]Due to this observation, we exclude sample dates before August 26, 2013 in the remainder of the analysis.

CMAT seems to be further bounded within even narrower ranges. For instance, the period from mid-2019 to the end of the sample roughly contains no values outside [-5,+30] milliseconds. There is also a tendency for the observations to cluster around certain values such as -20, +20, +30 and values slight below and above zero as indicated by Figure 8 and 9. Possibly this reflects the true lead-lag time or it may be the result of fluctuations in latencies over time. As indicated by the shaded areas of Figure 8, CMAT does not show any clear pattern under periods of high volatility. This contrasts with the LLT measure which exhibits a clear dependence on the VIX index as illustrated in Figure 6. The difference is also highlighted in Figure 9 where the range of LLT is significantly narrowed when conditioning on high VIX while the same does not occur for CMAT which continues to span the same range of values. The overweight of points above the 45-degree line indicates that on a given sample date CMAT is generally higher than LLT.

4.4 The dynamics of the lead-lag relationship

In order to understand the drivers of the lead-lag relationship, we run a regression where we choose each of the three measures of the lead-lag relation, LLR, LLT, and LLC as the dependent variable. Section 4.4.1 introduces the model and presents the argumentation for inclusion of the independent variables. Next, Section 4.4.2 presents the results of the regressions.

4.4.1 Regression model

Letting LLM_t denote the chosen measure of the lead-lag relation, the regression model is

$$\begin{aligned} LLM_t = &\beta_0 + \beta_1 PCMA_t^+ + \beta_2 PCMA_t^- + \beta_3 NGP_{t-1}^+ + \beta_4 NGP_{t-1}^- + \beta_5 LQR_t \\ &+ \beta_6 VIX_t + \beta_7 SPX_t + \beta_8 D_t^{news} + \beta_9 Expiry_t^{VX} + \beta_{10} Expiry_t^{ES} + \varepsilon_t. \end{aligned} \tag{8}$$

Below we present in detail the variables considered in the regression (8):

- Peak cross-market trading activity, $PCMA_t$: The trading activity in the two markets obviously could have some impact on the lead-lag relationship. In particular, trading activity which emerge from trading strategies involving both markets should matter. If price movements of VIX futures and SPX futures are sufficiently negatively correlated, high-frequency traders may employ trading strategies akin to statistical arbitrage. Trading in the two markets may also be linked if market makers hedge their VIX futures exposure using SPX futures. That is, after having provided liquidity in the VIX futures market, market makers implement their hedge by trading in the SPX futures market as detailed in Section 2. To proxy the part of the trading activity which is related to these type of activities, we use the cross-market activity measure introduced by Dobrev and Schaumburg (2017) and detailed in Section 3.3. For each sample date, our measure of cross-market trading is the peak of the cross-market activity curve computed from (7). When $CMAT_t < 0$ the maximum cross-market activity is associated with trades in VIX futures followed by trades in SPX futures and vice versa when $CMAT_t > 0$. According to Figure 1, cross-market trading associated with a negative $CMAT_t$ would be consistent with VIX futures hedging while cross-market trading with a positive $CMAT_t$ would correspond to cross-market trading motivated by other activities. Hence, we use the two variables $PCMA_t^+ = PCMA_t 1_{\{CMAT_t > 0\}}$ and $PCMA_t^- = PCMA_t 1_{\{CMAT_t \leq 0\}}$ in order to separate the cross-market trading into a component consistent with the VIX futures hedging strategy ($PCMA_t^-$) and a component which would generally not be consistent with hedging ($PCMA_t^+$).

On days where VIX futures hedging is the main source for cross-market activity, we expect that $CMAT_t < 0$ and that higher cross-market activity strengthens the lead of the VIX futures, i.e. $PCMA_t^-$ has a negative impact on LLR. We also expect that the hedging activities would introduce additional negative correlation between returns as buying in the VIX futures market is accompanied by selling in the SPX futures market and vice versa. Hence, we should expect to see stronger negative correlation on days of sizable VIX futures hedging activities, meaning that $PCMA_t^-$ has a negative impact on LLC. When $CMAT_t > 0$ the cross-market trading can to a lesser extent be attributed to VIX futures hedging activities of dealers. Instead a high level of cross-market trading can be an indicator of a significant amount of high-frequency traders present in the two markets.

For high-frequency trading, we have no expectation of which direction the lead-lag relation will be pushed but we expect that it will strengthen the negative correlation, i.e. $PCMA_t^+$ has a negative impact on LLC.

- Net gamma position of dealers, NGP_{t-1}: As illustrated in Figure 1, an aggregate negative gamma position of dealers has the potential to amplify market movements and possible feedback effects from VIX futures hedging as the option dealers delta hedge in the same direction as the market. In contrast, a positive position in gamma works in the opposite direction, since the position implies that in order to maintain the hedge after a market drop, dealers have to buy SPX futures, and reversely, when the stock index increases they should sell SPX futures. Hence, we include a variable NGP_{t-1}^- composed of all the days where the gamma position is negative and a variable NGP_{t-1}^+ for all the days where the position is positive. We lag the variable by one day so it measures the dealers' gamma position by the beginning of day t. We retrieve closing prices on options from OptionMetrics and estimate the daily net gamma position making some assumptions: First, we assume that end user demand in put options is long and on call options short. This claim is empirically justified by Garleanu et al. (2008); Goyenko and Zhang (2019). Using this assumption and similar to the analysis in Baltussen et al. (2021); Barbon and Buraschi (2020), we construct our proxy for the net gamma position at time t as

$$NGP_t = \sum_{i=1}^{N_t^C} \Gamma_t^{BS}(C^i) OI_t(C^i) - \sum_{i=1}^{N_t^P} \Gamma_t^{BS}(P^i) OI_t(P^i),$$

where N_t^C is the number of call options traded at time t with open interest greater than 0 and N_t^P the equivalent counterpart on the put side. $\Gamma_t^{BS}(C^i)$ denotes the Black and Scholes gamma of call option i and $OI_t(C^i)$ is the option's open interest. Similar notation holds for put options.

Figure 10 depicts the estimated gamma position throughout the sample period. The gamma of the options position fluctuates around zero. However, when markets are in turmoil, as measured by a high level of the VIX index, the position is mostly negative

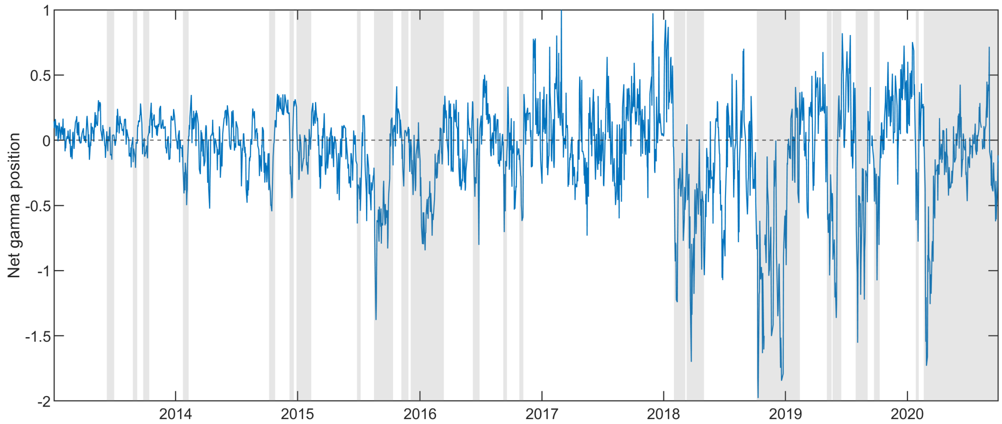

Figure 10: Dealer net gamma position. The shaded areas represent the dates with a level of the VIX index belonging to the 60% upper quantile.

and the gamma can move rapidly from positive to negative when volatility increases.

- The relative liquidity of the two markets, LQR_t: We define the relative liquidity of the two markets as the Amihud liquidity measure (Amihud, 2002) computed for the SPX futures market relative to the same measure computed for the VIX futures market, $LQR_t = AMH_t^{ES}/AMH_t^{VX}$. For each of the two markets, the Amihud measure is obtained as

$$AHM_t = \frac{1}{N}\sum_{i=1}^{N}\frac{|r_i|}{Vol_i^{\$}}$$

with N being the number of 5-minute intervals during the trading day, and r_i and $Vol_i^{\$}$ is the return and dollar volume, respectively, over the ith interval. Traditionally, price discovery tends to occur in the market with the highest level of liquidity (Kyle, 1985). Hence, the relative liquidity in the two markets could be an important driver of the lead-lag relationship. Since the Amihud measure is inversely related to the level of liquidity we expect that an increase (decrease) in LQR_t leads to a stronger (weaker) VIX futures lead.

- The VIX index, VIX_t: There are at least two reasons why the VIX index should be included as regressor. First, there is mixed evidence on whether informed trading occurs at the index level. Pan and Poteshman (2006) do not find evidence that index option trading is informative about future changes in the index while Li et al. (2017) find that informed SPX option trading take place during the financial crisis. Informed trading can arise due to better information processing skills or different views about the same publicly available information which may be more common during volatile periods (Ciner and Karagozoglu, 2008). If informed trading is present at the index level and increases with the amount of volatility then informed traders preferring to trade VIX (SPX) futures means that higher VIX will negatively (positively) impact LLR and LLT. Second, a short SPX futures position provide a hedge against stock market crashes and with a negative correlation between the SPX and the VIX index, so does a long VIX futures position (Moran and Dash, 2007; Szado, 2009; Hilal et al., 2011). Thus, high volatility or uncertainty could create SPX futures selling pressure and VIX futures buying pressure. Whether investors prefer to insure against crashes with one or the other of the futures contracts under high volatility will be revealed by the sign of the coefficient on VIX. With investors' increasing demand for protection at times of high VIX, we then expect that LLC decreases in response to an increase in VIX. Furthermore, other lead-lag studies have shown the importance of volatility. For instance, Chen et al. (2016) show a dependence on volatility as the relative informativeness of SPX futures and SPY is reversed under high volatility. In the present study, inspection of Figure 6 also indicates a clear pattern related to the volatility: Under high volatility, LLR is slightly below one while more erratic in periods of low volatility. LLT is generally close to zero under high volatility but otherwise extremely erratic. Moreover, LLC tend to be stronger and more negative in high volatility regimes. To account for all this, we also include the level of the VIX index in the set of regressors.

- SPX return, SPX_t: Ren et al. (2019) suggest that the lead-lag relation between index options and the index is reversed when the index is not stable or up-trending. Lee et al. (2017) show how the predictability of the VIX futures basis on SPX futures returns changes across the SPX return distribution. These results indicate the SPX return could influence the lead-lag relation. Hence, we include the daily SPX return in the regression.

Table 4: LLR

LLR	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Constant	0.893***	0.921***	0.928***	0.825***	0.851***	0.837***	0.883***	0.907***	0.917***	0.847***	0.861***	0.854***	0.882***	0.899***	0.917***
	(80.284)	(45.123)	(45.291)	(97.290)	(38.513)	(44.206)	(70.885)	(43.768)	(44.155)	(64.175)	(36.927)	(40.482)	(69.003)	(42.735)	(44.081)
$PCMA_t^+$	-1.627***	-1.938***	-1.715***				-1.515***	-1.904***	-1.647***				-1.530***	-2.062***	-1.697***
	(-6.026)	(-5.849)	(-6.242)				(-4.834)	(-5.297)	(-5.079)				(-4.698)	(-5.540)	(-5.063)
$PCMA_t^-$	-3.835***	-4.080***	-3.948***				-3.698***	-3.997***	-3.859***				-3.718***	-4.098***	-3.933***
	(-11.982)	(-11.748)	(-12.278)				(-10.088)	(-10.459)	(-10.367)				(-9.954)	(-10.629)	(-10.353)
NGP_{t-1}^+				0.093**	0.087**	0.093**	0.070*	0.072*	0.067				0.070*	0.075*	0.068
				(2.122)	(1.981)	(2.131)	(1.651)	(1.724)	(1.608)				(1.661)	(1.776)	(1.630)
NGP_{t-1}^-				0.074***	0.065***	0.074***	-0.004	-0.005	-0.008				-0.004	-0.013	-0.008
				(5.262)	(4.646)	(5.285)	(-0.280)	(-0.352)	(-0.601)				(-0.267)	(-0.876)	(-0.549)
LQR_t										-22.626**	-19.095*	-22.701**	0.798	-7.009	2.353
										(-2.334)	(-1.710)	(-2.394)	(0.167)	(-1.255)	(0.480)
VIX_t		0.001			-0.001			0.001*			-0.001			0.002**	
		(1.428)			(-1.361)			(1.802)			(-0.654)			(2.457)	
SPX_t			-0.512**			0.263			-0.485*			0.190			-0.647**
			(-1.986)			(1.250)			(-1.837)			(0.631)			(-2.490)
D_t^{news}		-0.001	-0.001		-0.001	-0.000		-0.001	-0.001		-0.000	-0.000		0.000	-0.000
		(-0.054)	(-0.073)		(-0.048)	(-0.025)		(-0.050)	(-0.078)		(-0.045)	(-0.039)		(0.005)	(-0.028)
$Expiry_t^{VX}$		-0.001*	-0.001*		-0.000	-0.000		-0.001*	-0.001*		-0.000	-0.000		-0.001*	-0.001*
		(-1.826)	(-1.778)		(-0.429)	(-0.407)		(-1.706)	(-1.663)		(-0.511)	(-0.491)		(-1.717)	(-1.693)
$Expiry_t^{ES}$		-0.000	-0.000		-0.000	-0.000		-0.000	-0.000		0.000	0.000		-0.000	-0.000
		(-0.680)	(-0.563)		(-0.444)	(-0.540)		(-0.726)	(-0.583)		(0.050)	(0.081)		(-0.642)	(-0.648)
Adj. R^2 (%)	8.69	8.74	8.77	2.64	2.58	2.48	8.92	8.97	8.96	1.42	1.24	1.22	8.84	8.98	8.92
No. of Obs.	1772	1772	1772	1768	1768	1768	1768	1768	1768	1766	1766	1766	1762	1762	1762

Newey-West t-statistics are in parenthesis. ***, **, * indicates 1%, 5% and 10% significance, respectively. Due to the break in CMAT shown in Figure 8, we exclude the first part of the sample making August 26, 2013 the first sample date. The regression model is shown in (8).

- News announcements, D_t^{news}: Frino et al. (2000) show that the leadership of stock index futures relative to the stock index itself is strengthened around the time of macroeconomic announcements, while Chen and Tsai (2017) find that VIX futures lead the VIX index more on the days of the release. Based on the possible variations in the lead-lag relationship around these announcements, we also include a dummy variable equal to one on the days of U.S. macroeconomic news release. The announcement dates are the dates of scheduled release of information on the Consumer Price Index, Producer Price Index, Employment Situation or Gross Domestic Product.

- Time to expiry, $Expiry_t^{VX}$ and $Expiry_t^{ES}$: We further control for the time to expiration of the VIX futures and SPX futures contracts used to compute the lead-lag measure.

4.4.2 Regression results

In this section we run the regression specified in equation (8) and report the results of the regressions with each of the lead-lag measures as the dependent variable.

The results of the regression with LLR as the dependent variable are shown in Table 4. All the coefficients on the cross-market activity variables are significant and suggest that a higher

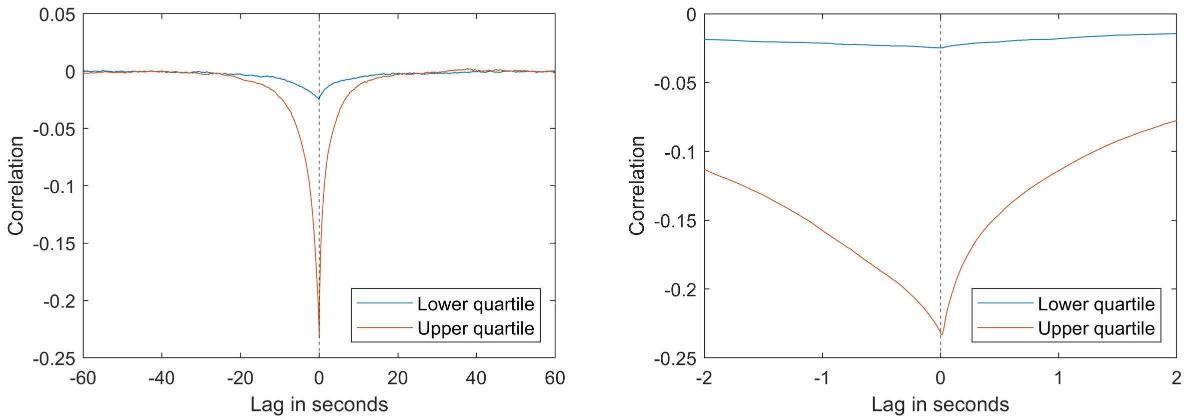

Figure 11: Average cross-correlation function for subsamples of positive CMAT-days. Quartiles of PCMA is found for the subsample where CMAT is positive. The average is taken over the cross-correlation function for the subsample where PCMA is above (below) the respective quartiles and where CMAT is positive. The two figures are identical except for the finer scale on the right figure.

proportion of cross-market trading relative to the total amount of trading strengthens the lead of the VIX futures. Since $PCMA_t^-$ captures cross-market activity with $CMAT_t < 0$, it measures activity only on days where VIX futures trading is followed by SPX futures trading. According to the hedging strategy, any changes in the VIX futures exposure of dealers would be hedged in the SPX futures market making the SPX futures trades lag the VIX futures trades. We take the strong significance of $PCMA_t^-$ as an indication that VIX futures hedging activities are driving the strengthening of the VIX futures lead on these days. The coefficient on the level of cross-market trading conditioned on $CMAT_t > 0$ is less intuitive. A positive $CMAT_t$ means that SPX futures trading is followed by VIX futures trading. This is an indication of SPX futures leading VIX futures. However, when the cross-market activity increases for days with $CMAT_t > 0$ we observe a negative impact on LLR meaning that the VIX futures lead is strenghtened (or the SPX futures lead is weakened). Figure 11 depicts the average cross-correlation function conditioned on $CMAT_t > 0$ and $PCMA_t$ being in the lower and upper quartile, respectively. Notice, when cross-market activity increases, the cross-correlation function shifts downwards and becomes more skewed to the left. In particular, the change in skew is reflected in a decrease in LLR. Hence, even in the case when cross-market activity is high and arise from SPX futures trades leading the VIX futures trading, the VIX futures returns are more informative about

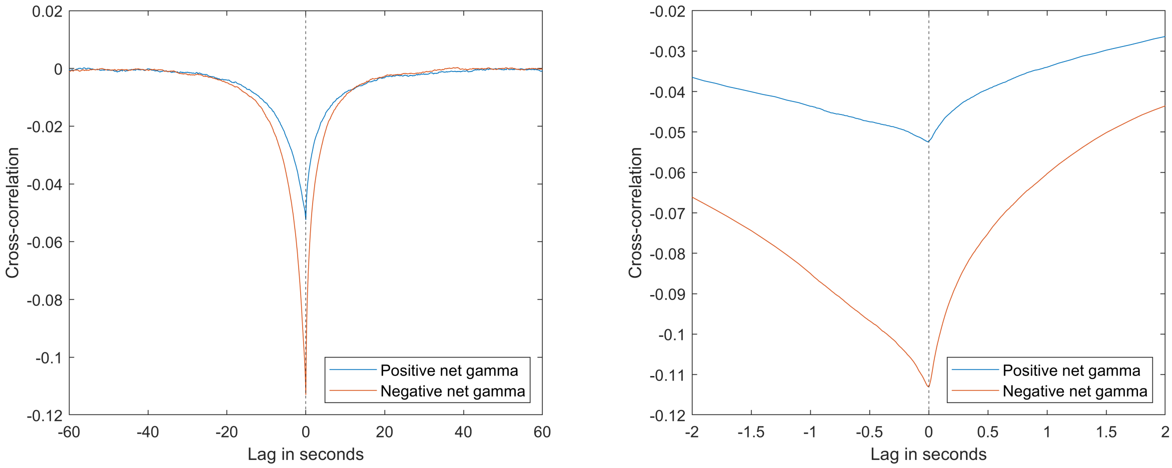

Figure 12: Average cross-correlation function for subsamples of positive and negative positions in gamma. The two figures are identical except for the finer scale on the right figure.

future SPX futures returns in comparison to the reverse direction.

We now focus on the two regressors associated with the net gamma position in column (4)-(6). A negative gamma position is associated with a stronger VIX futures lead while a positive position moves LLR in the opposite direction. Moreover, consistent with the observation that a negative gamma amplifies market movements, we observe from column (4)-(6) in Table 6 how LLC becomes more negative when the net gamma is negative. In contrast, when the gamma position is positive, the LLC measure increases and approaches zero from below. Both effects are clearly illustrated in Figure 12, where the average cross-correlation curves are depicted conditioned on $NGP_{t-1} > 0$ and $NGP_{t-1} \leq 0$, respectively. In particular, the stronger VIX futures lead for negative gamma positions seems to arise from the curve becoming even more skewed to the left when $NGP_{t-1} \leq 0$. This observation, is aligned with the negative gamma position amplifying market movements due to hedge positions being rebalanced as illustrated in Figure 1. However, after including the cross-market activity variables in column (7)-(9) of Table 4, especially the negative net gamma position is insignificant. This indicates that the source of the VIX futures leadership is more likely to be related to VIX futures hedging than the feedback loop associated with option market makers rebalancing of hedges.

The coefficient on the relative liquidity ratio in column (10)-(12) is negative and significantly different from zero. Hence, when the relative liquidity improves in favor of the VIX futures market, the VIX futures lead is strengthened while the opposite holds for the reverse scenario. This is a relation which also exists after controlling for the level of VIX and SPX returns and the pattern would be consistent with investors preferring to trade in the most liquid market. However, column (13)-(15) shows that once additional variables are included in the regression, the relative liquidity is no longer significant in explaining LLR.

The pattern in Figure 6 showed that periods of high VIX are characterized by LLR less than one. However, when other variables are accounted for in the regressions there is no clear relation between LLR and the level of the VIX index. The same holds for the SPX return which has a negative coefficient and is significant only in the regressions including the cross-market variables. Since the level of the VIX index could say something about the amount of private information, as described in Section 4.4.1, the lack of a clear relation between LLR and the level of the VIX index thus does not provide further evidence of informed investors preferring one market over the other. Likewise, for the inclusion of the VIX index motivated by an attempt to measure investors' preferences for obtaining protection at times of market turmoil, the regressions do not reveal that the market generally prefers one of the contracts over the other for different levels of VIX.

The regression results for LLT are shown in Table 5 and are less appealing to interpret. A quick glance at Figure 6 reveals that LLT is fluctuating wildly around zero during low volatility periods, while being close to zero when volatility is high. This can explain why the R-squared is close to zero across all the LLT regressions. Hence, as argued in Section 3.2, LLR is the more robust measure of the lead-lag relation and we therefore mainly rely on results from Table 4 when drawing conclusions regarding the lead-lag relation.

The LLC measure is different from the other two measures as it does not say anything about which asset is the leader. Instead it measures the value of the cross-correlation at the peak located at LLT (see Figure 3). If we focus on the regressions of column (1)-(3) in Table 6 we observe some interesting features: For both of the cross-market trading variables we see that when they increase, the negative correlation between VIX futures and SPX futures returns gets more pronounced. With investors buying (selling) VIX futures and VIX futures dealers

Table 5: LLT

LLT	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Constant	-1.677**	-3.025***	-2.918***	-0.679*	-2.382**	-1.656	-1.416**	-2.714**	-2.619**	-1.311***	-2.692***	-2.163**	-1.542**	-2.586**	-2.646**
	(-2.490)	(-2.816)	(-2.619)	(-1.885)	(-2.225)	(-1.607)	(-2.068)	(-2.384)	(-2.197)	(-2.718)	(-2.729)	(-2.224)	(-2.209)	(-2.344)	(-2.214)
$PCMA_t^{+}$	37.609**	36.199**	37.904**				35.768**	36.938**	35.961**				31.735*	38.875**	32.358**
	(2.516)	(2.172)	(2.544)				(2.250)	(2.085)	(2.262)				(1.923)	(2.143)	(1.981)
$PCMA_t^{-}$	35.329**	35.041**	35.533**				32.782*	34.706*	32.880*				28.953	34.948*	28.633
	(2.078)	(1.967)	(2.090)				(1.814)	(1.822)	(1.816)				(1.571)	(1.815)	(1.573)
NGP_{t-1}^{+}				-2.004	-1.679	-1.978	-1.796	-1.704	-1.749				-1.699	-1.694	-1.669
				(-0.911)	(-0.766)	(-0.897)	(-0.826)	(-0.785)	(-0.803)				(-0.780)	(-0.778)	(-0.768)
NGP_{t-1}^{-}				-0.864*	-0.412	-0.874*	0.202	0.277	0.198				0.412	0.500	0.320
				(-1.757)	(-1.014)	(-1.763)	(0.523)	(0.729)	(0.518)				(1.039)	(1.145)	(0.784)
LQR_t										500.540**	218.982	492.935**	214.915	245.201	182.957
										(2.172)	(0.885)	(2.187)	(1.168)	(1.009)	(1.044)
VIX_t		0.009			0.049**			0.004			0.049			-0.018	
		(0.423)			(2.078)			(0.198)			(1.528)			(-0.620)	
SPX_t			-13.740			-23.291**			-14.645			-26.982**			-20.851**
			(-1.485)			(-2.210)			(-1.561)			(-2.436)			(-2.009)
D_t^{news}		-0.102	-0.103		-0.081	-0.092		-0.095	-0.095		-0.082	-0.084		-0.094	-0.087
		(-0.181)	(-0.183)		(-0.143)	(-0.163)		(-0.168)	(-0.168)		(-0.147)	(-0.150)		(-0.167)	(-0.154)
$Expiry_t^{VX}$		0.030	0.030		0.023	0.022		0.029	0.029		0.024	0.023		0.029	0.028
		(1.378)	(1.390)		(1.041)	(1.009)		(1.335)	(1.322)		(1.089)	(1.047)		(1.329)	(1.292)
$Expiry_t^{ES}$		0.006	0.007		0.005	0.006		0.006	0.007		0.004	0.004		0.006	0.006
		(0.600)	(0.641)		(0.495)	(0.595)		(0.605)	(0.634)		(0.396)	(0.363)		(0.520)	(0.534)
Adj. R^2 (%)	0.25	0.12	0.14	0.09	0.02	-0.02	0.20	0.06	0.08	0.17	0.03	0.05	0.17	0.02	0.05
No. of Obs.	1772	1772	1772	1768	1768	1768	1768	1768	1768	1766	1766	1766	1762	1762	1762

Newey-West t-statistics are in parenthesis. ***, **, * indicates 1%, 5% and 10% significance, respectively. Due to the break in CMAT shown in Figure 8, we exclude the first part of the sample making August 26, 2013 the first sample date. The regression model is shown in (8).

Table 6: LLC

LLC	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Constant	0.018***	0.071***	0.034***	-0.073***	0.046**	-0.083***	0.011***	0.069***	0.027***	-0.026**	0.072***	-0.049***	0.023***	0.085***	0.029***
	(5.536)	(6.705)	(4.144)	(-11.768)	(2.387)	(-6.365)	(3.304)	(6.164)	(3.267)	(-2.518)	(6.094)	(-3.543)	(5.584)	(11.972)	(3.872)
$PCMA_t^{+}$	-4.431***	-2.609***	-4.443***				-4.344***	-2.642***	-4.352***				-3.898***	-2.354***	-3.887***
	(-20.308)	(-11.984)	(-20.199)				(-16.151)	(-11.133)	(-15.762)				(-17.590)	(-15.945)	(-17.155)
$PCMA_t^{-}$	-3.660***	-2.364***	-3.669***				-3.544***	-2.386***	-3.549***				-3.152***	-2.178***	-3.127***
	(-27.888)	(-12.464)	(-26.797)				(-20.883)	(-11.862)	(-19.649)				(-19.315)	(-18.262)	(-18.843)
NGP_{t-1}^{+}				0.063***	0.014	0.065***	0.040***	0.016**	0.040***				0.031***	0.011	0.031***
				(3.367)	(1.274)	(3.544)	(4.062)	(1.968)	(3.953)				(3.394)	(1.600)	(3.403)
NGP_{t-1}^{-}				0.124***	0.043***	0.125***	0.001	-0.005	0.001				-0.021**	0.006	-0.020**
				(8.996)	(4.866)	(9.247)	(0.110)	(-0.887)	(0.107)				(-2.178)	(1.067)	(-1.991)
LQR_t										-52.741***	7.614	-53.570***	-21.603***	9.868***	-21.444***
										(-5.995)	(1.096)	(-6.576)	(-4.810)	(2.856)	(-4.971)
VIX_t		-0.006***			-0.009***			-0.006***			-0.011***			-0.007***	
		(-6.028)			(-7.832)			(-5.976)			(-15.367)			(-13.835)	
SPX_t			0.075			1.089***			0.108			0.949***			0.242
			(0.464)			(3.823)			(0.628)			(3.868)			(1.520)
D_t^{news}		-0.001	0.001		-0.002	0.000		-0.001	0.001		-0.002	-0.001		-0.001	0.000
		(-0.419)	(0.367)		(-0.782)	(0.090)		(-0.504)	(0.320)		(-0.975)	(-0.250)		(-0.670)	(0.117)
$Expiry_t^{VX}$		-0.000	-0.000		0.000***	0.001		-0.000	-0.000		0.000**	0.001**		-0.000	-0.000
		(-0.263)	(-1.432)		(2.901)	(1.460)		(-0.240)	(-1.212)		(2.337)	(1.993)		(-0.225)	(-0.783)
$Expiry_t^{ES}$		-0.000	-0.000*		0.000	-0.000		-0.000	-0.000*		0.000	0.000		-0.000	-0.000
		(-0.789)	(-1.774)		(0.610)	(-0.700)		(-0.792)	(-1.793)		(0.511)	(1.262)		(-1.362)	(-0.636)
Adj. R^2 (%)	73.27	85.69	73.53	24.79	74.16	27.04	73.80	85.76	74.04	38.95	76.03	40.67	77.66	87.89	77.74
No. of Obs.	1772	1772	1772	1768	1768	1768	1768	1768	1768	1766	1766	1766	1762	1762	1762

Newey-West t-statistics are in parenthesis. ***, **, * indicates 1%, 5% and 10% significance, respectively. Due to the break in CMAT shown in Figure 8, we exclude the first part of the sample making August 26, 2013 the first sample date. The regression model is shown in (8).

responding to this by selling (buying) SPX futures, the hedging mechanisms would create a greater negative correlation explaining the negative coefficient on $PCMA_t^-$. This means that VIX futures hedging activities could be creating a tighter link between the two markets. We also observe that LLC has a significant negative relation to the level of the VIX index which confirms the relation between LLC and VIX from Figure 6. Not only for the regressions involving LLC but also the other two dependent variables, the dummy for the macro-economic announcement dates and the time to expiration of the contracts generally are insignificant.

Overall, we note that across all the regressions with LLR and LLC as the dependent variable, cross-market activity is the variable that provides the largest contribution to R-squared. This indicates its importance for the lead-lag relation. If hedging activity is captured by cross-market activity, this supports the role of hedging by VIX futures dealers in driving the lead-lag relation between VIX futures and SPX futures.

4.5 Market impact from uninformed VIX futures trading

While the above section indicates that VIX futures hedging influences the lead-lag relation, it remains unclear whether the hedging channel increases or decreases efficiency of the SPX futures market. In this section, we look into this issue by exploiting the presence of uninformed VIX futures trading by VIX ETPs.

If the leadership of the VIX futures is partly explained by hedging activities of dealers, this may be due to two different mechanisms: If VIX futures trading is informative the lead of the VIX futures indicates that the hedging activities of VIX futures market makers help transmit information from the VIX futures to the SPX futures markets. On the other hand, if VIX futures trading is uninformative the leadership of the VIX futures implies that hedging activities could push SPX futures prices away from their true value. Ultimately, the hedging activities exert a potential systemic risk through its adverse effect on the index futures market at times of large-scaled SPX futures selling by VIX futures market makers.

In the VIX futures market, the rebalancing trades by VIX ETPs is a potentially large source of uninformed trading. Based on the structure of the products, issuers of VIX ETPs have an incentive to rebalance their hedge as close as possible to close of the VIX futures market (Alexander and Korovilas, 2013).[3] This means that even if VIX ETP trading has an informative component throughout the day, the corresponding rebalancing trades by VIX ETP issuers are postponed to the end of the trading day and can therefore be viewed as a channel of uninformed trading. This view is supported by studies showing how the rebalancing by VIX ETPs impacts the VIX futures market. Fernandez-Perez et al. (2019) show that VIX futures market is less informationally efficient on days of large VIX ETP rebalancing flows especially leading up to close where VIX ETPs are expected to implement most of their hedging. Brøgger (2021) documents a transitory price impact in the VIX futures market from rebalancing by VIX ETPs. Todorov (2021) finds a connection between the size of the non-fundamental component in VIX futures prices and VIX ETP rebalancing.

Fortunately, VIX ETP rebalancing demand is relatively easy to compute. We follow the methodology of Todorov (2021) to estimate the aggregate demand for VIX futures among VIX ETPs. The VIX ETPs that use the first two VIX futures contracts as the main hedging vehicle track a 30-day constant maturity VIX futures index. We use K to denote the target maturity of the benchmark index and hence $K = 30$ for the VIX ETPs in our sample (see Table 2). To maintain a constant maturity of K, the weight in the front-month contract, ω_t, must satisfy the condition $\omega_t T_t^{VX1} + (1-\omega_t)\, T_t^{VX2} = K$ with T_t^{VX1} and T_t^{VX2} being the time to maturity of the front-month and second-month contract, respectively. VIX ETPs are also characterized by their leverage target which we denote by L. L is 1 when the VIX ETP simply tracks the given VIX futures index while it is 2 when they track twice the index return and -1 when tracking the inverse index return. The front-month and second-month VIX futures dollar demand of a given

[3]This incentive stems from the possibility of early redemption of ETN shares. Shares are redeemed at the closing indicative value computed from closing prices of VIX futures. Issuers of VIX ETPs wishing to hedge their exposure therefore attemp to trade at exactly this price which can be achieved by trading at close.

VIX ETP can be computed as

$$\begin{aligned} Demand_t^{\$,VX1} =& -\frac{L}{K}A_{t-1}\left(1+Lr_t^{\overline{VX}}\right)+\omega_{t-1}A_{t-1}L\left(L-1\right)r_t^{\overline{VX}} \\ &+\left(\omega_{t-1}-\frac{1}{K}\right)Lu_t+\omega_{t-1}\left(1-\hat{\omega}_{t-1}\right)LA_{t-1}\left(r_t^{VX2}-r_t^{VX1}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} Demand_t^{\$,VX2} =& \frac{L}{K}A_{t-1}\left(1+Lr_t^{\overline{VX}}\right)+\left(1-\omega_{t-1}\right)A_{t-1}L\left(L-1\right)r_t^{\overline{VX}} \\ &+\left(1-\omega_{t-1}+\frac{1}{K}\right)Lu_t-\hat{\omega}_{t-1}\left(1-\omega_{t-1}\right)LA_{t-1}\left(r_t^{VX2}-r_t^{VX1}\right) \end{aligned} \tag{10}$$

where A_t is AUM of the VIX ETP and u_t denotes the dollar-value of capital flows defined as $u_t = A_t - (1+r_t)A_{t-1}$ with r_t denoting the return on the VIX ETP based on the price of its shares. We also define $\hat{\omega}_t = \omega_t P_t^{VX1}/(\omega_t P_t^{VX1} + (1-\omega_t)P_t^{VX2})$ with P_t^{VXm} denoting the VIX futures price for $m = 1,2$. Finally, r_t^{VXm} is the return on the mth VIX futures contract, and the return of the VIX futures benchmark index is given by

$$r_t^{\overline{VX}} = \hat{\omega}_{t-1}r_t^{VX1} + \left(1-\hat{\omega}_{t-1}\right)r_t^{VX2}. \tag{11}$$

The total demand for each of the two contracts on date t is the sum over each of the H VIX ETP's demand, $Demand_t^{\$,VXm,total} = \sum_{j=1}^{H} Demand_t^{\$,VXm,j}$. We combine the VIX futures demand for the two contracts into a single number and measure it relative to either open interest of the front-month and second-month contract, OI_t^{VXm}

$$D_t^{OI} = \frac{Demand_t^{\$,VX1,total}}{cP_t^{VX1}OI_t^{VX1}} + \frac{Demand_t^{\$,VX2,total}}{cP_t^{VX2}OI_t^{VX2}} \tag{12}$$

or relative to the trading volume during the rebalancing window, Vol_t^{VXm},

$$D_t^{Vol} = \frac{Demand_t^{\$,VX1,total}}{cP_t^{VX1}Vol_t^{VX1}} + \frac{Demand_t^{\$,VX2,total}}{cP_t^{VX2}Vol_t^{VX2}}. \tag{13}$$

Here c denotes the contract multiplier of the VIX futures.

With an estimate of the amount of uninformed VIX futures trading taking place up to the close of the market, we can examine the effect of uninformed VIX futures trading on the SPX futures market during this part of the day. Thus, we analyze to which extend the VIX ETP rebalancing demand explains SPX futures returns, $r^{ES}_{t,16:00\rightarrow t,16:15}$, measured over the last 15 minutes of the regular VIX futures trading hours. The analysis is implemented using the following regression model

$$r^{ES}_{t,16:00\rightarrow t,16:15} = \alpha + \beta D^{VIX\,ETP}_t + \gamma Controls_t + u_t \tag{14}$$

where the variable $D^{VIX\,ETP}_t$ is as in (12) or (13). If β is significantly negative, uninformed buying in VIX futures depresses SPX futures returns. Such an effect would be consistent with the hedging mechanisms illustrated in Figure 1 as the investors' VIX futures buying is hedged by selling SPX futures. Since the VIX futures buying is mechanically driven by rebalancing needs rather than new information, this would indicate that (at least during the rebalancing window) VIX futures hedging makes the SPX futures market less informationally efficient. On the other hand, if there is no evidence that the VIX futures hedging trigged by uninformed trading carries over to the SPX futures market β should be insignificant. This is what should hold in the case where VIX futures hedging only has the function of assisting price discovery.

To limit simultaneity issues, the VIX ETP demand in (9) and (10) is computed as seen from 16:15 on day $t-1$ to 16:00 on day t. To account for other factors influencing late-day the SPX futures return, we control for the SPX futures return, $r^{ES}_{t,09:30\rightarrow t,16:00}$, and VIX futures index return, $r^{\overline{VX}}_{t,09:30\rightarrow t,16:00}$, computed up to the time where rebalancing is assumed to begin. These variables allow for the continuation of a trend throughout the remainder of the trading day and are thus related to market efficiency.

Table 7 shows that VIX ETP rebalancing negatively predicts SPX futures returns over the last 15 minutes. Hence, when VIX ETP rebalancing involves buying more VIX futures, then on average the SPX futures price decreases. This is what would be expected under price pressures from hedging and indicates that the VIX futures hedging activity can move the SPX futures market. The uninformed nature of the rebalancing by VIX ETPs supports the view that the

Table 7: Regression results from (14)

$r^{ES}_{t,16:00\to t,16:15}$	(1)	(2)	(3)	(4)
Constant	0.007**	0.008**	0.008**	0.008**
	(2.405)	(2.449)	(2.575)	(2.530)
D^{OI}_t	-0.129*	-0.174**		
	(-1.941)	(-2.371)		
D^{Vol}_t			-0.006*	-0.008**
			(-1.861)	(-2.119)
$r^{ES}_{t,09:30\to t,16:00}$		-0.042*		-0.042*
		(-1.906)		(-1.942)
$r^{\overline{VX}}_{t,09:30\to t,16:00}$		-0.004		-0.005
		(-0.854)		(-1.256)
Adj. R^2 (%)	1.27	4.54	0.37	3.33
No. of Obs.	1772	1772	1772	1772

Newey-West t-statistics are in parenthesis. ***, **, * indicates 1%, 5% and 10% significance, respectively. Due to the break in CMAT shown in Figure 8, we exclude the first part of the sample making August 26, 2013 the first sample date. All returns have been multiplied by 100 and should be interpreted in percentage terms. D^{OI}_t and D^{Vol}_t are defined in (12) and (13), respectively, and are measured from 16:15 on day $t-1$ to 16:00 on day t.

dealers' hedging activity is not simply a channel for price discovery.

If VIX ETP rebalancing is equivalent to uninformed trading we would expect that the corresponding price impact in the SPX futures market is only transitory. Therefore, the following regression model allows for examining how over-night SPX futures returns are related to the rebalancing from the previous day

$$r^{ES}_{t-1,16:15\to t,09:30} = \alpha + \beta D^{VIX\,ETP}_{t-1} + v_t, \tag{15}$$

with the lagged rebalancing variable, $D^{VIX\,ETP}_{t-1}$, being measured from 16:15 on day $t-2$ to 16:15 on day $t-1$. A transitory price impact from rebalancing would be consistent with a positive value of β as this corresponds to a reversal of the initial price impact.

Table 8 shows the results from the regression in (15). The coefficients on both of the lagged VIX ETP rebalancing variables are insignificant. The insignificant coefficients makes it less clear whether the price impact found in Table 7 is the result of transitory price pressures from VIX futures dealers.

Table 8: Regression results from (15)

$r^{ES}_{t-1,16:15\to t,09:30}$	(1)	(2)
Constant	0.015 (0.965)	0.017 (1.063)
D^{OI}_{t-1}	-0.170 (-1.074)	
D^{Vol}_{t-1}		-0.083 (-1.005)
Adj. R^2 (%)	0.03	-0.00
No. of Obs.	1740	1740

Newey-West t-statistics are in parenthesis. ***, **, * indicates 1%, 5% and 10% significance, respectively. Due to the break in CMAT shown in Figure 8, we exclude the first part of the sample making August 26, 2013 the first sample date. All returns have been multiplied by 100 and should be interpreted in percentage terms. D^{OI}_t and D^{Vol}_t are defined in (12) and (13), respectively, and are measured from 16:15 on day $t-2$ to 16:15 on day $t-1$.

5 Conclusion

We study the lead-lag relationship between VIX futures and SPX futures on a high-frequency sample of transactions over the period from January 2013 to September 2020. To analyze the lead-lag relation, we consider estimators of the cross-correlation function. The leadership strength is computed on a daily basis using various measures of lead-lag strength. The analysis reveals large time-variation in the lead-lag relation. Under high volatility, the markets exhibit stronger negative correlation and a short-lived lead-lag with a tendency for VIX futures to lead SPX futures. We find that the cross-market activity explains a major part of the lead-lag relation and that days of high activity are associated with a strengthened VIX futures lead over the SPX futures. Other variables such as the relative liquidity of the two markets also have some explanatory power but appear to be less important for explaining the lead-lag relation once the cross-market activity is taken into account. As VIX futures dealers can hedge their VIX futures position through delta-hedged SPX option positions, their hedging strategy would involve trading SPX futures after providing liquidity in the VIX futures market. We therefore argue in favor of the hypothesis that hedging activities of VIX futures dealers are an important source of cross-market activity and thus hedging activities could be driving part of the VIX futures lead over SPX futures. Generally, the hedging transactions could benefit the SPX futures market by increasing the flow of information from the VIX futures market to the SPX futures market while the mechanical nature of hedging also raise the concern that it could destabilize the stock market. Utilizing the presence of uninformed VIX futures trading by issuers of VIX ETPs, we show that uninformed VIX futures trading predicts SPX futures returns in the direction consistent with the VIX futures hedging strategy. This finding indicates that VIX futures hedging could move the SPX futures market via mechanisms unrelated to informed trading with potentially destabilizing impact in stressed market situations.

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References

Alexander, C. and Korovilas, D. (2013). Volatility exchange-traded notes: Curse or cure? *Journal of Alternative Investments*, 16(2):52–70.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31–56.

Augustin, P., Cheng, I.-H., and Van den Bergen, L. (2021). Volmageddon and the failure of short volatility products. *Financial Analysts Journal*, pages 1–17.

Avellaneda, M. and Lipkin, M. (2003). A market-induced mechanism for stock pinning. *Quantitative Finance*, 3(6):417–425.

Baltussen, G., Da, Z., Lammers, S., and Martens, M. (2021). Hedging demand and market intraday momentum. *Journal of Financial Economics*, 142(1):377–403.

Bank for International Settlements (2018). BIS Quarterly Review, March 2018: International banking and financial market developments. Technical report, available at https://www.bis.org.

Barbon, A. and Buraschi, A. (2020). Gamma fragility. Available at SSRN 3725454.

Bhansali, V. and Harris, L. (2018). Everybody's doing it: Short volatility strategies and shadow financial insurers. *Financial Analysts Journal*, 74(2):12–23.

Bollen, N. P., O'Neill, M. J., and Whaley, R. E. (2017). Tail wags dog: Intraday price discovery in VIX markets. *Journal of Futures Markets*, 37(5):431–451.

Bollerslev, T., Litvinova, J., and Tauchen, G. (2006). Leverage and volatility feedback effects in high-frequency data. *Journal of Financial Econometrics*, 4(3):353–384.

Bollerslev, T., Sizova, N., and Tauchen, G. (2012). Volatility in equilibrium: Asymmetries and dynamic dependencies. *Review of Finance*, 16(1):31–80.

Brøgger, S. B. (2021). The market impact of predictable flows: Evidence from leveraged VIX products. *Journal of Banking & Finance*, page 106280.

Buccheri, G., Corsi, F., and Peluso, S. (2021). High-frequency lead-lag effects and cross-asset linkages: A multi-asset lagged adjustment model. *Journal of Business & Economic Statistics*, 39(3):605–621.

Carr, P. and Wu, L. (2006). A tale of two indices. *Journal of Derivatives*, 13(3):13–29.

Chan, K., Chung, Y. P., and Fong, W.-M. (2002). The informational role of stock and option volume. *Review of Financial Studies*, 15(4):1049–1075.

Chang, J. (2017). Monthly Market Review, November 2017: A "vixing" puzzle. pages 1–3. Rockefeller Asset Management.

Chen, W.-P., Chung, H., and Lien, D. (2016). Price discovery in the S&P 500 index derivatives markets. *International Review of Economics & Finance*, 45:438–452.

Chen, Y.-L. and Tsai, W.-C. (2017). Determinants of price discovery in the VIX futures market. *Journal of Empirical Finance*, 43:59–73.

Chu, Q. C., Hsieh, W.-l. G., and Tse, Y. (1999). Price discovery on the S&P 500 index markets:

An analysis of spot index, index futures, and SPDRs. *International Review of Financial Analysis*, 8(1):21–34.

Ciner, C. and Karagozoglu, A. K. (2008). Information asymmetry, speculation and foreign trading activity: Emerging market evidence. *International Review of Financial Analysis*, 17(4):664–680.

Cont, R. and Kokholm, T. (2013). A consistent pricing model for index options and volatility derivatives. *Mathematical Finance*, 23(2):248–274.

Dao, T. M., McGroarty, F., and Urquhart, A. (2018). Ultra-high-frequency lead–lag relationship and information arrival. *Quantitative Finance*, 18(5):725–735.

Dobrev, D. and Schaumburg, E. (2017). High-frequency cross-market trading: Model free measurement and applications. Working paper.

Easley, D., O'hara, M., and Srinivas, P. S. (1998). Option volume and stock prices: Evidence on where informed traders trade. *Journal of Finance*, 53(2):431–465.

Fernandez-Perez, A., Frijns, B., Tourani-Rad, A., and Webb, R. I. (2019). Does increased hedging lead to decreased price efficiency? The case of VIX ETPs and VIX futures. *Financial Review*, 54(3):477–500.

Frijns, B., Tourani-Rad, A., and Webb, R. I. (2016). On the intraday relation between the VIX and its futures. *Journal of Futures Markets*, 36(9):870–886.

Frino, A., Walter, T., and West, A. (2000). The lead–lag relationship between equities and stock index futures markets around information releases. *Journal of Futures Markets*, 20(5):467–487.

Garleanu, N., Pedersen, L. H., and Poteshman, A. M. (2008). Demand-based option pricing. *Review of Financial Studies*, 22(10):4259–4299.

Golez, B. and Jackwerth, J. C. (2012). Pinning in the S&P 500 futures. *Journal of Financial Economics*, 106(3):566–585.

Gonzalo, J. and Granger, C. (1995). Estimation of common long-memory components in cointegrated systems. *Journal of Business & Economic Statistics*, 13(1):27–35.

Goyenko, R. and Zhang, C. (2019). Option returns: Closing prices are not what you pay. Unpublished working paper McGill University.

Hasbrouck, J. (1995). One security, many markets: Determining the contributions to price discovery. *Journal of Finance*, 50(4):1175–1199.

Hasbrouck, J. (2003). Intraday price formation in US equity index markets. *Journal of Finance*, 58(6):2375–2400.

Hayashi, T. and Yoshida, N. (2005). On covariance estimation of non-synchronously observed diffusion processes. *Bernoulli*, 11(2):359–379.

Hilal, S., Poon, S.-H., and Tawn, J. (2011). Hedging the black swan: Conditional heteroskedasticity and tail dependence in S&P500 and VIX. *Journal of Banking & Finance*, 35(9):2374–2387.

Hoffmann, M., Rosenbaum, M., and Yoshida, N. (2013). Estimation of the lead-lag parameter from non-synchronous data. *Bernoulli*, 19(2):426–461.

Hu, J. (2014). Does option trading convey stock price information? *Journal of Financial Economics*, 111(3):625–645.

Huth, N. and Abergel, F. (2014). High frequency lead/lag relationships-empirical facts. *Journal of Empirical Finance*, 26:41–58.

Kao, D.-X., Tsai, W.-C., Wang, Y.-H., and Yen, K.-C. (2018). An analysis on the intraday trading activity of VIX derivatives. *Journal of Futures Markets*, 38(2):158–174.

Kaul, G., Nimalendran, M., and Zhang, D. (2004). Informed trading and option spreads. Available at SSRN 547462.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1335.

Lee, H.-C., Liao, T.-H., and Tung, P.-Y. (2017). Investors' heterogeneity in beliefs, the VIX futures basis, and S&P 500 index futures returns. *Journal of Futures Markets*, 37(9):939–960.

Li, W.-X., French, J. J., and Chen, C. C.-S. (2017). Informed trading in S&P index options? Evidence from the 2008 financial crisis. *Journal of Empirical Finance*, 42:40–65.

Moran, M. T. and Dash, S. (2007). VIX futures and options: Pricing and using volatility products to manage downside risk and improve efficiency in equity portfolios. *Journal of Trading*, 2(3):96–105.

Ni, S. X., Pearson, N. D., and Poteshman, A. M. (2005). Stock price clustering on option expiration dates. *Journal of Financial Economics*, 78(1):49–87.

Pan, J. and Poteshman, A. M. (2006). The information in option volume for future stock prices. *Review of Financial Studies*, 19(3):871–908.

Ren, F., Ji, S.-D., Cai, M.-L., Li, S.-P., and Jiang, X.-F. (2019). Dynamic lead–lag relationship between stock indices and their derivatives: A comparative study between Chinese mainland, Hong Kong and US stock markets. *Physica A: Statistical mechanics and Its applications*, 513:709–723.

Schlag, C. and Stoll, H. (2005). Price impacts of options volume. *Journal of Financial Markets*, 8(1):69–87.

Shu, J. and Zhang, J. E. (2012). Causality in the VIX futures market. *Journal of Futures Markets*, 32(1):24–46.

Szado, E. (2009). VIX futures and options: A case study of portfolio diversification during the 2008 financial crisis. *Journal of Alternative Investments*, 12(2):68–85.

Todorov, K. (2021). Passive funds affect prices: Evidence from the most ETF-dominated asset classes. Available at SSRN 3504016.

Todorov, V. and Tauchen, G. (2011). Volatility jumps. *Journal of Business & Economic Statistics*, 29(3):356–371.

Whaley, R. E. (2009). Understanding the VIX. *Journal of Portfolio Management*, 35(3):98–105.

Zhang, F. (2010). High-frequency trading, stock volatility, and price discovery. Available at SSRN 1691679.